

Math 126
Practice Final

1) Find the slope and y-intercept of line represented by the following equation.

$$4x - 3y = 3$$

a) $m = 3, b = -4$

b) $m = -\frac{4}{3}, b = 1$

c) $m = \frac{4}{3}, b = -1$

d) $m = \frac{3}{4}, b = -4$

$$4x - 3y = 3$$

$$4x - 4x - 3y = -4x + 3$$

$$-3y = -4x + 3$$

$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{3}{-3}$$

$$y = \frac{4}{3}x - 1$$

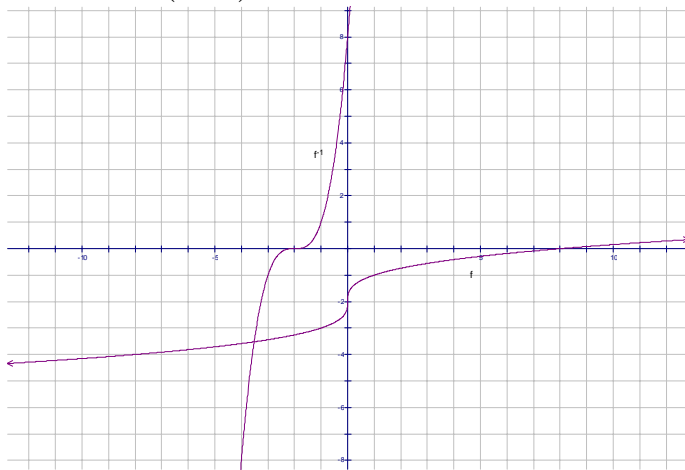
$$\Rightarrow m = \frac{4}{3}, b = -1$$

Solution: C

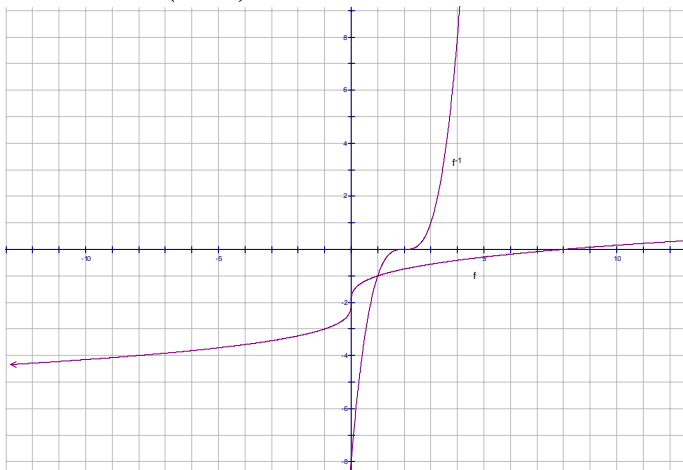
2) Find the inverse of the following function, and then graph $f(x)$ and $f^{-1}(x)$

$$f(x) = \sqrt[3]{x} - 2$$

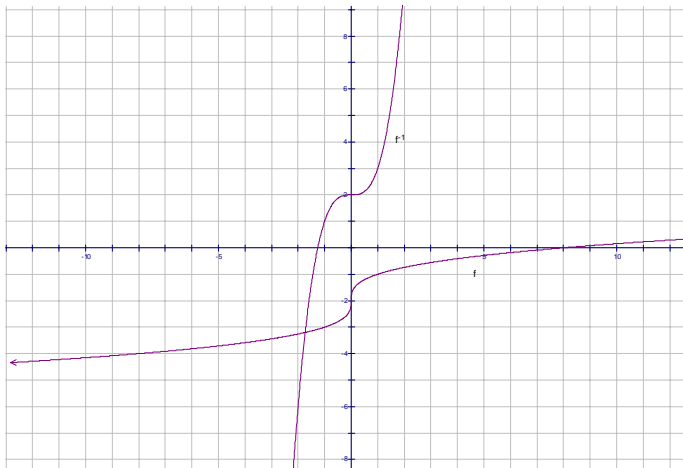
a) $f^{-1}(x) = (x + 2)^3$



b) $f^{-1}(x) = (x+2)^3$



c) $f^{-1}(x) = x^3 + 2$



d) None of the above

$$f(x) = \sqrt[3]{x} - 2$$

$$y = \sqrt[3]{x} - 2$$

$$x = \sqrt[3]{y} - 2$$

$$x + 2 = \sqrt[3]{y}$$

$$(x + 2)^3 = (\sqrt[3]{y})^3$$

$$(x + 2)^3 = f^{-1}(x)$$

Solution: A

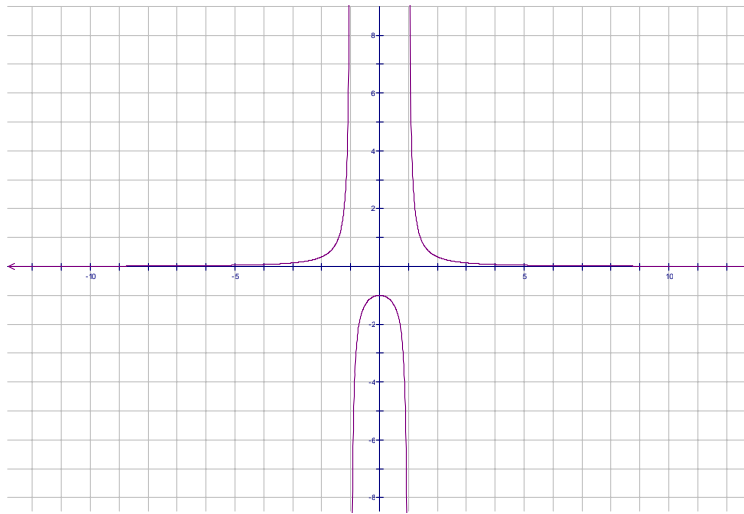
3) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2}$

- a) 2
- b) -2
- c) Undefined
- d) -1

$$\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 4)(x - 2)}{x - 2} = \lim_{x \rightarrow 2} x - 4 = 2 - 4 = -2$$

Solution: B

4) Which statement about continuity is correct



- a) The function is continuous everywhere.
- b) The function is discontinuous everywhere
- c) The function is continuous everywhere except the values $x = -2$ and $x = 2$
- d) The function is continuous everywhere except the values $x = -1$ and $x = 1$

Solution: D

5) Given $f(x) = 2x^3 + 5x^2 + 3$, find the derivative.

- a) $f'(x) = 4x^3 + 6x$
- b) $f'(x) = 6x^2 + 10x$
- c) $f'(x) = 12x^2 + 10x$
- d) None of the above

$$f(x) = 2x^3 + 5x^2 + 3$$

$$f'(x) = 6x^2 + 10x$$

Solution: B

6) Find the derivative of $f(x) = (x^2 - 8x)^4$

a) $f'(x) = 4(2x - 8)^3$

b) $f'(x) = (x^2 - 8x)^3$

c) $f'(x) = 4(x^2 - 8x)^3(2x - 8)$

d) None of the above

$$f(x) = (x^2 - 8x)^4$$

$$u = x^2 - 8x$$

$$du = 2x - 8$$

$$f(x) = u^4$$

$$f'(x) = 4u^3 du = 4(x^2 - 8x)^3(2x - 8)$$

Solution: C

7) The Homer Simpson Tire Corporation determined that the daily cost of producing lawn tractor tires can be modeled by the cost function $C(x) = 130 + 60x - 0.02x^2$, where $0 \leq x \leq 360$. Determine the marginal cost for producing 300 tires.

$$C(x) = 130 + 60x - 0.02x^2$$

$$C'(x) = 60 - .04x$$

$$C'(300) = 60 - .04(300) = 60 - 12 = \$48.00$$

Solution: \$48

8) Determine the intervals where the function is increasing. $f(x) = x^3 - 2$

- a) The function is increasing on $(-\infty, \infty)$
- b) The function is increasing on $(0, \infty)$
- c) The function is increasing on $(-\infty, 0)$
- d) The function is increasing on $(-\infty, 0) \cup (0, \infty)$

$$f(x) = x^3 - 2$$

$$f'(x) = 3x^2$$

$$3x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	+	+
Conclusion	Increasing	Increasing

Solution: D

9) Find the value of x , the number of units sold, that will produce a maximum profit. $P(x) = 2000 + 120x - 0.03x^2$

- a) 200 units
- b) 2000 units
- c) 4000 units
- d) No solution

$$P(x) = 2000 + 120x - 0.03x^2$$

$$P'(x) = 120 - .06x$$

$$0 = 120 - .06x$$

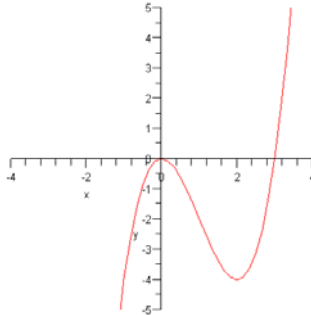
$$.06x = 120$$

$$x = 2000$$

Solution: B

10) Find the extrema points of the following function on a domain of all real numbers. $f(x) = x^3 - 3x^2$

- a) Relative maximum at (0,0): Relative minimum at (2,-4)
- b) Relative maximum at (2,-4): Relative minimum at (0,0)
- c) Absolute maximum at (0,0)
- d) Absolute minimum at (2,0)



$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x$$

$$0 = 3x^2 - 6x$$

$$0 = 3x(x - 2)$$

$$3x = 0 \text{ or } x - 2 = 0$$

$$x = 0 \quad x = 2$$

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Test Value	$x = -1$	$x = 1$	$x = 3$
Sign of $f''(x)$	+	-	+
Conclusion	Increasing	Decreasing	Increasing

Relative maximum at (0,0)

Relative minimum at (2,-4)

Solution: A

11) Which statement is true about the concavity of the function $f(x) = x^2 - 3$

- a) The function is concave up on $(-\infty, \infty)$
- b) The function is concave down on $(-\infty, \infty)$
- c) The function is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$
- d) The function has no concavity.

$$f(x) = x^2 - 3$$

$$f'(x) = 2x$$

$$f''(x) = 2 > 0$$

Concave up $(-\infty, \infty)$

Solution: A

12) Solve $4^{2x-1} = \frac{1}{64}$

- a) $x = -2$
- b) $x = 1$
- c) $x = -1$
- d) $x = 2$

$$4^{2x-1} = \frac{1}{64}$$

$$4^{2x-1} = \frac{1}{4^3}$$

$$4^{2x-1} = 4^{-3}$$

$$2x - 1 = -3$$

$$2x = -3 + 1$$

$$2x = -2$$

$$x = -1$$

Solution: C

13) Solve $e^{x+2} = 5$

a) $x = \frac{\ln(5) + 1}{2}$

b) $x = \frac{\ln(5) - 1}{2}$

c) $x = \ln(5) - 2$

d) $x = \ln(5) + 2$

$$e^{x+2} = 5$$

$$\ln(e^{x+2}) = \ln(5)$$

$$x + 2 = \ln(5)$$

$$x = \ln(5) - 2$$

Solution: C

14) Find the derivative of $f(x) = \frac{x^2}{e^{2x}}$

a) $f'(x) = \frac{2x}{2e^{2x}}$

b) $f'(x) = \frac{2xe^x - x^2e^x}{e^{2x}}$

c) $f'(x) = \frac{2x}{e^x}$

d) $f'(x) = \frac{2xe^{2x} - 2x^2e^{2x}}{e^{4x}}$

$$f(x) = \frac{x^2}{e^{2x}}$$

$$f'(x) = \frac{(e^{2x})(x^2)' - (x^2)(e^{2x})'}{(e^{2x})^2} = \frac{2xe^{2x} - 2x^2e^{2x}}{e^{4x}}$$

Solution: D

15) Find the derivative of $f(x) = \ln(x^4)$

a) $f'(x) = 4x^3 \ln(x)$

b) $f'(x) = \frac{4}{x}$

c) $f'(x) = \ln(4x^3)$

d) $f'(x) = 4x$

$$f(x) = \ln(x^4)$$

$$f(x) = \ln u, \text{ where } u = x^4 \Rightarrow du = 4x^3$$

$$f'(x) = \frac{du}{u}$$

$$f'(x) = \frac{4x^3}{x^4} = \frac{4}{x}$$

Solution: B

16) Evaluate $\int x^2 dx$

- a) $x^3 + C$
- b) $2x + C$
- c) $\frac{1}{3}x^3 + C$
- d) $3x^3 + C$

$$\int x^2 dx = \frac{1}{2+1}x^{2+1} + C = \frac{1}{3}x^3 + C$$

Solution: C

17) Evaluate $\int_0^2 3x^2 + 1 dx$

- a) -10
- b) 10
- c) 5
- d) 20

$$\int_0^2 3x^2 + 1 dx = \left(\frac{3}{2+1}x^{2+1} + \frac{1}{1+0}x^{0+1} \right) \Big|_0^2 = (x^3 + x) \Big|_0^2 = (2^3 + 2) - (0^3 - 0) = 10 - 0 = 10$$

Solution: B

18) Evaluate $\int_0^1 e^x dx$

- a) 1
- b) $\frac{1}{2}e^2$
- c) $e - 1$
- d) e

$$\int_0^1 e^x dx = (e^x) \Big|_0^1 = e^1 - e^0 = e - 1$$

Solution: C

19) Evaluate $\int_1^4 \frac{1}{x} dx$

a) $\ln(4) - \ln(2)$

b) $\ln(4)$

c) $\frac{1}{4}$

d) 1

$$\int_1^4 \frac{1}{x} dx = \ln x \Big|_1^4 = \ln 4 - \ln 1 = \ln 4$$

Solution: B

Other Questions

20) Find the equation of tangent line to the curve $y = (2x - 1)e^x$ at the point $(0, -1)$.

$$y = (2x - 1)e^x$$

$$y' = (2x - 1)' e^x + (e^x)' (2x - 1)$$

$$y' = 2e^x + (2x - 1)e^x$$

$$m = 2e^0 + (2(0) - 1)e^0 = 2(1) + (-1)(1) = 2 - 1 = 1$$

$$y - (-1) = 1(x - 0)$$

$$y + 1 = 0$$

$$y = -1$$

21) The marginal cost producing calculators is $\frac{dC}{dx} = x^2 - 300x$. Find the cost function, if the fixed cost is \$350.

$$\int x^2 - 300x dx = \left(\frac{1}{2+1} x^{2+1} + \frac{1}{1+1} x^{1+1} + C \right) = \frac{1}{3} x^3 + x + C = \frac{1}{3} x^3 + x + 350$$

22) Find the derivative of $y = \ln(t^3 e^t)$

$$y = \ln(t^3 e^t)$$

$$y = \ln(u), \text{ where } u = t^3 e^t \Rightarrow du = 3t^2 e^t + t^3 e^t$$

$$y' = \frac{du}{u}$$

$$y' = \frac{3t^2 e^t + t^3 e^t}{t^3 e^t}$$

23) The demand function for a product is $p = 900 - 4x$ where x is the number of units sold. How many units must be sold to maximize revenue?

$$p = 900 - 4x$$

$$R = xp = x(900 - 4x) = 900x - x^2$$

$$R' = 900 - 2x$$

$$900 - 2x = 0$$

$$2x = 900$$

$$x = 450$$

24) Find the absolute extrema for the function $f(x) = x^2 - 4$ on the closed interval $[0,2]$

$$f(x) = x^2 - 4$$

$$f'(x) = 2x$$

$$2x = 0$$

$$x = 0$$

Critical Values

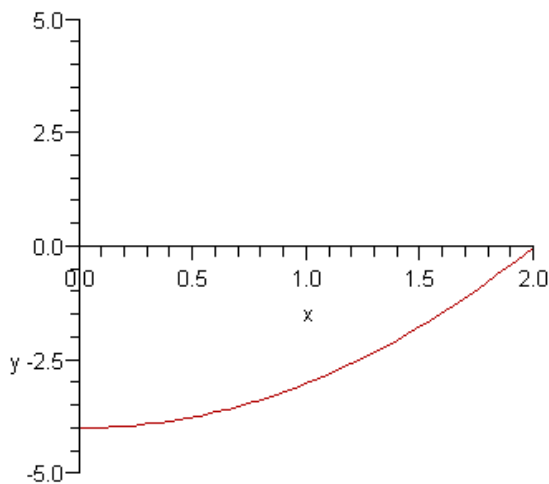
Absolute maximum at (2,0): Abs. Min (0,-4)

$$f(0) = 0^2 - 4 = -4$$

(0,-4)

$$f(2) = 2^2 - 4 = 4 - 4 = 0$$

(2,0)



25) What is the price elasticity of the demand for the demand function

$$p = 20 - .01x \text{ at } x = 300?$$

$$p' = -.01$$

$$\eta = \frac{p}{p'} = \frac{20 - .01x}{-.01} = \frac{20 - .01x}{-.01x}$$

$$\eta = \frac{20 - .01(300)}{-.01(300)} = \frac{20 - 3}{-3} = -\frac{17}{3}$$

26) Evaluate $\int_1^2 4x^3 dx$

$$\int_1^2 4x^3 dx = \left(\frac{4}{3+1} x^{3+1} \right) \Big|_1^2 = (x^4) \Big|_1^2 = 2^4 - 1^4 = 16 - 1 = 15$$

27) Find the derivative of $y = 2x^3 e^{5x}$

$$y = 2x^3 e^{5x}$$

$$y' = (2x^3)' (e^{5x}) + 2x^3 (e^{5x})'$$

$$y' = 6x^2 e^{5x} + 2x^3 (5e^{5x})$$

$$y' = 6x^2 e^{5x} + 10x^3 e^{5x}$$

28) $\int 4x dx$

$$\int 4x dx = \frac{4}{1+1} x^{1+1} + C = 2x^2 + C$$

29) Find the derivative $y = 4x^2 \ln(3x)$

$$y' = (4x^2)' \ln(3x) + (\ln(3x))' (4x^2)$$

$$y' = 12x \ln(3x) + \frac{3}{3x} (4x^2)$$

$$y' = 12x \ln(3x) + 4x$$

30) Find the slope of a line that passes between the points (2,3) and (5,0)

$$m = \frac{0-3}{5-2} = \frac{-3}{2} = -\frac{3}{2}$$

31) Find the distance between the points (2,4) and (5,7)

$$d = \sqrt{(5-2)^2 + (7-4)^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18}$$

32) Find the inverse of $f(x) = 6x - 2$

$$f(x) = 6x - 2$$

$$y = 6x - 2$$

$$x = 6y - 2$$

$$x - 2 = 6y$$

$$y = \frac{x-2}{6} \Rightarrow f^{-1}(x) = \frac{x-2}{6}$$

33) Write $\ln A + \ln B - 2 \ln C$ as a single logarithm.

$$\ln A + \ln B - 2 \ln C$$

$$\ln AB - \ln C^2$$

$$\ln\left(\frac{AB}{C^2}\right)$$

34) Write $5^4 = 625$ as single logarithmic expression.

$$\log_5 625 = 4$$

35) Write $\log_6 216 = 3$ as exponential expression.

$$6^3 = 216$$

36) Evaluate $\lim_{x \rightarrow 1} 2x^2 + 4x$

$$\lim_{x \rightarrow 1} 2x^2 + 4x = 2(1)^2 + 4(1) = 2 + 4 = 6$$

37) Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \rightarrow 1} x + 1 = 1 + 1 = 2$$