

Math 126

Section 5.1 Exponential Functions

Definition:

If a is a real number with $a > 0$ and $a \neq 1$ then the function $f(x) = a^x$ is an exponential function with base a .

Example 1

Let $f(x) = 3^x$. Evaluate each function.

a) Evaluate $f(4)$

$$f(4) = 3^4 = 81$$

b) Evaluate $f(-3)$

$$f(-3) = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

c) Evaluate $f\left(\frac{1}{2}\right)$

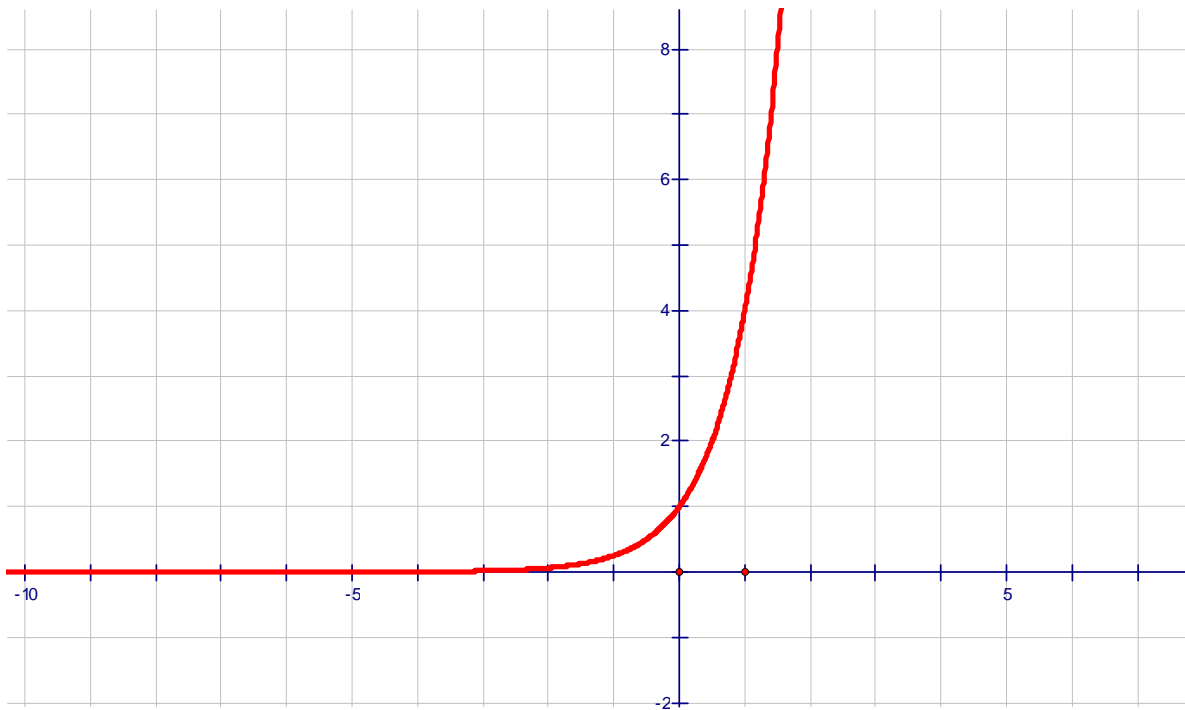
$$f\left(\frac{1}{2}\right) = 3^{\frac{1}{2}} = \sqrt{3}$$

d) Evaluate $f\left(\frac{3}{2}\right)$

$$f\left(\frac{3}{2}\right) = 3^{\frac{3}{2}} = \sqrt{3^3} = \sqrt{27} = 3\sqrt{3}$$

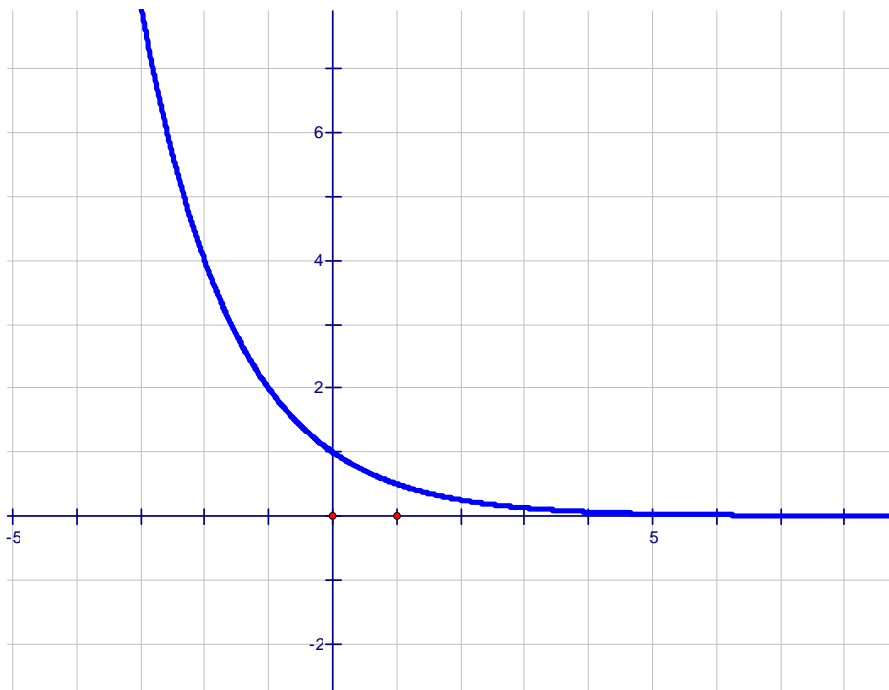
Example 2Graph $f(x) = 4^x$

x	$f(x) = 4^x$
-2	$f(-2) = 4^{-2} = \frac{1}{4^2} = \frac{1}{16} = .06$
-1	$f(-1) = 4^{-1} = \frac{1}{4^1} = \frac{1}{4} = .25$
0	$f(0) = 4^0 = 1$
1	$f(1) = 4^1 = 4$
2	$f(2) = 4^2 = 16$



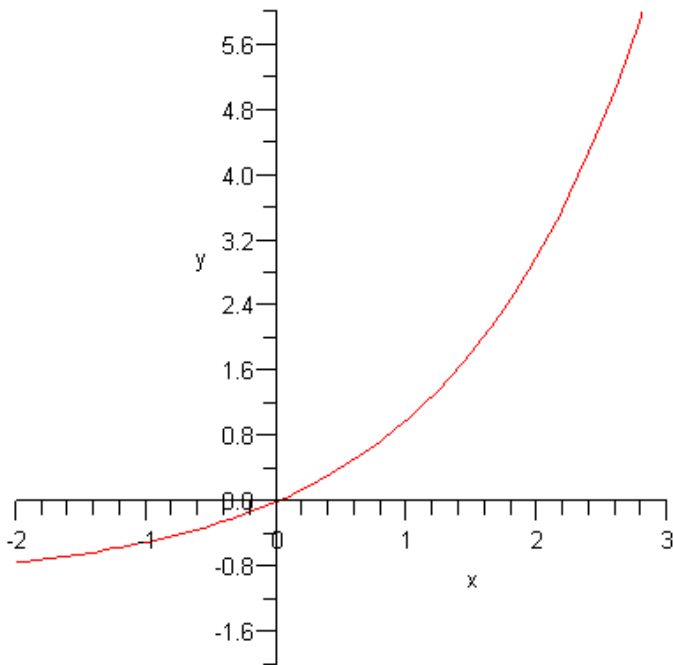
Example 3Graph $f(x) = 2^{-x}$

x	$f(x) = 2^{-x}$
-2	$f(-2) = 2^{-(-2)} = 2^2 = 4$
-1	$f(-1) = 2^{-(-1)} = 2$
0	$f(0) = 2^{-0} = 2^0 = 1$
1	$f(1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2} = .5$
2	$f(2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$



Example 4**Graph $y = 2^x - 1$**

x	$y = 2^x - 1$
-2	$y = 2^{-2} - 1 = .25 - 1 = -.75$
-1	$y = 2^{-1} - 1 = .5 - 1 = -.5$
0	$y = 2^0 - 1 = 1 - 1 = 0$
1	$y = 2^1 - 1 = 2 - 1 = 1$
2	$y = 2^2 - 1 = 4 - 1 = 3$



The exponential function

Limit definition of e^x

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

x	.1	.01	.001
$f(x) = (1+x)^{\frac{1}{x}}$	$f(.1) = (1+.1)^{\frac{1}{.1}}$ $f(.1) = (1.1)^{10}$ $f(.1) = 2.59$	$f(.01) = (1+.01)^{\frac{1}{.01}}$ $f(.01) = (1.01)^{100}$ $f(.01) = 2.70$	$f(.001) = (1+.001)^{\frac{1}{.001}}$ $f(.001) = (1.001)^{1000}$ $f(.001) = 2.717$

x	-.1	-.01	-.001
$f(x) = (1+x)^{\frac{1}{x}}$	$f(-.1) = (1-.1)^{-\frac{1}{.1}}$ $f(-.1) = (.9)^{-10}$ $f(-.1) = 2.86$	$f(-.01) = (1-.01)^{-\frac{1}{.01}}$ $f(-.01) = (.99)^{-100}$ $f(-.01) = 2.73$	$f(-.001) = (1-.001)^{-\frac{1}{.001}}$ $f(-.001) = (.999)^{-1000}$ $f(-.001) = 2.719$

Actual approximation for $e \approx 2.718$

Example 4

Evaluate each exponential function

a) Evaluate e^2

$$e^2 = 7.39$$

b) Evaluate e^{-3}

$$e^{-3} = .05$$

Example 5

Solve $2^x = 32$

$$2^x = 32$$

$$2^x = 2^5$$

$$x = 5$$

Example 6

Solve $5^x = 625$

$$5^x = 625$$

$$5^x = 5^4$$

$$x = 4$$

Example 7

Solve $5^x = \frac{1}{125}$

$$5^x = \frac{1}{125}$$

$$5^x = \frac{1}{5^3}$$

$$5^x = 5^{-3}$$

$$x = -3$$

Example 8

$$\text{Solve } 4^x = \frac{1}{256}$$

$$4^x = \frac{1}{256}$$

$$4^x = \frac{1}{4^4}$$

$$4^x = 4^{-4}$$

$$x = -4$$

Example 9

$$\text{Solve } e^{2x} = e^9$$

$$e^{2x} = e^9$$

$$2x = 9$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$x = \frac{9}{2}$$

Example 10

$$\text{Solve } e^{\sqrt{x}} = e^2$$

$$e^{\sqrt{x}} = e^2$$

$$\sqrt{x} = 2$$

$$(\sqrt{x})^2 = 2^2$$

$$x = 4$$

Example 11

The population in millions of Chicago can be modeled by the function $P = 2.77e^{.0015x}$ where x is time in years and P is the population in millions. Use the model to predict the population of Chicago in 10 years.

$$P = 2.77e^{0.0015x} = 2.77e^{.0015(10)} = 2.77e^{.015} = 2.81 \text{ million}$$

Example 12

If \$10,000 is invested in an account for x years where interest is compound quarterly at a rate of 8%, the future value is given by the following function. Use the model to find the future value of the account for 10 years.

$$A = 10000(1.02)^{4x}$$

$$A = 10000(1.02)^{4x} = 10000(1.02)^{4(10)} = 10000(1.02)^{40} = \$22080.40$$

Example 13

The model to compute the future value of a saving bond worth \$20,000 that compound interest continuously at an interest rate of 4% is given by $A = 20000e^{.04t}$. Use the model to calculate the future value of the account after 5 years.

$$A = \$20000e^{.04t} = \$20000e^{.04(5)} = \$20,000e^{.2} = \$24,428.06$$

Example 14

Suppose \$4000 is placed in a savings bond for 4 years that compounds interest semiannually at a rate of 2% per year. Find the future value of the saving bond in 4 years.

$$P = \$4000$$

$$T = 4$$

$$R = .02$$

$$n = 2$$

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = \$4000 \left(1 + \frac{.02}{2} \right)^{2 \cdot 4}$$

$$A = \$4000(1 + .01)^8$$

$$A = \$4000(1.01)^8$$

$$A = \$4000(1.082856)$$

$$A = \$4331.42$$

Loan (Financing a Home or Car)

Buying a car or house is a financial decision that most people will encounter in their life time. In this section, we will look at how banks and financial companies compute the loan payment that a customer's will pay when they finance a home or car. We will also answer common questions about getting a loan from a bank such as how much money will you pay in interest when you finance a home with a loan, or how will interest rate and term of loan can effect your loan payment.

The Loan Formula

P = principle

r = Rate

n = Number of Times Interest is Computed

T = Time

PMT = Payment

$$PMT = \frac{P \cdot \left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Example 15

What would your monthly house payment be on a \$140,000 house if you finance the house for 30 years at an interest rate of 6%?

$P = \$140,000$

$R = 6\%$

$n = 12$

$T = 30$

$$PMT = \frac{(\$140,000) \left(\frac{.06}{12}\right)}{1 - \left(1 + \frac{.06}{12}\right)^{-12(30)}} = \frac{(\$140,000)(.005)}{1 - (1 + .005)^{-360}} = \frac{\$700}{1 - (1.005)^{-360}} = \frac{\$700}{1 - .166041} = \frac{\$700}{.833958} = \$839.37$$