Math 126

Section 9.9

Business Functions

Cost Function - C(x)

Revenue Function – R(x)

Profit Function – P(x)

$P(x) = R(x) - C(x)$

$R(x) = xp$

Marginal Measures

Marginal Cost - $C'(x)$

Marginal Revenue - $R'(x)$

Marginal Profit - $P'(x)$

Example 1

Find the marginal cost function of the cost function $C = 4500 + 1.47x$

$C(x) = 4500 + 1.47x$
$C'(x) = 1.47$

Example 2

Find the marginal cost function of the cost function $C = 55000 + 470x - 0.25x^2$

$C(x) = 55000 + 470x - 0.25x^2$
$C'(x) = 470 - .5x$

Example 3

Find the marginal revenue function given $R = 50x - .5x^2$

$$R(x) = 50x - .5x^2$$
$$R'(x) = 50 - 1.0x$$

Example 4

Find the marginal revenue function given $R = -6x^3 + 8x^2 + 200x$

$$R(x) = -6x^3 + 8x^2 + 200x$$
$$R'(x) = -18x^2 + 16x + 200$$

Example 5

Find the marginal profit of the function $P(x) - 2x^2 + 72x - 145$

$$P(x) = -2x^2 + 72x - 145$$
$$P'(x) = -4x + 72$$

Example 6

Suppose that the total revenue of a commodity is $R(x) = 25x - 0.05x^2$

1) Find $R(50)$ and tell what it represents.

$$R(50) = 25(50) - 0.05(50)^2 = 1500 - 0.05(2500) = 1500 - 125 = \$1375.00$$

This value represents the profit for sells 50 units.

2) Find the marginal revenue function.

$$R(x) = 25x - 0.05x^2$$
$$R'(x) = 25 - 2(.05)x^{2-1}$$
$$R'(x) = 25 - .10x$$

3) Find the marginal revenue at $x = 50$ and tell what it predicts.

$$R'(x) = 25 - .10x$$
$$R'(50) = 25 - .10(50) = 25 - 5 = 20$$

This value predicts the revenue earns for selling the 50th unit.

4) Find $R(51) - R(50)$ and explain what this value represents.

$$\begin{aligned} R(51) - R(50) &= \left(25(51) - .05(51)^2\right) - \left(25(50) - .05(50)^2\right) \\ &= (\$1275 - \$130.05) - (\$1250 - \$125) \\ &= \$1144.95 - \$1125 \\ &= \$19.95 \end{aligned}$$

The value represents the profit for selling the 50th unit.

Definition: Break Even Point

The value of x (number of units sold) where cost equals revenue

$$P(x) = C(x)$$

Example 7

Find the break even point for the given cost function and revenue function.

$$C(x) = 6x + 50{,}000$$
$$R(x) = 35x$$
$$35x = 6x + 50000$$
$$35x - 6x = 6x - 6x + 50000$$
$$29x = 50000$$
$$\frac{29x}{29} = \frac{50000}{29}$$
$$x = 1724.14$$
$$x = 1725 \ \textit{units}$$

Example 8

Find the break even point for the given cost function and revenue function.

$$C(x) = 4x + 10000$$
$$R(x) = 20x$$
$$20x = 4x + 10000$$
$$20x - 4x = 4x - 4x + 10000$$
$$16x = 10000$$
$$\frac{16x}{16} = \frac{10000}{16}$$
$$x = 625 \; units$$

Example 9

A manufacture's total cost is $C(x) = 0.1x^3 - .05x^2 + 500x + 200$ dollars when the level production is x units. Find the marginal cost for producing 40 units.

$$C(x) = 0.1x^3 - .05x^2 + 500x + 200$$
$$C'(x) = .3x^{3-1} - .10x^{2-1} + 500$$
$$C(x) = .3x^2 - .1x + 500$$
$$Let \; x = 40$$
$$Find \; C'(40) = .3(40)^2 - .1(40) + 500 = 480 - 4 + 500 = \$976.00$$

Example 10

Suppose that the total revenue in dollars of manufacturing t units is given by $R(t) = 240t - 0.05t^2$ dollars when t units are produced and sold during the month. Currently, the manufacture is producing 80 units per month and is planning to increase production by 81 units. Find the marginal revenue for producing 81 units or the increase in revenue for increasing from 80 units to 81 units.

$$R(t) = 240t - 0.05t^2$$
$$R'(t) = 240 - .10t$$
$$R'(81) = 240 - .10(81) = 240 - 8.1 = \$241.9$$

Example 11

Suppose that in a monopoly market, the demand function for a product is given by $p = 160 - .02x$ where x is the number of units and p is the price in dollars.

a) Find the total revenue from the sale of 70 units.

$$R(x) = xp$$
$$R(x) = x(160 - .02x)$$
$$R(x) = 160x - .02x^2$$

Revenue for 70 units sold.

$$R(70) = 160(70) - .02(70)^2$$
$$R(70) = 1120 - .02(4900)$$
$$R(70) = 1120 - 98 = \$1022.00$$

b) Find the marginal revenue at 400 units

$$R'(x) = 160 - .04x$$
$$R'(70) = 160 - .04(70)$$
$$R'(70) = 160 - 2.8$$
$$R'(70) = \$157.20$$

Example 12

Suppose that in a monopoly market, the demand function for a product is given by $p = 200 - .05x$ where x is the number of units and p is the price in dollars.

a) Find the total revenue from the sale of 50 units.

$$R(x) = xp$$
$$R(x) = x(200 - .05x)$$
$$R(x) = 200x - .05x^2$$

Revenue for 50 units sold.

$$R(70) = 200(50) - .05(50)^2$$
$$R(70) = 1000 - .05(2500)$$
$$R(70) = 1000 - 125 = \$875.00$$

b) Find the marginal revenue at 500 units

$$R'(x) = 200 - 2(.05)x^{2-1}$$
$$R'(x) = 200 - .1x$$
$$R'(70) = 200 - .10(500)$$
$$R'(70) = 200 - 50$$
$$R'(70) = \$150.00$$