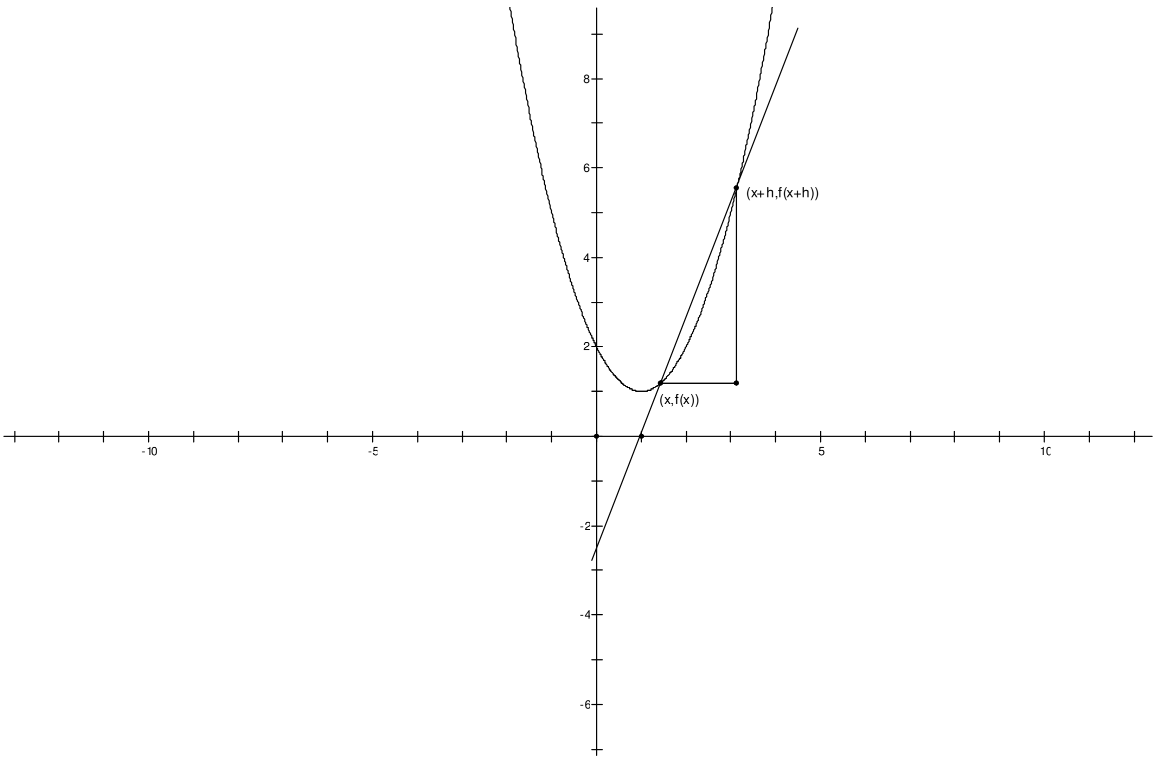
Section 9.3

Derivatives

The derivative is the slope tangent line to the curve

Limit Definition of a Derivative



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = slope = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 1

Given $f(x) = 3x - 2$, *find* $f'(x)$ using the limit definition of a derivative.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)-2-(3x-2)}{h} = \lim_{h\to 0}\frac{3x+3h-2-3x+2}{h}$$
$$= \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = 3$$

Example 2

Given $f(x) = 5x + 3$, *find* $f'(x)$ using the limit definition of a derivative.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{5(x+h)+3-(5x+3)}{h} = \lim_{h\to 0}\frac{5x+5h+3-5x-3}{h} = \lim_{h\to 0}\frac{5h}{h} = \lim_{h\to 0} 5 = 5$$

Example 3

Given $f(x) = x^2 - 2$, *find* $f'(x)$ using the limit definition of a derivative.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)^2-2-(x^2-2)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)(x+h)-2-x^2+2}{h}$$
$$= \lim_{h\to 0}\frac{x^2+xh+xh+h^2-2-x^2+2}{h}$$
$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-2-x^2+2}{h}$$
$$= \lim_{h\to 0}\frac{2xh+h^2}{h}$$
$$= \lim_{h\to 0}\frac{h(2x+h)}{h} = \lim_{h\to 0} 2x+h = 2x+0 = 2x$$

Example 4

Given $f(x) = x^2 + 5$, *find* $f'(x)$ using the limit definition of a derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 5 - (x^2 + 5)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+h) + 5 - x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + xh + xh + h^2 + 5 - x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x + 0$$

$$= 2x$$

Power Rule

Function	Derivative
$f(x) = 3x - 2$	$f'(x) = 3$
$f(x) = x^2 - 2$	$f'(x) = 2x$
$f(x) = x^3 - 3x^2$	$f'(x) = 3x^2 - 6x$
$f(x) = x^4$	$f(x) = 4x^3$

When looking at the changes between the function and the derivative, it can be observed that the power decreases by a power one and the leading coefficient is multiplied by the exponent. Using this relationship, the power rule for derivative can be developed.

General Power Rule

$$Given\ f(x) = ax^n,\ f'(x) = nax^{n-1}$$

Example 5

$$Given\ f(x) = 2x^5,\ find\ f'(x)$$
$$f'(x) = 5 \cdot 2x^{5-1} = 10x^4$$

Example 6

$$Given\ f(x) = 3x^4,\ find\ f'(x)$$
$$f'(x) = 4 \cdot 3x^{4-1} = 12x^3$$

Example 7

$$Given\ f(x) = x^3 + 2x^2 + 3x,\ find\ f'(x)$$
$$f'(x) = 3x^{3-1} + 2 \cdot 2x + 3x^{1-1} = 3x^2 + 4x + 3$$

Constant Rule for Derivatives

If $f(x) = c$ where c is a constant, then $f'(x) = 0$

Example 8

Given $f(x) = x^4 + 4x^3 + 7x^2 + 6,$ *find* $f'(x)$

$f'(x) = 4x^{4-1} + 4 \cdot 3x^{3-1} + 7 \cdot 2x^{2-1} + 0 = 4x^3 + 12x^2 + 14x$