

Math 121

Section 1.6

Continuity

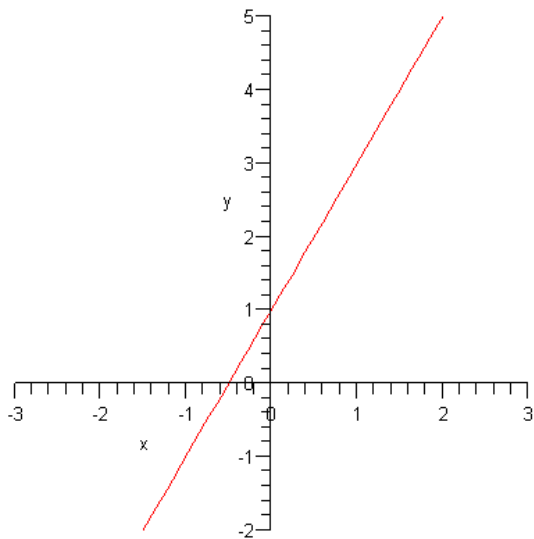
Definition of a Continuous Function: Let C be a number in the interval (a,b) , and let f be a function whose domain contains the interval (a,b) . The function f is continuous at point C if the following condition is true.

- 1) $\lim_{x \rightarrow c} f(x)$ exist
- 2) $f(c)$ exist
- 3) $\lim_{x \rightarrow c} f(x) = f(c)$

A function f is continuous on an interval (a,b) , if f is continuous at every point on the interval (a,b)

Example 1

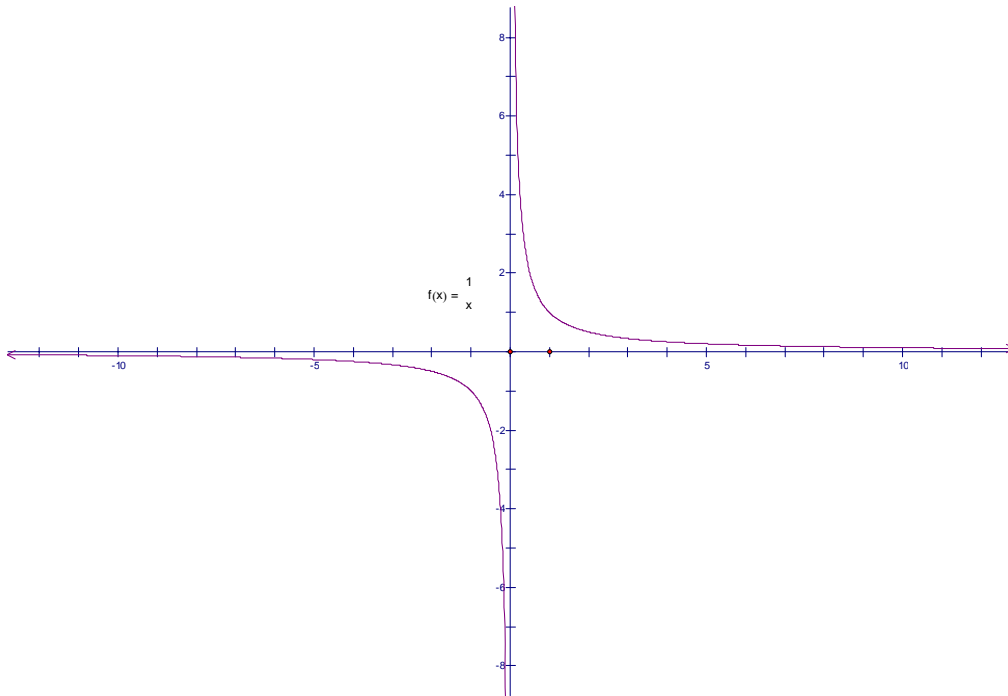
Give the intervals where the function is continuous



f is continuous on $(-\infty, \infty)$

The graph of a continuous function is piecewise smooth.

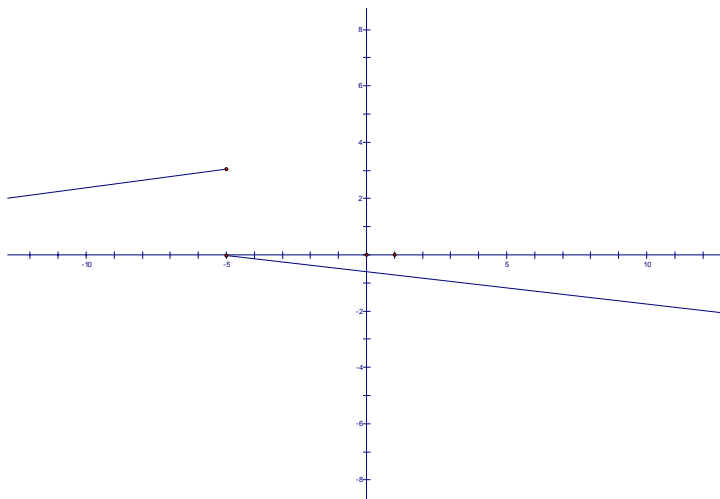
Example 2 Give the intervals where the function is continuous



f is continuous on $(-\infty, 0) \cup (0, \infty)$

Example 4

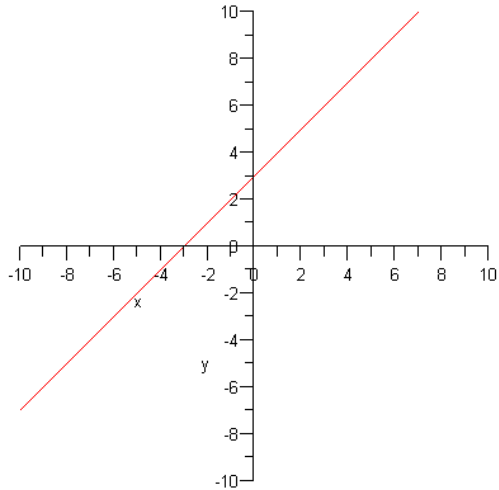
Give the intervals where the function is continuous



f is continuous on $(-\infty, -5) \cup (-5, \infty)$

Example 5

Give the intervals where the function is continuous



f is continuous on $(-\infty, \infty)$

Three types of situations that make a graph discontinuous

- 1) A hole
 - 2) A break in the graph or gap in the graph
 - 3) A asymptote
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Continuity of Rational and Irrational Functions

- 1) A polynomial function is continuous everywhere
 - 2) A rational function is continuous on its domain
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Example 6

Give the intervals where the function is continuous

$$f(x) = x^3 - 4$$

f is a polynomial $\Rightarrow f$ is continuous

Example 7

Give the intervals where the function is continuous

$$f(x) = 5x^3 - 2x^2 + 4$$

f is a polynomial $\Rightarrow f$ is continuous on $(-\infty, \infty)$

Example 8

Give the intervals where the function is continuous

$$f(x) = \frac{5}{x-2}$$

f is undefined at $x = 2 \Rightarrow f$ is discontinuous at $x = 2$

$\Rightarrow f$ is continuous on $(-\infty, 2) \cup (2, \infty)$

Example 9

Give the intervals where the function is continuous

$$f(x) = \frac{x-3}{x^2-9}$$

$$f(x) = \frac{x-3}{(x-3)(x+3)}$$

f is undefined at $x = -3$ and $x = 3 \Rightarrow f$ is discontinuous at $x = -3$ and $x = 3$

$\Rightarrow f$ is continuous on $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Example 10

Give the intervals where f is continuous

$$f(x) = \sqrt{x-3}$$

f is undefined on $(-\infty, 3) \Rightarrow f$ is continuous on $(3, \infty)$

Example 11

Give the intervals where the function is continuous

$$f(x) = x^2 + 3x$$

f is a polynomial \Rightarrow f is continuous
