Integration Unit

The integral or the anti derivative is the inverse operation of the derivative.

Integral Notation of the Anti derivative

Let $\int f(x)dx = F(x) + C$, where C is an arbitrary constant and F is the antiderivative of f

Power Rule for the Integration

$$\int cx^n dx = \frac{c}{n+1}x^{n+1} + C,\ where\ n \neq -1$$

Example 1

Evaluate $\int x^2 dx$

$$\int x^2 dx = \frac{1}{2+1}x^{2+1} + C$$
$$= \frac{1}{3}x^3 + C$$

Example 2

Evaluate $\int x^4 + x^3\ dx$

$$\int x^4 + x^3\ dx = \frac{1}{4+1}x^{4+1} + \frac{1}{3+1}x^3 + C$$
$$= \frac{1}{5}x^5 + \frac{1}{4}x^4 + C$$

Example 3

Evaluate $\int \frac{1}{x^3} dx$

$$\int \frac{1}{x^3} dx = \int x^{-3} dx$$
$$= \frac{1}{-3+1} x^{-3+1} + C$$
$$= -\frac{1}{2} x^{-2} + C$$
$$= -\frac{1}{2x^2} + C$$

Example 4

Evaluate $\int \frac{3}{x^2} dx$

$$\int \frac{3}{x^2} = \int 3x^{-2}$$
$$= \frac{3}{-2+1} x^{-2+1} + C$$
$$= -\frac{3}{1} x^{-1} + C$$
$$= -\frac{3}{x} + C$$

Example 5

Evaluate $\int 5x^5 dx$

$$\int 5x^4 dx = \frac{5}{4+1} x^{4+1} + C = \frac{5}{5} x^5 + C = x^5 + C$$

The anti derivative of the exponential function

$$\int e^x dx = e^x + C$$

Example 1

Evaluate $\int 4e^x dx$

$$\int 4e^x dx = 4e^x + C$$

Example 2

Evaluate $\int 6e^x dx$

$$\int 6e^x dx = 6e^x + C$$

Definition

$$\int e^u \frac{du}{dx} dx = \int e^u du = e^u + C$$

Example 3

Evaluate $\int 2xe^{x^2} dx$

$$\int 2xe^{x^2} dx$$

$Let\ u = x^2$

$du = 2x$

$$\int 2xe^{x^2} = \int du\, e^u = e^u + C = e^{x^2} + C$$

Example 4

Evaluate $\int xe^{2x^2}\,dx$

$$\int xe^{2x^2}\,dx$$

$$Let\ u = 2x^2$$

$$du = 4x \Rightarrow \frac{du}{4} = x$$

$$\int xe^{2x^2} = \int \frac{du}{4} e^u = \frac{1}{4} e^u + C = \frac{1}{4} e^{x^2} + C$$

The Antiderivative of the Natural Logarithmic Function

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Example 5

Evaluate $\int \frac{3}{x}\,dx$

$$\int \frac{3}{x}\,dx = 3\ln|x| + C$$

Definition

$$\int \frac{\frac{du}{dx}}{u}\,dx = \int \frac{du}{u} = \ln|u| + C$$

Example 6

$$\int \frac{x}{x^2} dx$$

$$\int \frac{x}{x^2} dx$$

$u = x^2$

$du = 2x$

$\frac{du}{2} = x$

$$\int \frac{x}{x^2} dx = \int \frac{\frac{du}{2}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2| + C$$

Definite Integers

The Fundamental Theorem of Calculus

Let $f(x)$ be a function whose anti derivative is $F(x)$, then

$$\int_b^a f(x) = F(a) - F(b)$$

Examples of Definite Integrals

Example 7

$$\int_1^3 3x^2 dx$$

$$\int_1^3 3x^2 dx = \frac{3}{2+1} x^{2+1} \Big|_1^3 = \frac{3}{3} x^3 \Big|_1^3 = x^3 \Big|_1^3 = 3^3 - 1^2 = 9 - 1 = 8$$

Example 8

$$\int_1^2 x^3 dx$$

$$\int_1^2 x^3 dx = \frac{1}{3+1} x^{3+1} \Big|_1^2 = \frac{1}{4} x^4 \Big|_1^2 = \frac{1}{4} 2^4 - \frac{1}{4} 1^4 = \frac{1}{4} 16 - \frac{1}{4} 1 = 4 - \frac{1}{4} = \frac{15}{4}$$

Example 9

$$\int_1^2 e^x dx$$

$$\int_1^2 e^x dx = e^x \Big|_1^2 = e^2 - e^1$$

Example 10

$$\int_1^{10} \frac{1}{x} dx$$

$$\int_1^{10} \frac{1}{x} dx = \ln(x) \Big|_1^{10} = \ln(10) - \ln(1) = \ln(10) - 0 = \ln(10)$$

Example 11

$$\int_1^2 5x^4 dx$$

$$\int_0^2 5x^4 dx = \frac{5}{4+1} x^{4+1} \Big|_0^2 = \frac{5}{5} x^5 \Big|_0^2 = x^5 \Big|_0^2 = 2^5 - 0^5 = 32 - 0 = 32$$