Math 126
Test 1 Review

1) Find the distance between the points (-3,3) and (5,7)

Solution:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(5-(-3))^2 + (7-3)^2}$$
$$d = \sqrt{8^2 + 4^2}$$
$$d = \sqrt{64+16}$$
$$d = \sqrt{80} \approx 8.94$$

2) Find the slope of a line passing through (3,2) and (2,5)

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{2-3} = \frac{3}{-1} = -3$$

3) Find the equation of line in slope-intercept form, given it has a slope of $\frac{3}{4}$ and a y-intercept of 2.

Solution:

$$m = \frac{3}{4}$$
$$b = 2$$
$$y = \frac{3}{4}x + 2$$

4) Find the equation of line in slope-intercept form given that it passes through the points (1,-1) and (3,3).

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{3-1} = \frac{4}{2} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 2(x-1)$$
$$y + 1 = 2x - 2$$
$$y + 1 - 1 = 2x - 2 - 1$$
$$y = 2x - 3$$

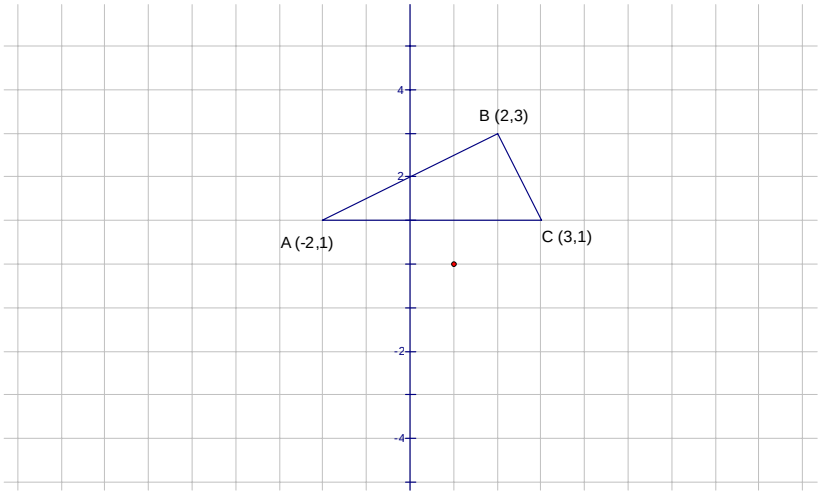
5) Find the slope and y-intercept of line with the given equation: $2x+3y=6$.

Solution:

$$2x+3y=6$$
$$2x-2x+3y=-2x+6$$
$$3y=-2x+6$$
$$\frac{3y}{3}=\frac{-2x}{3}+\frac{6}{3}$$
$$y=-\frac{2}{3}x+2$$

6) Show that ΔABC is a right triangle, given that $A=(-2,1)$, $B=(2,3)$, and $C=(3,1)$

Solution:



$$d(AB)=\sqrt{(2-(-2))^2+(3-1)^2}=\sqrt{4^2+2^2}=\sqrt{16+4}=\sqrt{20}$$
$$d(AC)=\sqrt{(3-(-2))^2+(1-1)^2}=\sqrt{5^2+0^2}=\sqrt{25}=5$$
$$d(BC)=\sqrt{(3-2)^2+(1-3)^2}=\sqrt{1^2+(-2)^2}=\sqrt{1+4}=\sqrt{5}$$

Check using the Pythagorean Theorem

$$c^2=a^2+b^2$$
$$(5)^2=\left(\sqrt{20}\right)^2+\left(\sqrt{5}\right)^2$$
$$25=20+5$$
$$25=25$$

Since the sides check in the Pythagorean Theorem, the triangle is a right triangle.

7) Find the inverse of $f(x) = 2x - 3$

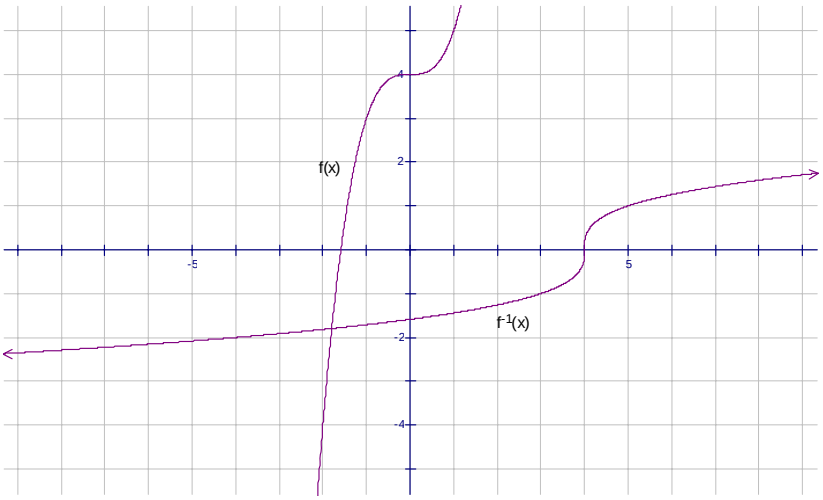
Solution:

$$f(x) = 2x - 3$$
$$y = 2x - 3$$
$$x = 2y - 3$$
$$x + 3 = 2y - 3 + 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = \frac{2y}{2}$$
$$y = \frac{x+3}{2}$$
$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$

8) Find the inverse of $f(x) = x^3 + 4$, and then graph $f(x)$ and $f'(x)$

Solution:

$$f(x) = x^3 + 4$$
$$y = x^3 + 4$$
$$x = y^3 + 4$$
$$x - 4 = y^3$$
$$\sqrt[3]{x-4} = \sqrt[3]{y^3}$$
$$\sqrt[3]{x-4} = y \Rightarrow f^{-1}(x) = \sqrt[3]{x-4}$$



9) Find $\lim_{x\to 2} x^3 + 2x$

Solution:

$$\lim_{x\to 2} x^3 + 2x = 2^3 + 2(2) = 8 + 4 = 12$$

10) Find $\lim_{x\to 3} \frac{x}{x-3}$

Solution:

$$\lim_{x\to 3} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0} \quad (\textit{Limit is undefined})$$

11) Find $\lim_{x\to 5} \frac{x-5}{x^2-25}$

Solution:

$$\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x\to 5} \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$$

12) Find $\lim_{x\to 5} \sqrt{x-8}$

Solution:

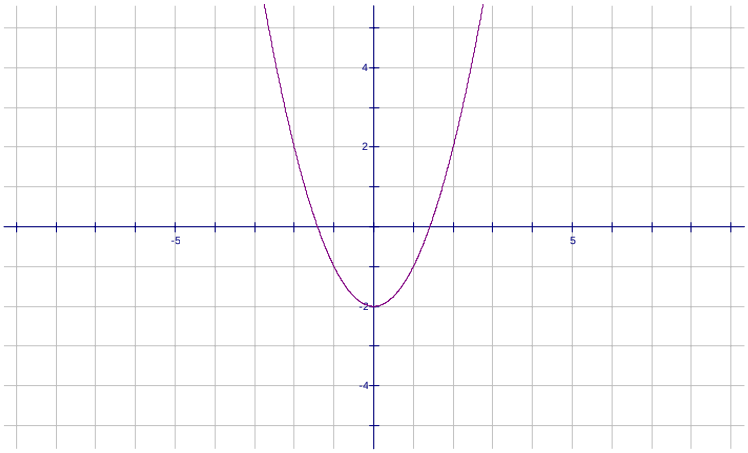
$$\lim_{x\to 5} \sqrt{x-8} = \sqrt{5-8} = \sqrt{-3}$$ **Limit does not exist**

13) Find $\lim_{x\to 2} \frac{x^2-4x+4}{x-2}$

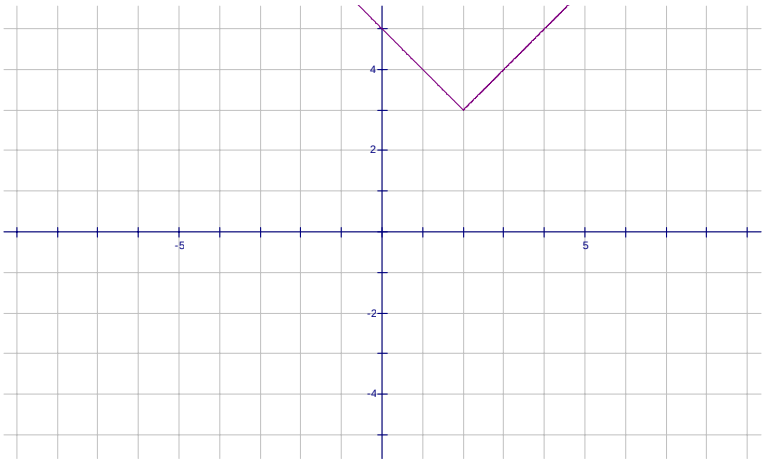
Solution:

$$\lim_{x\to 2} \frac{x^2-4x+4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x-2)}{x-2} = \lim_{x\to 2} x-2 = 2-2 = 0$$

14) Graph $y = x^2 - 2$



15) Graph $y = |x - 2| + 3$



16) Factor $x^2 - 49$

Solution:

$x^2 - 49$

$(x - 7)(x + 7)$

17) Factor $x^2 - 7x - 8$

Solution:

$x^2 - 7x - 8$

$(x - 8)(x + 1)$

18) Factor $x^3 - 8$

Solution:

$x^3 - 8$

$x^3 - 2^3$

$(x-2)(x^2 + 2x + 2^2)$

$(x-2)(x^2 + 2x + 4)$

19) Simplify $(3x^2)^4$

Solution:

$(3x^2)^4$

$3^4 x^8$

$81x^8$

20) Simplify $(2x^3)(4x^6)$

Solution:

$(2x^3)(4x^6)$

$2 \cdot 4x^{3+6}$

$8x^9$