Section 9.1

Limits

Example of limits

Find the limit: $\lim_{x\to 1} x^2 + 1$

Find the limit of the function as x approaches 1

x	.9	.99	.999	1.1	1.01	1.001
$f(x)$	1.810	1.980	1.998	2.210	2.020	2.002

The values in the table seem to approach 2 as x approaches 1. There the limit of function as x approaches 1 is 2. This is written symbolically as $\lim_{x\to 1} x^2 + 1 = 2$

Definition 1: $\lim_{x\to c} f(x) = f(c)$

Thus the limit in the above example can be found by using definition 1,
$\lim_{x\to 1} f(x) = f(1) = 1^2 + 1 = 1 + 1 = 2$

Example 1

Find $\lim_{x\to 3} x^3$

Solution: $\lim_{x\to 3} x^3 = 3^3 = 27$

Example 2

Find $\lim_{x\to -4} 3x^2 + 8x$

Solution: $\lim_{x\to -4} 3x^2 + 8x = 3(-4)^2 + 8(-4) = 3(16) - 32 = 48 - 32 = 16$

Example 3

Find $\lim_{x\to 1}\frac{3}{x-1}$

Solution: $\lim_{x\to 1}\frac{3}{x-1}=\frac{3}{1-1}=\frac{3}{0}$ *Undefined*

Limit does not exist

Example 4

Find $\lim_{x\to 5}\sqrt{x-7}$

Solution: $\lim_{x\to 5}\sqrt{x-7}=\sqrt{5-7}=\sqrt{-2}$ *Undefined*

Limit does not exist

Example 5

Find $\lim_{x\to 3}\frac{x-3}{x^2-9}$

Solution: $\lim_{x\to 3}\frac{x-3}{x^2-9}=\lim_{x\to 3}\frac{x-3}{(x-3)(x+3)}=\lim_{x\to 3}\frac{1}{x+3}=\frac{1}{3+3}=\frac{1}{6}$

In example 5, if you substitute 3 in for x before you reduce the polynomial down, you will get an undetermined form. $\frac{0}{0}$

Example 6

Find $\lim_{x\to 2}\dfrac{2-x}{x^2-4}$

Solution: $\lim_{x\to 2}\dfrac{2-x}{x^2-4}=\lim_{x\to 3}\dfrac{-(x-2)}{(x-2)(x+2)}=\lim_{x\to 3}\dfrac{-1}{x+2}=\dfrac{-1}{2+2}=-\dfrac{1}{4}$

Example 7

Find $\lim_{t\to 1}\dfrac{t^2+t-2}{t^2-1}$

Solution: $\lim_{t\to 1}\dfrac{t^2+t-2}{t^2-1}=\lim_{t\to 1}\dfrac{(t+2)(t-1)}{(t-1)(t+1)}=\lim_{t\to 3}\dfrac{t+2}{t+1}=\dfrac{1+2}{1+1}=\dfrac{3}{2}$

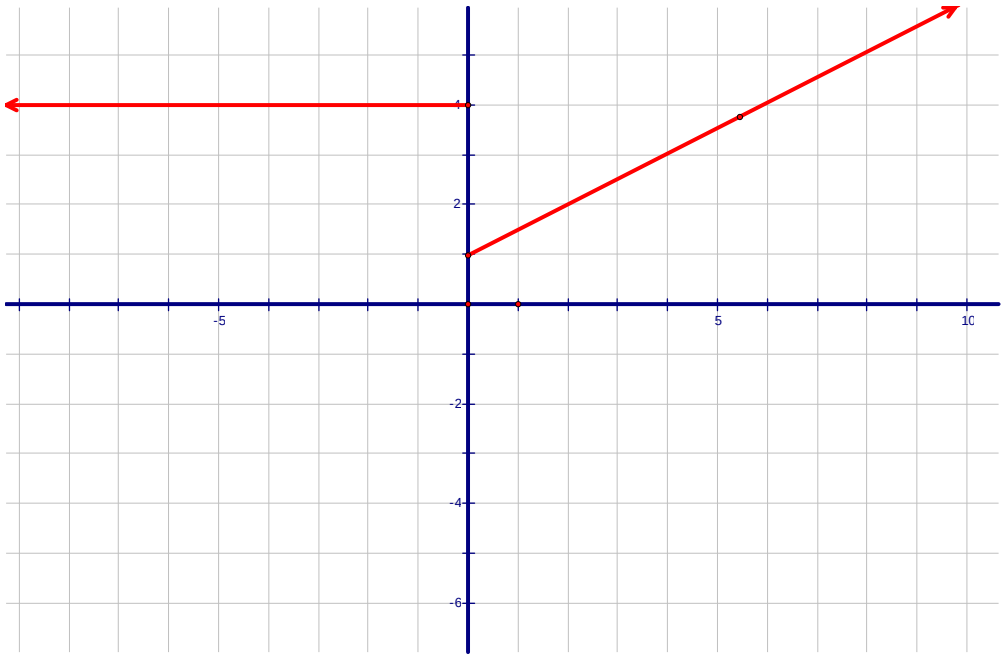
Example 8

Find $\lim_{x\to -1}\dfrac{4x-5}{3-x}$

Solution: $\lim_{x\to -1}\dfrac{4x-5}{3-x}=\dfrac{4(-1)-5}{3-(-1)}=\dfrac{-4-5}{4}=-\dfrac{9}{4}$

Evaluating a limit from a graph

Example 9



a) Find $\lim_{x\to 2} f(x)$ **Solution:** $\lim_{x\to 2} f(x) = 2$

b) Find $\lim_{x\to -3} f(x)$ **Solution:** $\lim_{x\to -3} f(x) = 4$

c) Find $\lim_{x\to 0^+} f(x)$

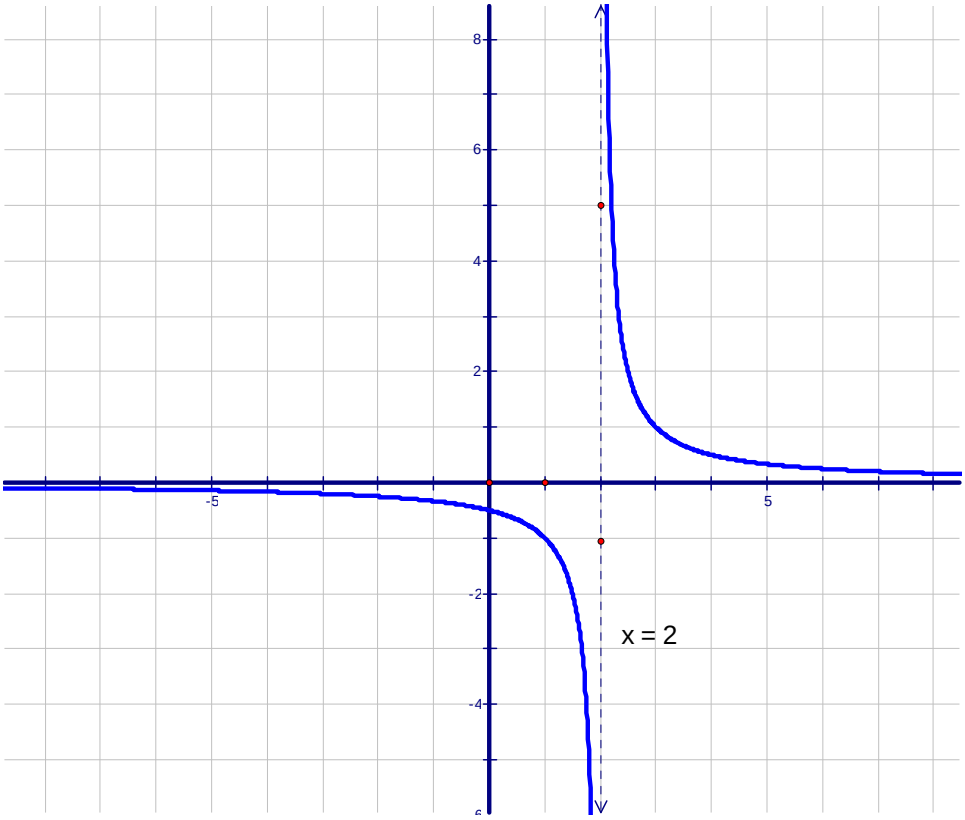
Solution $\lim_{x\to 0^+} f(x) = 1$ **Left hand limit (The y value x approaches from the left)**

d) Find $\lim_{x\to 0^-} f(x)$

Solution: $\lim_{x\to 0^-} f(x) = 4$ **Right hand limit (The y value x approaches from the right)**

e) Find $\lim_{x\to 0} f(x)$: *Does Not Exist* **The limit doesn't exist since the right hand and the left hand limits are different.**

Example 10



a) Find $\lim_{x \to 2} f(x)$:

Solution: No Limit (Limit does not exist)

b) Find $\lim_{x \to 0} f(x)$:

Solution: $\lim_{x \to 0} f(x) = \frac{1}{2}$

c) Find $\lim_{x \to 3} f(x)$

Solution: $\lim_{x \to 3} f(x) = 1$