

Math 121

Section 0.4

Factoring Polynomials

Types of Factoring

- 1) Factor by taking out a Greatest Common Factor (GFC)
- 2) Factor a trinomial as two binomials.
- 3) Factor a binomial as a difference of two squares
- 4) Factor a binomial as a difference of cubes or a sum of cubes
- 5) Factor by grouping

Factoring out a Greatest Common Factor

Taking out a greatest common factor is essentially the same as working the distributive property backwards.

Review of distributive property

$$3(x + 5) = 3x + 3 \cdot 5 = 3x + 15$$

Example 1

Factor out a greatest common factor $4x + 16$

Solution: $4x + 16 = 4(x + 4)$

Example 2

Factor out a greatest common factor $5x - 35$

Solution: $5x - 35 = 5(x - 7)$

Example 3

Factor out a greatest common factor $2x^3 - 8x^2 + 16x$

Solution: $2x^3 - 8x^2 + 16x = 2x(x^2 - 4x + 8)$

Example 4

Factor out a greatest common factor $12x^3y + 24x^2y^2 + 48xy$

Solution: $12x^3y + 24x^2y^2 + 48xy = 12xy(x^2 + 2xy + 4)$

Factoring a trinomial as two binomials

Factoring a trinomial is the same as working the **FOIL** process

So, here is a short review of **FOIL**

$$(x + 3)(x + 4)$$
$$x \cdot x + 4x + 3x + 3 \cdot 4$$
$$x^2 + 3x + 4x = 12$$
$$x^2 + 7x + 12$$

Example of factoring a trinomial as two binomials

Factor $x^2 + 10x + 25$

In this example you want to find two numbers that multiply to get 25 and add to get 10.

By using x as the first entry in each binomial you get:

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Here are some similar examples

Example 5

Factor $x^2 - 10x + 24$

Answer: $(x - 6)(x - 4)$ Hint: Find two numbers that multiply to get 24 and add to get -10

Example 6

Factor $x^2 - 2x - 35$

Answer: $(x - 7)(x + 5)$ Hint: Find two integers that multiply to get -35 and add to get -2 which are the factors -7 and 5

Example 7

Factor $x^2 + 4x - 12$

Answer: $(x - 2)(x + 6)$

Example 8

Factor $3x^2 + 7x + 2$

Answer: $(3x + 1)(x + 2)$

Factor a binomial as a difference of squares

Factor $x^2 - 4$

Answer: $(x - 2)(x + 2)$ Hint: Basically use the **FOIL** process backwards again and find two integers that multiply to get -4 and add to get zero. This process will cancel out the x-terms.

Check:

$$(x - 2)(x + 2)$$

$$x^2 + 2x - 2x - 4$$

$$x^2 - 4$$

Other similar examples

Example 9

Factor $m^2 - 36$

Answer: $(x - 6)(x + 6)$

Example 10

Factor $x^2 - 144$

Answer: $(x + 12)(x - 12)$

Example 11

Factor $9x^2 - 25y^2$

Answer:

$$9x^2 - 25y^2$$

$$(3x)^2 - (5y)^2$$

$$(3x + 5y)(3x - 5y)$$

General Form of a Difference of Squares: $A^2 - B^2 = (A + B)(A - B)$

Difference of Cubes and sum of Cubes**Main formulas****Difference of Two Cubes**

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

Sum of Two Cubes

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

Example 12Factor $x^3 - 27$

Answer:

$$\begin{aligned} & x^3 - 3^3 \\ & (x-3)(x^2 + 3x + 3^2) \\ & (x-3)(x^2 + 3x + 9) \end{aligned}$$

Example 13Factor $x^3 + 64$

Answer:

$$\begin{aligned} & x^3 + 64 \\ & x^3 + 4^3 \\ & (x+4)(x^2 - 4x + 4^2) \\ & (x+4)(x^2 - 4x + 16) \end{aligned}$$

Example 14Factor $x^3 + 8$

Answer;

$$\begin{aligned} & x^3 + 8 \\ & x^3 + 2^3 \\ & (x+2)(x^2 - 2x + 2^2) \\ & (x+2)(x^2 - 2x + 4) \end{aligned}$$

Example 15Factor $8x^3 - 27y^3$

Answer;

$$\begin{aligned} & 8x^3 - 27y^3 \\ & (2x)^3 - (3y)^3 \\ & (2x - 3y)((2x)^2 - (2x)(3y) + (3y)^2) \\ & (2x - 3y)(4x^2 - 6xy + 9y^2) \end{aligned}$$

Factoring by Grouping

Example

Factor $x^3 - 2x^2 - 4x + 8$

Solution:

$$\begin{aligned} & x^3 - 2x^2 - 4x + 8 \\ & x^2(x - 2) - 4(x - 2) \\ & (x^2 - 4)(x - 2) \\ & (x - 2)(x + 2)(x - 2) \\ & (x - 2)^2(x + 2) \end{aligned}$$

Factor out a x^2 from the first two terms and -4 from the last two terms
Factor out $(x - 2)$ from both terms

Similar types of examples

Example 16Factor $m^3 - 3m^2 - 12m + 36$

Solution:

$$\begin{aligned} & m^3 - 3m^2 - 12m + 36 \\ & m^2(m - 3) - 12(m - 3) \\ & (m^2 - 12)(m - 3) \end{aligned}$$

Definition: The **zero of a polynomial** $P(x)$ is a value x such that $P(x) = 0$ when x is substituted into the polynomial $P(x)$.

Example 17

Factor $c^3 - 4c^2 - 16c + 64$

Solution:

$$\begin{aligned}c^3 - 4c^2 - 16c + 64 \\c^2(c - 4) - 16(c - 4) \\(c^2 - 16)(c - 4) \\(c - 4)(c + 4)(c - 4) \\or (c - 4)^2(c + 4)\end{aligned}$$

Don't forget that $(c^2 - 16)$ is a difference of squares

Applications of factoring: Finding the zeros of a polynomial

Example 18

Find all zeros of the polynomial: $x^2 - 7x + 12$

Solve $x^2 - 7x + 12 = 0$

$$\begin{aligned}x^2 - 7x + 12 &= 0 \\(x - 3)(x - 4) &= 0 \\x - 3 &= 0 \quad or \quad x - 4 = 0 \\x - 3 + 3 &= 0 + 3 \quad x - 4 + 4 = 0 + 4 \\x &= 3 \quad \quad \quad x = 4\end{aligned}$$

Example 19

Find all zeros of the polynomial: $x^2 - x - 20$

Solution: Solve $x^2 - x - 20 = 0$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x - 5 + 5 = 0 + 5 \quad x + 4 - 4 = 0 - 4$$

$$x = 5 \qquad x = -4$$

Example 20

Find all zero of the polynomial: $m^2 - 81$

Solution: Solve $m^2 - 81 = 0$

$$m^2 - 81 = 0$$

$$(m + 9)(m - 9) = 0$$

$$m + 9 = 0 \quad \text{or} \quad m - 9 = 0$$

$$m + 9 - 9 = 0 - 9 \quad m - 9 + 9 = 0 + 9$$

$$m = -9 \qquad m = 9$$

Example 21

Find all zeros of the polynomial: $x^2 - 5x = 0$

Solution: Solve $x^2 - 5x = 0$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$x = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = 5$$

Example 22

Find all zeros of the polynomial: $x^2 - 3x^2 - 4x + 12$

Solution: Solve $x^2 - 3x^2 - 4x + 12 = 0$

$$x^3 - 3x^2 - 4x + 12 = 0$$

$$x^2(x-3) - 4(x-3) = 0$$

$$(x^2 - 4)(x-3) = 0$$

$$(x-2)(x+2)(x-3) = 0$$

$$x-2=0 \text{ or } x+2=0 \text{ or } x-3=0$$

Thus the solutions are : $x = 2, x = -2, x = 3$

Example 23

Find all zeros of the polynomial: $m^4 - 16$

Solution: Solve $m^4 - 16 = 0$

$$m^4 - 16 = 0$$

$$(m^2 + 4)(m^2 - 4) = 0$$

$$(m^2 + 4)(m-2)(m+2) = 0$$

$$m^2 + 4 = 0 \Rightarrow \text{No Solution}$$

$$m-2 = 0 \Rightarrow m = 2$$

$$m+2 = 0 \Rightarrow m = -2$$

Thus the solutions are : $m = -2, m = 2$

Example 24

Solve $3x^2 - 5x - 7 = 0$

$$a = 3, b = -5, c = -7$$

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-7)}}{2(3)} = \frac{5 \pm \sqrt{25 + 84}}{6} = \frac{5 \pm \sqrt{109}}{6}$$