Math 126
Practice Final

1) Find the slope and y-intercept of line represented by the following equation.
$4x-3y=3$

a) $m=3, b=-4$

b) $m=-\frac{4}{3}, b=1$

c) $m=\frac{4}{3}, b=-1$

d) $m=\frac{3}{4}, b=-4$

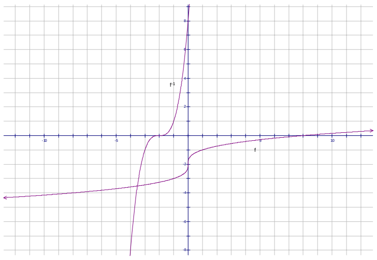
$$4x-3y=3$$
$$4x-4x-3y=-4x+3$$
$$-3y=-4x+3$$
$$\frac{-3y}{-3}=\frac{-4x}{-3}+\frac{3}{-3}$$
$$y=\frac{4}{3}x-1$$
$$\Rightarrow m=\frac{4}{3}, b=1$$

Solution: C

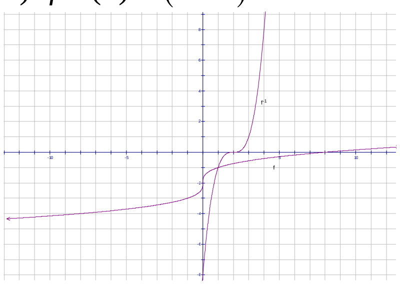
2) Find the inverse of the following function, and then graph $f(x)$ and $f^{-1}(x)$

$f(x) = \sqrt[3]{x} - 2$

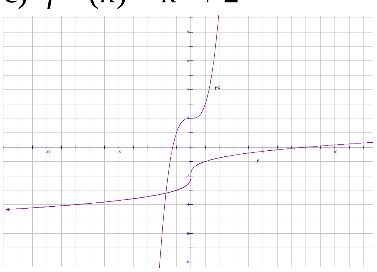
a) $f^{-1}(x) = (x+2)^3$



b) $f^{-1}(x) = (x+2)^3$



c) $f^{-1}(x) = x^3 + 2$



d) None of the above

$f(x) = \sqrt[3]{x} - 2$

$y = \sqrt[3]{x} - 2$

$x = \sqrt[3]{y} - 2$

$x + 2 = \sqrt[3]{y}$

$(x+2)^3 = \left(\sqrt[3]{y}\right)^3$

$(x+2)^3 = f^{-1}(x)$

Solution: A

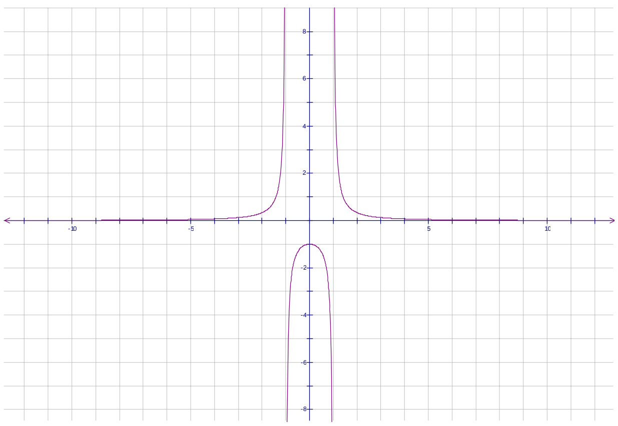
3) Evaluate $\lim_{x\to 2}\frac{x^2-6x+8}{x-2}$

a) 2
b) -2
c) Undefined
d) -1

$$\lim_{x\to 2}\frac{x^2-6x+8}{x-2}=\lim_{x\to 2}\frac{(x-4)(x-2)}{x-2}=\lim_{x\to 2}x-4=2-4=-2$$

Solution: B

4) Which statement about continuity is correct



a) The function is continuous everywhere.
b) The function is discontinuous everywhere
c) The function is continuous everywhere except the values x = -2 and x = 2
d) The function is continuous everywhere except the values x = -1 and x = 1

Solution: D

5) Given $f(x)=2x^3+5x^2+3$, find the derivative.

a) $f'(x)=4x^3+6x$
b) $f'(x)=6x^2+10x$
c) $f'(x)=12x^2+10x$
d) None of the above

$$f(x)=2x^3+5x^2+3$$
$$f^{-1}(x)=6x^2+10x$$

Solution: B

6) Find the derivative of $f(x) = (x^2 - 8x)^4$

a) $f'(x) = 4(2x-8)^3$
b) $f'(x) = (x^2 - 8x)^3$
c) $f'(x) = 4(x^2 - 8x)^3(2x-8)$
d) None of the above

$$f(x) = (x^2 - 8x)^4$$
$$u = x^2 - 8x$$
$$du = 2x - 8$$
$$f(x) = u^4$$
$$f(x) = 4u^3 du = 4(x^2 - 8x)^3(2x-8)$$

Solution: C

7) The Homer Simpson Tire Corporation determined that the daily cost of producing lawn tractor tires can be modeled by the cost function $C(x) = 130 + 60x - 0.02x^2$, where $0 \le x \le 360$. Determine the marginal cost for producing 300 tires.

a) \$4.80
b) \$48.00
c) \$480.00
d) None of the above

$$C(x) = 130 + 60x - 0.02x^2$$
$$C'(x) = 60 - .04x$$
$$C'(300) = 60 - .04(300) = 60 - 12 = \$48.00$$

Solution: B

8) Determine the intervals where the function is increasing. $f(x) = x^3 - 2$

a) The function is increasing on $(-\infty, \infty)$

b) The function is increasing on $(0, \infty)$

c) The function is increasing on $(-\infty, 0)$

d) The function is increasing on $(-\infty, 0) \cup (0, \infty)$

$f(x) = x^3 - 2$

$f(x) = 3x^2$

$3x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f^{-1}(x)$	+	+
Conclusion	Increasing	Increasing

Solution: D

9) Find the value of x, the number of units sold, that will produce a maximum profit. $P(x) = 2000 + 120x - 0.03x^2$

a) 200 units

b) 2000 units

c) 4000 units

d) No solution

$P(x) = 2000 + 120x - 0.03x^2$

$P'(x) = 120 - .06x$

$0 = 120 - .06x$

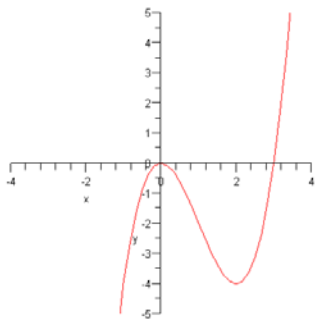
$.06x = 120$

$x = 2000$

Solution: B

10) Find the extrema points of the following function on a domain of all real numbers. $f(x) = x^3 - 3x^2$

a) Relative maximum at (0,0): Relative minimum at (2,-4)
b) Relative maximum at (2,-4): Relative minimum at (0,0)
c) Absolute maximum at (0,0)
d) Absolute minimum at (2,0)



$f(x) = x^3 - 3x^2$

$f(x) = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$3x = 0 \; or \; x - 2 = 0$

$x = 0 \qquad x = 2$

Interval	$(-\infty, 0)$	$(0,2)$	$(0,\infty)$
Test Value	$x = -1$	$x = 1$	$x = 3$
Sign of $f^{-1}(x)$	+	-	+
Conclusion	Increasing	Decreasing	Increasing

Relative maximum at (0,0)
Relative minimum at (2,-4)

Solution: A

11) Which statement is true about the concavity of the function $f(x) = x^2 - 3$

a) The function is concave up on $(-\infty, \infty)$
b) The function is concave down on $(-\infty, \infty)$
c) The function is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$
d) The function has no concavity.

$f(x) = x^2 - 3$

$f'(x) = 2x$

$f''(x) = 2 > 0$

$Concave\ up\ (-\infty, \infty)$

Solution: A

12) Solve $4^{2x-1} = \frac{1}{64}$

a) $x = -2$
b) $x = 1$
c) $x = -1$
d) $x = 2$

$$4^{2x-1} = \frac{1}{64}$$

$$4^{2x-1} = \frac{1}{4^3}$$

$$4^{2x-1} = 4^{-3}$$

$$2x - 1 = -3$$

$$2x = -3 + 1$$

$$2x = -2$$

$$x = -1$$

Solution: C

13) Solve $e^{x+2} = 5$

a) $x = \frac{\ln(5)+1}{2}$
b) $x = \frac{\ln(5)-1}{2}$
c) $x = \ln(5) - 2$
d) $x = \ln(5) + 2$

$$e^{x+2} = 5$$

$$\ln\left(e^{x+2}\right) = \ln(5)$$

$$x + 2 = \ln(5)$$

$$x = \ln(5) - 2$$

Solution: C

14) Find the derivative of $f(x) = \dfrac{x^2}{e^{2x}}$

a) $f'(x) = \dfrac{2x}{2e^{2x}}$

b) $f'(x) = \dfrac{2xe^{x} - x^2 e^{x}}{e^{2x}}$

c) $f'(x) = \dfrac{2x}{e^{x}}$

d) $f'(x) = \dfrac{2xe^{2x} - 2x^2 e^{2x}}{e^{4x}}$

$$f(x) = \frac{x^2}{e^{2x}}$$

$$f(x) = \frac{\left(e^{2x}\right)\left(x^2\right)' - \left(x^2\right)\left(e^{2x}\right)'}{\left(e^{2x}\right)^2} = \frac{2xe^{2x} - 2x^2 e^{2x}}{e^{4x}}$$

Solution: D

15) Find the derivative of $f(x) = \ln(x^4)$

a) $f'(x) = 4x^3 \ln(x)$

b) $f'(x) = \dfrac{4}{x}$

c) $f'(x) = \ln(4x^3)$

d) $f'(x) = 4x$

$$f(x) = \ln(x^4)$$

$$f(x) = \ln u,\ where\ u = x^4 \Rightarrow du = 4x^3$$

$$f'(x) = \frac{du}{u}$$

$$f'(x) = \frac{4x^3}{x^4} = \frac{4}{x}$$

Solution: B

Other Questions

16) Find the equation of tangent line to the curve $y = (2x-1)e^x$ at the point (0,-1).

$$y = (2x-1)e^x$$
$$y' = (2x-1)' e^x + (e^x)'(2x-1)$$
$$y' = 2e^x + (2x-1)e^x$$
$$m = 2e^0 + (2(0)-1)e^0 = 2(1) + (-1)(1) = 2 - 1 = 1$$

$$y - (-1) = 1(x-0)$$
$$y + 1 = 0$$
$$y = -1$$

17) Find the derivative of $y = \ln(t^3 e^t)$

$$y = \ln(t^3 e^t)$$
$$y = \ln(u),\ where\ \ u = t^3 e^t \Rightarrow du = 3t^2 e^t + t^3 e^t$$
$$y' = \frac{du}{u}$$
$$y' = \frac{3t^2 e^t + t^3 e^t}{t^3 e^t}$$

18) The demand function for a product is $p = 900 - 4x$ where x is the number of units sold. How many units must be sold to maximize revenue?

$$p = 900 - 4x$$
$$R = xp = x(900-4x) = 900x - x^2$$

$$R' = 900 - 2x$$
$$900 - 2x = 0$$
$$2x = 900$$
$$x = 450$$

19) Find the absolute extrema for the function $f(x) = x^2 - 4$ on the closed interval $[0,2]$

$f(x) = x^2 - 4$

$f'(x) = 2x$

$2x = 0$

$x = 0$

Critical Values

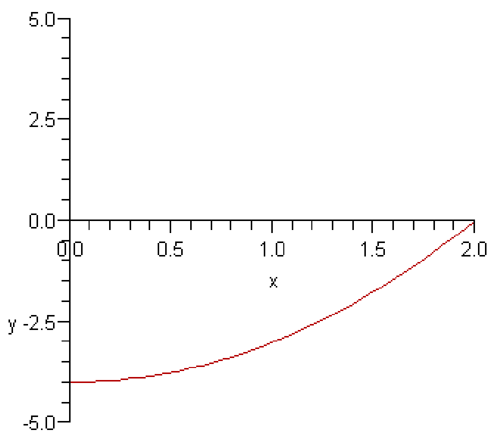
Absolute maximum at (2,0): Abs. Min (0,-4)

$f(0) = 0^2 - 4 = -4$

$(0,-4)$

$f(2) = 2^2 - 4 = 4 - 4 = 0$

$(2,0)$



20) What is the price elasticity of the demand for the demand function $p = 20 - .01x$ at x = 300?

$p' = -.01$

$$\eta = \frac{\frac{p}{x}}{p'} = \frac{\frac{20 - .01x}{x}}{-.01} = \frac{20 - .01x}{-.01x}$$

$$\eta = \frac{20 - .01(300)}{-.01(300)} = \frac{20 - 3}{-3} = -\frac{17}{3}$$

21) Find the derivative of $y = 2x^3 e^{5x}$

$$y = 2x^3 e^{5x}$$
$$y' = \left(2x^3\right)'\left(e^{5x}\right) + 2x^3\left(e^{5x}\right)$$
$$y' = 6x^2 e^{5x} + 2x^3\left(5e^{5x}\right)$$
$$y' = 6x^2 e^{5x} + 10x^3 e^{5x}$$

22) *Find the derivative* $y = 4x^2 \ln(3x)$

$$y' = \left(4x^2\right)' \ln(3x) + \left(\ln(3x)\right)'\left(4x^3\right)$$
$$y' = 12x\ln(3x) + \frac{3}{3x}\left(4x^2\right)$$
$$y' = 12x\ln(3x) + 4x$$

23) Find the slope of a line that passes between the points (2,3) and (5,0)

$$m = \frac{0-3}{5-2} = \frac{-3}{2} = -\frac{3}{2}$$

24) Find the distance between the points (2,4) and (5,7)

$$d = \sqrt{(5-2)^2 + (7-4)^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18}$$

25) Find the inverse of $f(x) = 6x - 2$

$$f(x) = 6x - 2$$
$$y = 6x - 2$$
$$x = 6y - 2$$
$$x - 2 = 6y$$
$$y = \frac{x-2}{6} \Rightarrow f'(x) = \frac{x-2}{6}$$

26) Write $\ln A + \ln B - 2\ln C$ as a single logarithm.

$$\ln A + \ln B - 2\ln C$$
$$\ln AB - \ln C^2$$
$$\ln\left(\frac{AB}{C^2}\right)$$

27) Write $5^4 = 625$ as single logarithmic expression.

$\log_5 625 = 4$

28) Write $\log_6 216 = 3$ as exponential expression.

$6^3 = 216$

29) Evaluate $\lim_{x \to 1} 2x^2 + 4x$

$\lim_{x \to 1} 2x^2 + 4x = 2(1)^2 + 4(1) = 2 + 4 = 6$

30) Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1} x + 1 = 1 + 1 = 2$

31) Find the derivative of $y = e^{10x^2}$

$$
\begin{aligned}
y &= e^{10x^2} \\
y &= e^u \quad \textit{where } u = 10x^2 \Rightarrow du = 20x \\
y' &= due^u \\
y' &= 20xe^{10x^2}
\end{aligned}
$$

32) Find the equation of a tangent line to the function $f(x) = x^2 + 1$ at the point $(1,2)$.

$$
\begin{aligned}
f(x) &= x^2 + 1 \\
f'(x) &= 2x \\
m = f'(1) &= 2(1) = 2
\end{aligned}
$$

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 2 &= 1(x - 1) \\
y - 2 &= x - 1 \\
y &= x + 1
\end{aligned}
$$

33) Find the equation of a tangent line to the function $f(x) = e^x - 2$ at the point $(0, -1)$.

$$f(x) = e^x - 2$$
$$f'(x) = e^x$$
$$m = f'(0) = e^0 = 1$$

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 1(x - 0)$$
$$y + 1 = x$$
$$y = x - 1$$