

## Math 126

### Section 9.4

#### Derivatives and the slope of a tangent line

##### Review of the Power Rule

$$\text{Given } f(x) = ax^n, f'(x) = nax^{n-1}$$

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##### Example 1

$$\text{Given } f(x) = x^3 + 5x, \text{ find } f'(x)$$

$$f'(x) = 3x^{3-1} + 5$$

$$f'(x) = 3x^2 + 5$$

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##### Example 2

$$\text{Given } f(x) = 2x^3 - 4x^2 - 5x, \text{ find } f'(x)$$

$$f'(x) = 3 \cdot 2x^{3-1} - 4 \cdot 2x^{2-1} - 5 \cdot 1x^{1-1}$$

$$f'(x) = 6x^2 - 8x - 5x^0$$

$$f'(x) = 6x^2 - 8x - 5$$

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##### Example 3

$$\text{Given } f(x) = \frac{1}{x^4}, \text{ find } f'(x)$$

$$f(x) = x^{-4}$$

$$f'(x) = -4x^{-4-1} = -4x^{-5} = \frac{-4}{x^5}$$

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##### Example 4

$$\text{Given } f(x) = \frac{3}{x^3} + \frac{2}{x^4}, \text{ find } f'(x)$$

$$f(x) = 3x^{-3} + 2x^{-4}$$

$$f'(x) = -3 \cdot 3x^{-3-1} - 4 \cdot 2x^{-4-1} = -9x^{-4} - 8x^{-5} = -\frac{9}{x^4} - \frac{8}{x^5}$$

**Example 5**

$$f(x) = \sqrt{x}, \text{ find } f'(x)$$

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

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**Example 6**

$$f(x) = 7\sqrt{x}, \text{ find } f'(x)$$

$$f(x) = 7x^{\frac{1}{2}}$$

$$f'(x) = \frac{7}{2}x^{\frac{1}{2}-1} = \frac{7}{2}x^{-\frac{1}{2}} = \frac{7}{2x^{\frac{1}{2}}} = \frac{7}{2\sqrt{x}}$$

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**Example 7**

Find the equation of the tangent line to the function  $f(x) = x^2 + 2$  at the point  $(1,3)$

$$f'(x) = 2x^{2-1} = 2x$$

$$m = f'(1) = 2(1) = 2$$

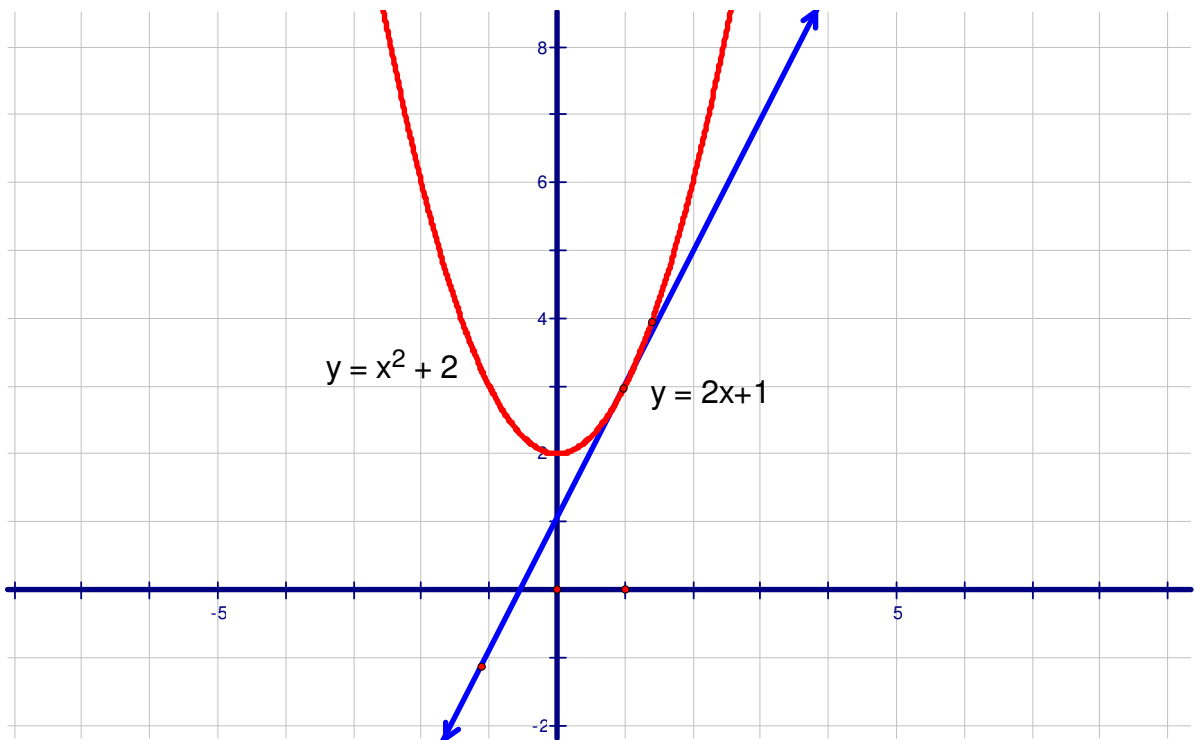
$$y - y_1 = m(x - x_1)$$

$$y - 3 = 2(x - 1)$$

$$y - 3 = 2x - 2$$

$$y - 3 + 3 = 2x - 2 + 3$$

$$y = 2x + 1$$



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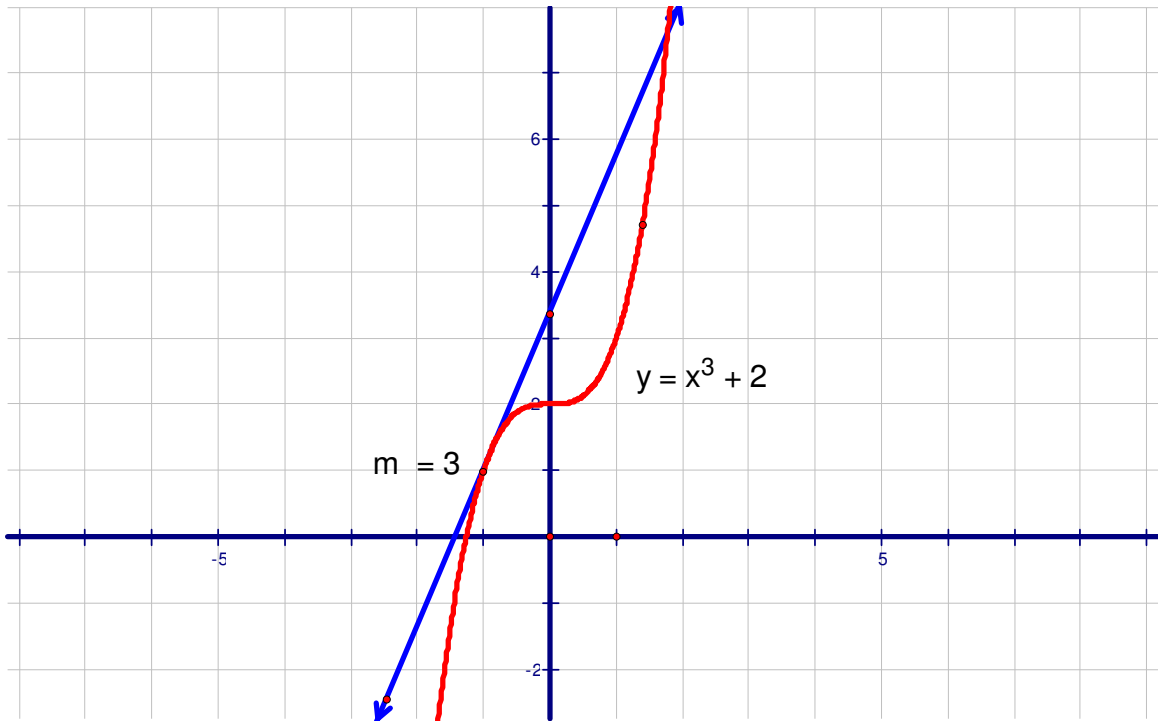
**Example 8**

Find the slope of the tangent line to the function  $f(x) = x^3 + 2$  at the point  $(-1, 1)$

$$f(x) = x^3 + 2$$

$$f'(x) = 3x^{3-2} = 3x^2$$

$$m = f'(-1) = 3(-1)^2 = 3(1) = 3$$



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### Example 9

Find the equation of the tangent line to the function  $f(x) = x^2 - x$  that passes through the point (2,2)

$$f(x) = x^2 - x$$

$$f'(x) = 2x^{2-1} - 1x^{1-1} = 2x - 1$$

$$m = f'(2) = 2(2) - 1 = 4 - 1 = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 3(x - 2)$$

$$y - 2 = 3x - 6$$

$$y - 2 + 2 = 3x - 6 + 2$$

$$y = 3x - 4$$

### Graph of Example 9

