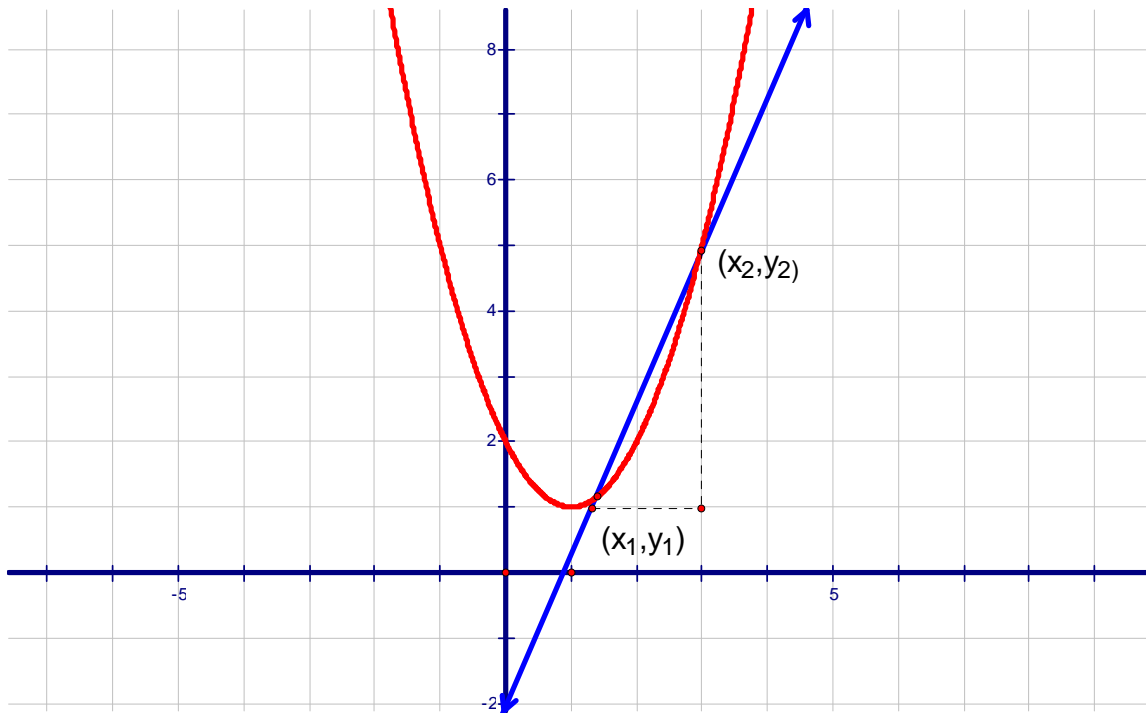


Section 9.3

Derivatives

The derivative is the slope tangent line to the curve

Limit Definition of a Derivative



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \text{slope} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Example 1

Given $f(x) = 3x - 2$, find $f'(x)$ using the limit definition of a derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h) - 2 - (3x - 2)}{h} = \lim_{h \rightarrow 0} \frac{3x + 3h - 2 - 3x + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h} = \lim_{h \rightarrow 0} 3 = 3 \end{aligned}$$

Example 2

Given $f(x) = 5x + 3$, find $f'(x)$ using the limit definition of a derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{5(x+h) + 3 - (5x + 3)}{h} = \lim_{h \rightarrow 0} \frac{5x + 5h + 3 - 5x - 3}{h} = \lim_{h \rightarrow 0} \frac{5h}{h} = \lim_{h \rightarrow 0} 5 = 5$$

Example 3

Given $f(x) = x^2 - 2$, find $f'(x)$ using the limit definition of a derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2 - (x^2 - 2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)(x+h) - 2 - x^2 + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + xh + xh + h^2 - 2 - x^2 + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h} = \lim_{h \rightarrow 0} 2x + h = 2x + 0 = 2x \end{aligned}$$

Example 4

Given $f(x) = x^2 + 5$, find $f'(x)$ using the limit definition of a derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 5 - (x^2 + 5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)(x+h) + 5 - x^2 - 5}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + xh + xh + h^2 + 5 - x^2 - 5}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h} \\ &= \lim_{h \rightarrow 0} 2x + h \\ &= 2x + 0 \\ &= 2x \end{aligned}$$

Power Rule

Function	Derivative
$f(x) = 3x - 2$	$f'(x) = 3$
$f(x) = x^2 - 2$	$f'(x) = 2x$
$f(x) = x^3 - 3x^2$	$f'(x) = 3x^2 - 6x$
$f(x) = x^4$	$f'(x) = 4x^3$

When looking at the changes between the function and the derivative, it can be observed that the power decreases by a power one and the leading coefficient is multiplied by the exponent. Using this relationship, the power rule for derivative can be developed.

General Power Rule

$$\text{Given } f(x) = ax^n, f'(x) = nax^{n-1}$$

Example 5

$$\text{Given } f(x) = 2x^5, \text{ find } f'(x)$$

$$f'(x) = 5 \cdot 2x^{5-1} = 10x^4$$

Example 6

$$\text{Given } f(x) = 3x^4, \text{ find } f'(x)$$

$$f'(x) = 4 \cdot 3x^{4-1} = 12x^3$$

Example 7

$$\text{Given } f(x) = x^3 + 2x^2 + 3x, \text{ find } f'(x)$$

$$f'(x) = 3x^{3-1} + 2 \cdot 2x + 3x^{1-1} = 3x^2 + 4x + 3$$

Constant Rule for Derivatives

If $f(x) = c$ where c is a constant, then $f'(x) = 0$

Example 8

Given $f(x) = x^4 + 4x^3 + 7x^2 + 6$, find $f'(x)$

$$f'(x) = 4x^{4-1} + 4 \cdot 3x^{3-1} + 7 \cdot 2x^{2-1} + 0 = 4x^3 + 12x^2 + 14x$$

Example 9

Find the derivative of $y = 3x^3 + 3x + 4$

Solution:

$$y = 3x^3 + 3x + 4$$

$$y' = 2 \cdot 3x^{3-1} + 3x^{1-1} + 0$$

$$y' = 6x^2 + 3$$

Example 10

Find the derivative of $y = 6x^2 + 14x$

Solution:

$$y = 6x^2 + 14x$$

$$y' = 2 \cdot 6x^{2-1} + 14x^{1-1}$$

$$y' = 12x + 14$$
