

# Math 121

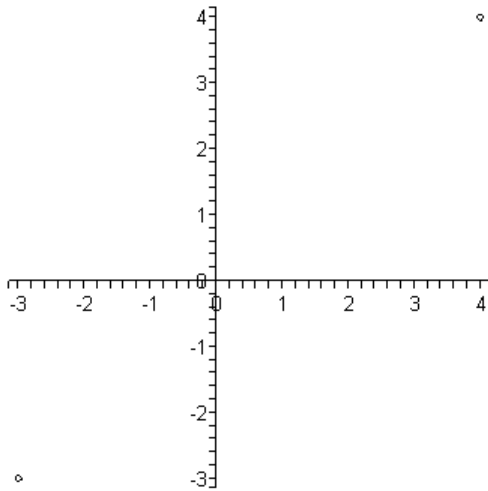
## Section 1.1

### The Cartesian Plane and the Distance Formula

#### The Cartesian Plane

Quadrant I

Quadrant II



Quadrant III

Quadrant IV

Independent axis is the x-axis

Dependent axis is the y-axis

Ordered pairs – (x,y)

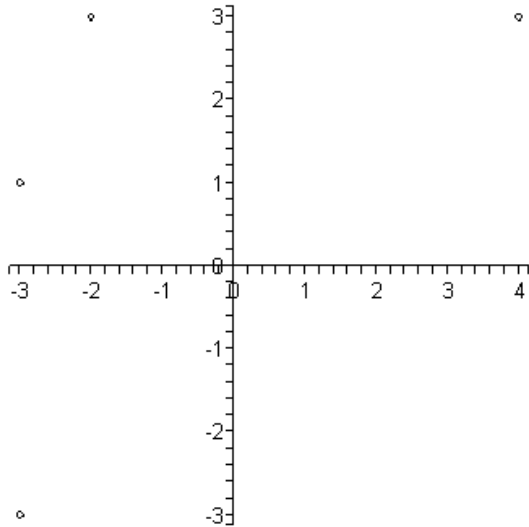
x = x-coordinate

y = y-coordinate



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**Example 1** Graph the following ordered pairs:  $(-3,1)$ ,  $(-2,3)$ ,  $(-3,-4)$ ,  $(4,3)$



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### Distance formula

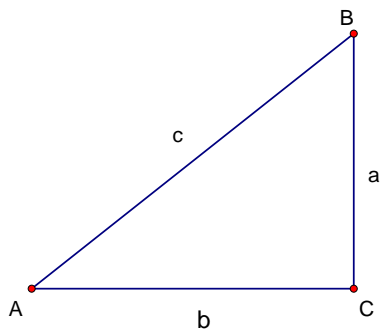
The distance between the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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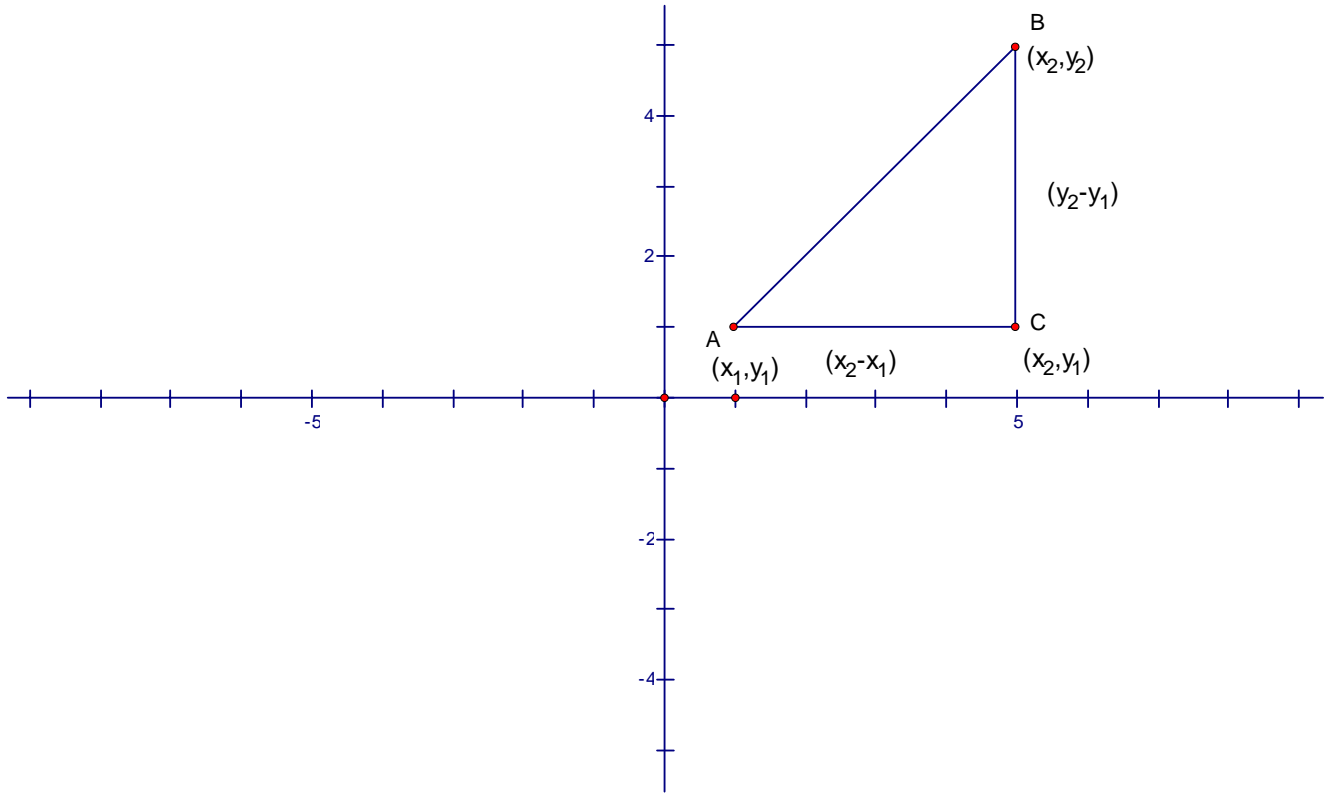
### Pythagorean Theorem

If  $\triangle ABC$  is a right triangle, then the sum of the squares of the two legs of the right triangle is equal to the square of the hypotenuse of the right triangle. ( $c^2 = a^2 + b^2$ )



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## Proof of the distance formula



Using the Pythagorean Theorem gives the following result.

$$c^2 = a^2 + b^2$$

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\sqrt{c^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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**Example 2**

Find the distance between the points (2,5) and (3,7).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - 2)^2 + (7 - 5)^2}$$

$$d = \sqrt{(1)^2 + (2)^2}$$

$$d = \sqrt{1 + 4}$$

$$d = \sqrt{5}$$

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**Example 3**

Find the distance between the points (-3,1) and (3,-2).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - (-3))^2 + (-2 - 1)^2}$$

$$d = \sqrt{(6)^2 + (-3)^2}$$

$$d = \sqrt{36 + 9}$$

$$d = \sqrt{45}$$

$$d = \sqrt{9 \cdot 5}$$

$$d = 3\sqrt{5}$$

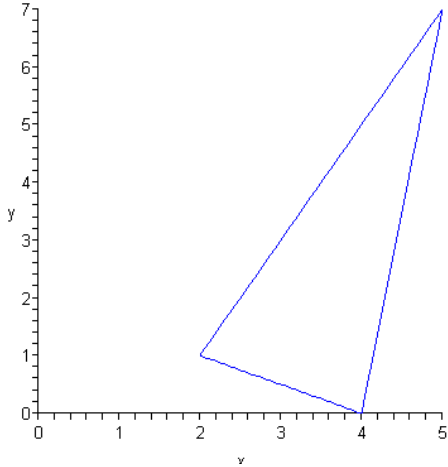
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**Example 4** Right Triangle Problem

Show that the points  $A = (2,1)$ ,  $B = (4,0)$ , and  $C = (5,7)$  form a right triangle, then find the area of the triangle.

i) **Make a plot of the points A,B, and C**



ii) **Find the distances between the points.**

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$d(B,C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1^2 + 7^2} = \sqrt{1+49} = \sqrt{50}$$

$$d(A,C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}$$

iii) **Use Pythagorean Theorem to check distances**

$$c^2 = a^2 + b^2$$

$$(\sqrt{50})^2 = (\sqrt{45})^2 + (\sqrt{5})^2$$

$$50 = 45 + 5$$

$$50 = 50$$

Thus, triangle ABC is a right triangle

**iv) Area of the triangle**

$$\text{base} = \sqrt{5} \text{ and height} = \sqrt{45}$$

$$A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{5})(\sqrt{45}) = \frac{1}{2}\sqrt{225} = \frac{1}{2}(15) = 7.5$$

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**Example 5**

Find  $x$  such that the distance between the point is 5 units, using the points  $(2,-1)$  and  $(x,2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$5 = \sqrt{(x - 2)^2 + (2 - (-1))^2}$$

$$5 = \sqrt{(x - 2)^2 + 3^2}$$

$$5 = \sqrt{(x - 2)^2 + 9}$$

$$5^2 = \left(\sqrt{(x - 2)^2 + 9}\right)^2$$

$$25 = (x - 2)^2 + 9$$

$$25 - 9 = (x - 2)^2 + 9 - 9$$

$$16 = (x - 2)^2$$

$$\sqrt{16} = \sqrt{(x - 2)^2}$$

$$\pm 4 = x - 2$$

$$x = 2 \pm 4 \Rightarrow x = -2 \text{ or } x = 6$$

