

## Math 126

### Section 1.6

#### Continuity

**Definition of a Continuous Function:** Let  $C$  be a number in the interval  $(a,b)$ , and let  $f$  be a function whose domain contains the interval  $(a,b)$ . The function  $f$  is continuous at point  $C$  if the following condition is true.

- 1)  $\lim_{x \rightarrow c} f(x)$  exist
- 2)  $f(c)$  exist
- 3)  $\lim_{x \rightarrow c} f(x) = f(c)$

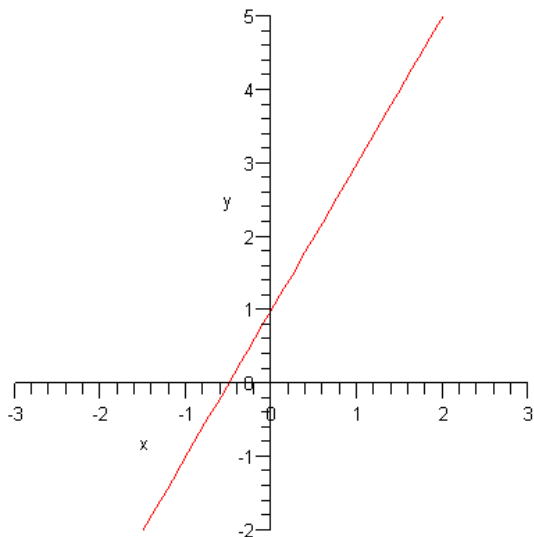
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**A function  $f$  is continuous on an interval  $(a,b)$ , if  $f$  is continuous at every point on the interval  $(a,b)$**

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#### Example 1

Give the intervals where the function is continuous



$f$  is continuous on  $(-\infty, \infty)$

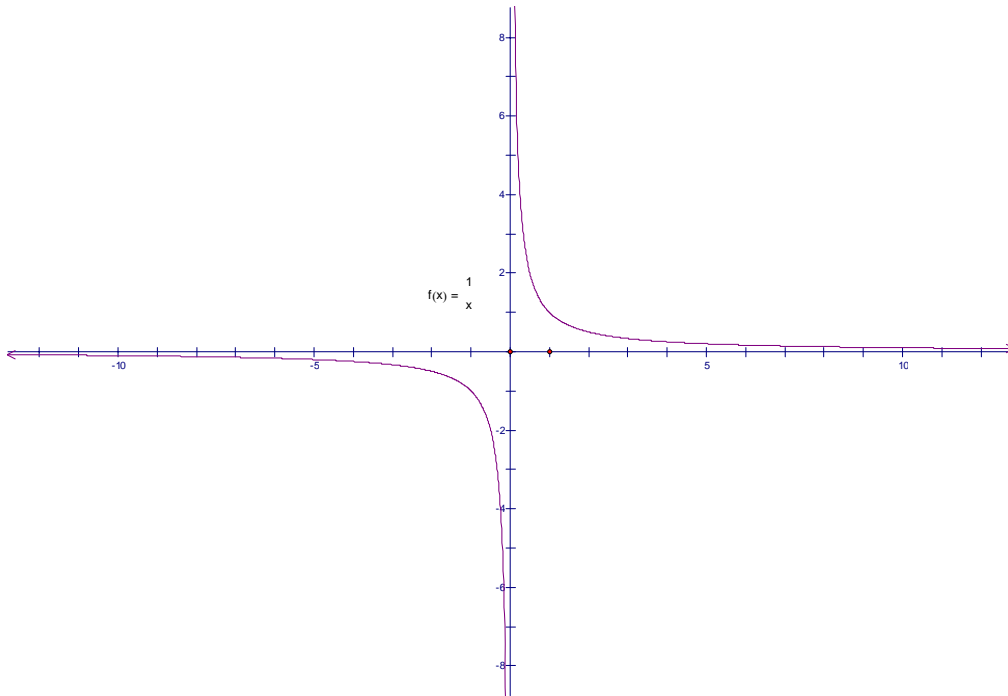
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**The graph of a continuous function is piecewise smooth.**

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**Example 2** Give the intervals where the function is continuous

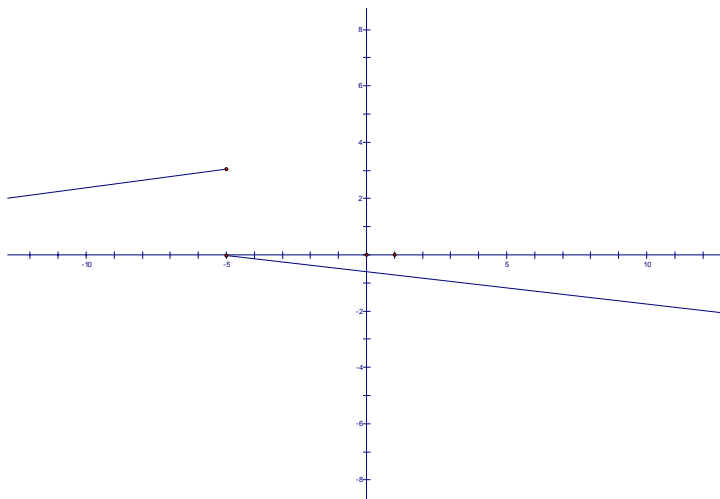


f is continuous on  $(-\infty, 0) \cup (0, \infty)$

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**Example 4**

Give the intervals where the function is continuous



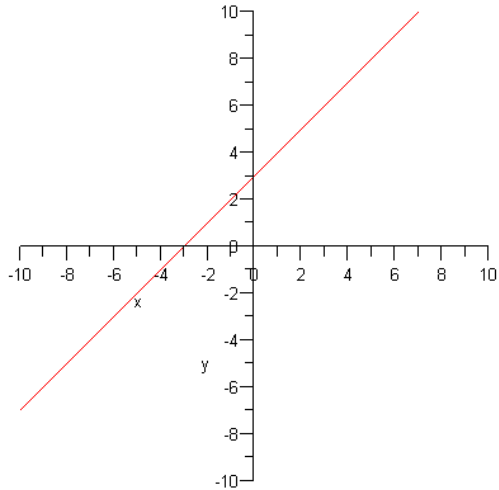
f is continuous on  $(-\infty, -5) \cup (-5, \infty)$

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**Example 5**

Give the intervals where the function is continuous



$f$  is continuous on  $(-\infty, \infty)$

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**Three types of situations that make a graph discontinuous**

- 1) A hole
  - 2) A break in the graph or gap in the graph
  - 3) A asymptote
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**Continuity of Rational and Irrational Functions**

- 1) A polynomial function is continuous everywhere
  - 2) A rational function is continuous on its domain
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**Example 6**

Give the intervals where the function is continuous

$$f(x) = x^3 - 4$$

$f$  is a polynomial  $\Rightarrow f$  is continuous

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**Example 7**

Give the intervals where the function is continuous

$$f(x) = 5x^3 - 2x^2 + 4$$

*f is a polynomial  $\Rightarrow f$  is continuous on  $(-\infty, \infty)$*

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**Example 8**

Give the intervals where the function is continuous

$$f(x) = \frac{5}{x-2}$$

*f is undefined at  $x = 2 \Rightarrow f$  is discontinuous at  $x = 2$*

*$\Rightarrow f$  is continuous on  $(-\infty, 2) \cup (2, \infty)$*

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**Example 9**

Give the intervals where the function is continuous

$$f(x) = \frac{x-3}{x^2-9}$$

$$f(x) = \frac{x-3}{(x-3)(x+3)}$$

*f is undefined at  $x = -3$  and  $x = 3 \Rightarrow f$  is discontinuous at  $x = -3$  and  $x = 3$*

*$\Rightarrow f$  is continuous on  $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$*

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**Example 10**

Give the intervals where  $f$  is continuous

$$f(x) = \sqrt{x-3}$$

*f is undefined on  $(-\infty, 3) \Rightarrow f$  is continuous on  $(3, \infty)$*

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**Example 11**

Give the intervals where the function is continuous

$$f(x) = x^2 + 3x$$

*f is a polynomial  $\Rightarrow$  f is continuous*

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