

Section 4.5

The derivative of the natural logarithm

Definition

If $f(x) = \ln(x)$, then $f'(x) = \frac{1}{x}$

Example 1

Find the derivative of $y = 5 \ln x$

$$y = 5 \ln x$$

$$y' = 5 \left(\frac{1}{x} \right)$$

$$y' = \frac{5}{x}$$

Chain Rule for the natural logarithm

Let $y = \ln u$

$$y' = \frac{1}{u} du \text{ or } y' = \frac{du}{u}$$

Example 2

Find the derivative of $y = \ln(3x^2)$

Let $y = \ln u$ where $u = 3x^2 \Rightarrow du = 6x$

$$y' = \frac{du}{u}$$

$$y' = \frac{6x}{3x^2}$$

$$y' = \frac{2}{x}$$

Example 3

Find the derivative of $y = \ln(x^2 + 6x)$

Let $y = \ln u$ where $u = x^2 + 6x \Rightarrow du = 2x + 6$

$$y' = \frac{du}{u}$$

$$y' = \frac{2x + 6}{x^2 + 6x}$$

Example 4

Find the derivative of $y = x^2 \ln(4x)$

$$y = x^2 \ln(4x)$$

Use the product rule

$$y' = 2x \ln(4x) + \left(\frac{4}{4x}\right)(x^2)$$

$$y' = 2x \ln(4x) + x$$

Example 5

Find the derivative of $y = x^3 \ln(x^2)$

$$y = x^3 \ln(x^2)$$

Use the product rule

$$y' = 3x^2 \ln(x^2) + \left(\frac{2x}{x^2}\right)(x^3)$$

$$y' = 3x^2 \ln(x^2) + 2x^2$$

Practice Examples

1) Find the derivative of $y = (x^2 + 3x)\ln(2x^2)$

Solution:

$$y = (x^2 + 3x)\ln(2x^2)$$

Use the product rule

$$y' = (2x + 3)\ln(2x^2) + \left(\frac{4x}{2x^2}\right)(x^2 + 3x)$$

$$y' = (2x + 3)\ln(2x^2) + \frac{2}{x}(x^2 + 3x)$$

2) Find the derivative of $y = \ln(x^2 + 2x + 1)$

Solution:

$$\text{Let } y = \ln u \text{ where } u = x^2 + 2x + 1 \Rightarrow du = 2x + 2$$

$$y' = \frac{du}{u}$$

$$y' = \frac{2x + 2}{x^2 + 2x + 1}$$