

Math 126
Section 9.5

Product and Quotient Rule

The Product Rule

If $f(x) = F(x)S(x)$, then $f'(x) = F'(x)S(x) + S'(x)F(x)$

Example 1

Given $f(x) = (x^2 - 4x)(x^2 - 3x)$, find $f'(x)$

$$\begin{aligned} f'(x) &= (2x - 4)(x^2 - 3x) + (2x - 3)(x^2 - 4x) = 2x^3 - 6x^2 - 4x^2 - 12x + 2x^3 - 8x^2 - 3x^2 + 12x \\ &= 4x^3 - 21x^2 + 24x \end{aligned}$$

Example 2

Given $f(x) = (5x^3 + 4x^2 + 4x)(2x^2 + 6x)$, find $f'(x)$

$$f'(x) = (15x^2 + 8x + 4)(2x^2 + 6x) + (4x + 6)(5x^3 + 4x^2 + 4x)$$

Example 3

Given $f(x) = (6x^3 - 5x + 2)(x^5 + x^4)$, find $f'(x)$

$$\begin{aligned} f'(x) &= (18x^2 - 5)(x^5 + x^4) + (5x^4 + 4x^3)(6x^3 - 5x + 2) \\ f'(x) &= 18x^7 + 18x^6 - 5x^5 - 5x^4 + 30x^7 - 25x^5 + 10x^4 + 24x^6 - 20x^4 + 8x^3 \\ f'(x) &= 48x^7 + 42x^6 - 30x^5 - 25x^4 + 8x^3 \end{aligned}$$

Example 4

Given $y = (3x^4 - 2x^3 + 10x^2 + 7x)(3x^6 + 2x^3)$, find $f'(x)$

$$y' = (12x^3 - 6x^2 + 20x + 7)(3x^6 + 2x^3) + (18x^5 + 6x^2)(3x^4 - 2x^3 + 10x^2 + 7x)$$

Quotient Rule

$$\text{If } f(x) = \frac{T(x)}{B(x)}, \text{ then } f'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{(B(x))^2}$$

Example 5

Given $f(x) = \frac{x^2}{3x^3 - 4x + 5}$, find $f'(x)$

Solution:

$$f'(x) = \frac{(3x^3 - 4x + 5)(2x) - (x^2)(9x^2 - 4)}{(3x^3 - 4x + 5)^2}$$

$$f'(x) = \frac{6x^4 - 8x^2 + 10x - 9x^4 + 4x^2}{(3x^3 - 4x + 5)^2}$$

$$f'(x) = \frac{-3x^4 - 4x^2 + 10x}{(3x^3 - 4x + 5)^2}$$

Find the derivative $\frac{dy}{dx}$

$$y = \frac{x^2 - 5x}{x^2 - 3x + 3}$$

Solution:

$$\frac{dy}{dx} = \frac{(x^2 - 3x + 3)(x^2 - 5x)' - (x^2 - 5x)(x^2 - 3x + 3)'}{(x^2 - 3x + 3)^2}$$

$$\frac{dy}{dx} = \frac{(x^2 - 3x + 3)(2x - 5) - (x^2 - 5x)(2x - 3)}{(x^2 - 3x + 3)^2}$$

Example 7

Find the derivative $\frac{ds}{dx}$

$$s(x) = \frac{4x^2 + 5x}{x^4}$$

Solution:

$$\frac{ds}{dx} = \frac{(x^4)(4x^2 + 5x)' - (4x^2 + 5x)(x^4)'}{(x^4)^2}$$

$$\frac{ds}{dx} = \frac{x^4(8x + 5) - (4x^2 + 5x)(4x^3)}{(x^4)^2}$$

$$\frac{ds}{dx} = \frac{8x^5 + 5x^4 - 16x^5 + 20x^4}{x^8}$$

$$\frac{ds}{dx} = \frac{-8x^5 + 25x^4}{x^8}$$

Example 8

Find the slope of a tangent line to the curve given by the function $f(x) = (x^2 + 1)(2x + 5)$ through the point $(-1, 6)$

First find the derivative of the function: $f'(x) = (2x)(2x + 5) + (2)(x^2 + 1)$

Next, find the value of the derivative at $x = -1$

$$f'(-1) = (2(-1))(2(-1) + 5) + 2((-1)^2 + 1) = (-2)(3) + 2(1 + 1) = -6 + 2(2) = -6 + 4 = -2$$

$$m = -1$$

Example 9

Find the slope of a tangent line to the curve given by the function $f(x) = \frac{x^2}{x+3}$ through the point $\left(-1, \frac{1}{2}\right)$

First find the derivative of the function: $f(x) = \frac{x^2}{x+3}$

$$f'(x) = \frac{(x+3)(2x) - (x^2)(1)}{(x+3)^2} = \frac{2x^2 + 6x - x^2}{(x+3)^2} = \frac{x^2 + 6x}{(x+3)^2}$$

Find the value of the derivative $x = -1$

$$f'(-1) = \frac{(-1) + 6(-1)}{(-1+3)^2} = \frac{-1-6}{2^2} = -\frac{7}{4}$$

Example 10

Find the derivative of the function: $h(t) = (t^5 - 1)(4t^2 - 7t - 3)$

Solution:

$$h(t) = (t^5 - 1)(4t^2 - 7t - 3)$$

$$h'(t) = (5t^4)(4t^2 - 7t - 3) + (8t - 7)(t^5 - 1)$$

$$h'(t) = 20t^6 - 35t^5 - 15t^4 + 8t^6 - 8t - 7t^5 + 7$$

$$h'(t) = 28t^6 - 35t^5 - 22t^4 - 8t + 7$$

Example 11

Find the derivative of the function: $f(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$

Solution:

$$f(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$$

$$f'(x) = \frac{(x^3 + 3x + 2)(2x) - (x^2 - 1)(3x^2 + 3)}{(x^2 - 1)^2}$$

Example 12

Find the equation of a tangent to the function at the given point.

$$f(x) = (x^3 - 4x)(x^2 + 1): (1, -6)$$

Solution:

$$f(x) = (3x^2 - 4)(x^2 + 1) + (x^3 - 4x)(2x)$$

$$f'(1) = (3(1)^2 - 4)(1^2 + 1) + (1^3 - 4(1))(2(1))$$

$$f'(1) = (3 - 4)(1 + 1) + (1 - 4)(2)$$

$$f'(1) = (-1)(2) + (-3)(2)$$

$$f'(1) = -2 + -6 = -8$$

$$\Rightarrow m = -8$$

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = -8(x - 1)$$

$$y + 6 = -8x + 8$$

$$y = -8x + 2$$

Extra Problems: Find the derivative

1) $f(x) = (x^3 - 3x^2)(x^2 + 2x)$ **Simplify**

2) $f(x) = (x^2 - 4x)(2x^2 - 4x + 6)$ **Do Not Simplify**

3) $h(x) = \frac{x^2 - 3x}{x^2}$ **Do Not Simplify**

4) $f(x) = (x^2 + 7x + 3)(x^3 + 2x + 1)$ **Do Not Simplify**

5) $f(x) = \frac{x^2}{x^3 + 4}$ **Simplify**