Math 126

Section 12.1

The Indefinite Integral

The integral of function does the opposite of the derivative.

Review of the derivative:

The derivative of $f(x) = x^3 - 2x^2$ is $f'(x) = 3x^2 - 4x$

The integral of $3x^2 - 4x$ would be $x^3 - 2x^2$

The power of x rule for integration

$$\int cx^n dx = \frac{cx^{n+1}}{n+1} + C$$

Example 1

Evaluate $\int x^2 dx$

$$\int x^2 dx = \frac{1 \cdot x^{2+1}}{2+1} + C = \frac{1}{3}x^3 + C$$

Example 2

Evaluate $\int (3x^2 + 4x) dx$

$$\int (3x^2 + 4x) dx = \frac{3 \cdot x^{2+1}}{2+1} + \frac{4x^{1+1}}{1+1} + C = \frac{3}{3}x^3 + \frac{4}{2}x^{1+1} + C = x^3 + 2x^2 + C$$

Example 3

Evaluate $\int \left(4x^3 + 6x^2\right)dx$

$$\int \left(4x^3 + 6x^2\right)dx = \frac{4 \cdot x^{3+1}}{3+1} + \frac{6x^{2+1}}{2+1} + C = \frac{4}{4}x^4 + \frac{6}{3}x^{2+1} + C = x^4 + 2x^3 + C$$

Example 4

Evaluate $\int \left(x^3 + x^2 + 5\right)dx$

$$\int \left(x^3 + x^2 + 5\right)dx = \frac{1 \cdot x^{2+1}}{3+1} + \frac{1 \cdot x^{2+1}}{2+1} + \frac{5x^{0+1}}{1+0} + C = \frac{1}{4}x^4 + \frac{1}{3}x^3 + 5x + C$$

Example 5

Evaluate $\int 2x^5 dx$

$$\int \left(2x^5\right)dx = \frac{2 \cdot x^{5+1}}{5+1} + C = \frac{2}{6}x^6 + C = \frac{1}{3}x^6 + C$$

Definition

The integral of the exponential function

$$\int e^x = e^x + C$$

Example 6

Evaluate $\int 3e^{x}dx$

$$\int 3e^{x}dx = 3e^{x} + C$$

Example 7

Evaluate $\int \left(10e^{x} + 3x^{2}\right)dx$

$$\int \left(10e^{x} + 3x^{2}\right)dx = 10e^{x} + \frac{3}{2+1}e^{2+1} + C = 10e^{x} + \frac{3}{3}x^{3} + C = 10e^{x} + x^{3} + C$$

Definition

$$\int \frac{1}{x}dx = \ln x + C$$

Example 8

Evaluate $\int \frac{2}{x}dx$

$$\int \frac{2}{x}dx = 2\int \frac{1}{x}dx = 2\ln x + C$$

Example 9

$$\int\left(\frac{1}{x}+x\right)dx$$

$$\int\left(\frac{1}{x}+x\right)dx=\ln(x)+\frac{1x^{1+1}}{1+1}+C=\ln(x)+\frac{1}{2}x^{2}+C$$