

Math 126

Section 9.8

Higher-Order Derivatives

First Derivative

Second Derivative

Third Derivative

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2$$

$$f'''(x) = 24x$$

1 st derivative	$f'(x)$	$\frac{dy}{dx}$
2 nd derivative	$f''(x)$	$\frac{d^2 y}{dx^2}$
3 rd derivative	$f'''(x)$	$\frac{d^3 y}{dx^3}$
4 th derivative	$f^{(4)}(x)$	$\frac{d^4 y}{dx^4}$
n th derivative	$f^n(x)$	$\frac{d^n y}{dx^n}$

Example 1

Given $f(x) = 4x^5 - 3x^2 + 8$, find $f''(x)$

$$f'(x) = 20x^4 - 6x$$

$$f''(x) = 80x^3 - 6$$

Example 2

Given $f(x) = \frac{3}{x^2}$, find $f'''(x)$

$$f(x) = 3x^{-2}$$

$$f'(x) = -2 \cdot 3x^{-2-1} = -6x^{-3}$$

$$f''(x) = 18x^{-3-1}$$

$$f''(x) = 18x^{-4}$$

$$f''(x) = \frac{18}{x^4}$$

Example 3

Given $f(x) = 3x^2 + 4x$, find $f''(x)$

$$f(x) = 3x^2 + 4x$$

$$f'(x) = 6x + 4$$

$$f''(x) = 6$$

Example 4

Given $y = x^5 - 3x^4$, find $f'''(x)$

$$y = x^5 - 3x^4$$

$$y' = 5x^4 - 12x^3$$

$$y'' = 20x^3 - 36x^2$$

$$y''' = 60x^2 - 72x$$

Example 5 Given $f(x) = 9 - x^2$, find $f''(\sqrt{5})$

$$f(x) = 9 - x^2$$

$$f'(x) = -2x$$

$$f''(x) = -2$$

$$f''(\sqrt{5}) = -2$$

Example 6

Given $f(x) = 3x^3 - 9x + 1$, find the second derivative and solve for $f''(x) = 0$

Solve for $f''(x) = 0$

$$f(x) = 3x^3 - 9x + 1$$

$$f'(x) = 9x^2 - 9$$

$$f''(x) = 18x$$

$$18x = 0$$

$$\frac{18x}{18} = \frac{0}{18}$$

$$x = 0$$

Example 7

Given $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 2$, find the second derivative and solve for $f''(x) = 0$

Solve for $f''(x) = 0$

$$f(x) = x^4 - 8x^3 + 18x^2 - 16x + 2$$

$$f'(x) = 4x^3 - 24x^2 + 36x - 16$$

$$f''(x) = 12x^2 - 48x + 36$$

$$12x^2 - 48x + 36 = 0$$

$$12(x^2 - 4x + 3) = 0$$

$$12(x - 3)(x - 1) = 0$$

$$x - 3 = 0 \text{ or } x - 1 = 0$$

$$x = 3 \quad x = 1$$

Example 8

Find $f^{(4)}(x)$ for the function $f(x) = \frac{1}{x^2}$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

$$f''(x) = (-3)(-2)x^{-3-1} = 6x^{-4}$$

$$f'''(x) = (-4)(6)x^{-4-1} = -24x^{-5}$$

$$f^{(4)}(x) = (-5)(-24)x^{-5-1} = 120x^{-6} = \frac{120}{x^6}$$

Example 9

Find $y^{(4)}$ for the function $y = x^5 + 4x^2$

$$y = x^5 + 4x^2$$

$$y' = 5x^4 + 8x$$

$$y'' = 20x^3 + 8$$

$$y''' = 60x^2$$

$$y^{(4)} = 120x$$

Example 10

The equation of motion of a particle is $s(t) = t^3 - 6t$ where s is in meters and t is seconds.

- a) Find the velocity and acceleration as a function of t .

$$v(t) = s'(t) = 3t^{3-1} - 6t^0 = 3t^2 - 6$$

$$a(t) = s''(t) = 6t$$

- b) Find the velocity after 2 seconds.

$$v(2) = 3(2)^2 - 6 = 12 - 6 = 6 \frac{m}{s}$$

- c) Find the acceleration after 2 seconds.

$$a(t) = s''(t) = 6t$$

$$a(2) = 6(2) = 12 \frac{m}{s^2}$$
