Math 121

Section 2.3

Business Functions

Cost Function - C(x)

Revenue Function – R(x)

Profit Function – P(x)

$P(x) = R(x) - C(x)$

Marginal Measures

Marginal Cost - $C'(x)$

Marginal Revenue - $R'(x)$

Marginal Profit - $P'(x)$

Example 1

Find the marginal cost function of the cost function $C = 4500 + 1.47x$

$$C = 4500 + 1.47x$$
$$C' = 1.47$$

Example 2

Find the marginal cost function of the cost function $C = 55000 + 470x - 0.25x^2$

$$C = 55000 + 470x - 0.25x^2$$
$$C' = 470 - .5x$$

Example 3

Find the marginal revenue function given $R = 50x - .5x^2$

$$R = 50x - .5x^2$$
$$R' = 50 - 1.0x$$

Example 4

Find the marginal revenue function given $R = -6x^3 + 8x^2 + 200x$

$$R = -6x^3 + 8x^2 + 200x$$
$$R' = -18x^2 + 16x + 200$$

Example 5

Find the marginal profit of the function $P(x) - 2x^2 + 72x - 145$

$$P = -2x^2 + 72x - 145$$
$$P' = -4x + 72$$

Example 6

The profit in dollars from renting x apartments can be modeled by $P = -.1x^2 + 2x - 100$

a) Find the additional profit when the number of units is increased from 7 renters to 8 renters.

$$P(7) = -.1(7)^2 + 2(7) - 100 = -4.9 + 14 - 100 = -\$90.90$$
$$P(8) = -.1(8)^2 + 2(8) - 100 = -6.4 + 16 - 100 = -\$90.40$$
$$Change\ in\ \text{Pr}\,ofit = -90.40 - (-90.90) = \$.50$$

b) Find the marginal profit for x=7

$$P'(x) = -.2x + 2$$
$$P'(7) = -.2(7) + 2 = \$.60$$

c) Compare the results from parts a and b: **The values are about the same**

Definition: Break Even Point

The value of x (number of units sold) where cost equals revenue

$P(x) = C(x)$

Example 7

Find the break even point for the given cost function and revenue function.

$$C(x) = 6x + 50{,}000$$
$$R(x) = 35x$$
$$35x = 6x + 50000$$
$$35x - 6x = 6x - 6x + 50000$$
$$29x = 50000$$
$$\frac{29x}{29} = \frac{50000}{29}$$
$$x = 1724.14$$
$$x = 1725 \ \ units$$

Example 8

Find the break even point for the given cost function and revenue function.

$$C(x) = 4x + 10000$$
$$R(x) = 20x$$
$$20x = 4x + 10000$$
$$20x - 4x = 4x - 4x + 10000$$
$$16x = 10000$$
$$\frac{16x}{16} = \frac{10000}{16}$$
$$x = 625 \ units$$

Example 9

A manufacture's total cost is $C(x) = 0.1x^3 - .05x^2 + 500x + 200$ dollars when the level production is x units. Find the marginal cost for producing 40 units.

$$C(x) = 0.1x^3 - .05x^2 + 500x + 200$$
$$C'(x) = .3x^{3-1} - .10x^{2-1} + 500$$
$$C(x) = .3x^2 - .1x + 500$$
$$Let \;\; x = 40$$
$$Find \;\; C'(40) = .3(40)^2 - .1(40) + 500 = 480 - 4 + 500 = \$976.00$$

Example 10

Suppose that the total revenue in dollars of manufacturing t units is given by $R(t) = 240t - 0.05t^2$ dollars when t units are produced and sold during the month. Currently, the manufacture is producing 80 units per month and is planning to increase production by 81 units. Find the marginal revenue for producing 81 units or the increase in revenue for increasing from 80 units to 81 units.

$$R(t) = 240t - 0.05t^2$$
$$R'(t) = 240 - .10t$$
$$R'(81) = 240 - .10(81) = 240 - 8.1 = \$241.9$$