**Section 9.1**

**Limits**

Example of limits

Find the limit: $\lim_{x\to 1} x^2 + 1$

Find the limit of the function as x approaches 1

| x | .9 | .99 | .999 | 1.1 | 1.01 | 1.001 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1.810 | 1.980 | 1.998 | 2.210 | 2.020 | 2.002 |

The values in the table seem to approach 2 as x approaches 1. There the limit of function as x approaches 1 is 2. This is written symbolically as $\lim_{x\to 1} x^2 + 1 = 2$

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**Definition 1:** $\lim_{x\to c} f(x) = f(c)$

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Thus the limit in the above example can be found by using definition 1,
$\lim_{x\to 1} f(x) = f(1) = 1^2 + 1 = 1 + 1 = 2$

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**Example 1**

Find $\lim_{x\to 3} x^3$

**Solution:** $\lim_{x\to 3} x^3 = 3^3 = 27$

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**Example 2**

Find $\lim_{x\to -4} 3x^2 + 8x$

**Solution:** $\lim_{x\to -4} 3x^2 + 8x = 3(-4)^2 + 8(-4) = 3(16) - 32 = 48 - 32 = 16$

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**Example 3**

Find $\lim_{x\to 1}\frac{3}{x-1}$

**Solution:** $\lim_{x\to 1}\frac{3}{x-1}=\frac{3}{1-1}=\frac{3}{0}$ *Undefined*

**Limit does not exist**

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**Example 4**

Find $\lim_{x\to 5}\sqrt{x-7}$

**Solution:** $\lim_{x\to 5}\sqrt{x-7}=\sqrt{5-7}=\sqrt{-2}$ *Undefined*

**Limit does not exist**

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**Example 5**

Find $\lim_{x\to 3}\frac{x-3}{x^2-9}$

**Solution:** $\lim_{x\to 3}\frac{x-3}{x^2-9}=\lim_{x\to 3}\frac{x-3}{(x-3)(x+3)}=\lim_{x\to 3}\frac{1}{x+3}=\frac{1}{3+3}=\frac{1}{6}$

**In example 5, if you substitute 3 in for x before you reduce the polynomial down, you will get an undetermined form.** $\frac{0}{0}$

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**Example 6**

Find $\lim_{x\to 2}\dfrac{2-x}{x^2-4}$

**Solution:** $\lim_{x\to 2}\dfrac{2-x}{x^2-4}=\lim_{x\to 3}\dfrac{-(x-2)}{(x-2)(x+2)}=\lim_{x\to 3}\dfrac{-1}{x+2}=\dfrac{-1}{2+2}=-\dfrac{1}{4}$

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**Example 7**

Find $\lim_{t\to 1}\dfrac{t^2+t-2}{t^2-1}$

**Solution:** $\lim_{t\to 1}\dfrac{t^2+t-2}{t^2-1}=\lim_{t\to 1}\dfrac{(t+2)(t-1)}{(t-1)(t+1)}=\lim_{t\to 3}\dfrac{t+2}{t+1}=\dfrac{1+2}{1+1}=\dfrac{3}{2}$

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**Example 8**

Find $\lim_{x\to -1}\dfrac{4x-5}{3-x}$

**Solution:** $\lim_{x\to -1}\dfrac{4x-5}{3-x}=\dfrac{4(-1)-5}{3-(-1)}=\dfrac{-4-5}{4}=-\dfrac{9}{4}$

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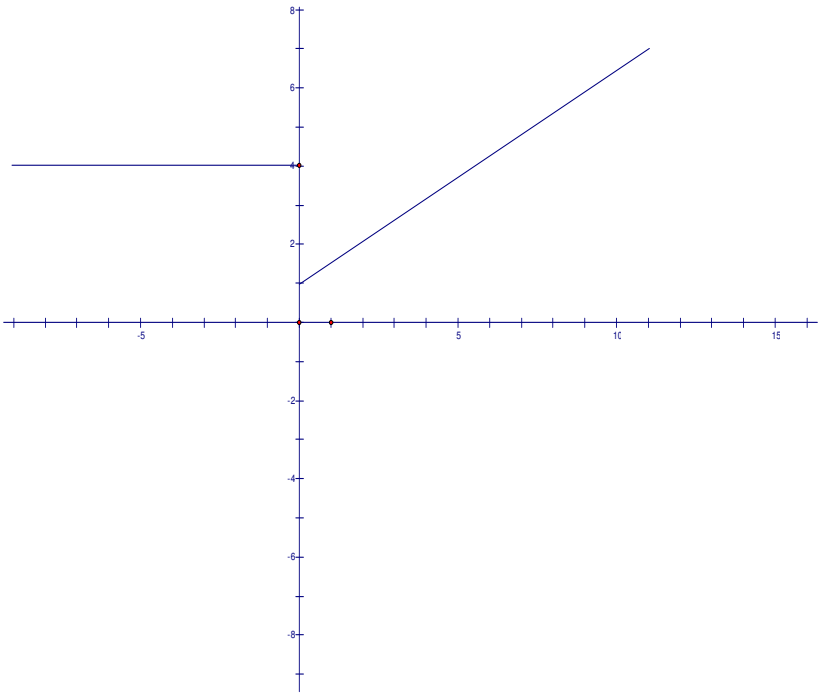
**Evaluating a limit from a graph**

**Example 9**




a) Find $\lim_{x\to 2} f(x)$ **Solution:** $\lim_{x\to 2} f(x) = 2$

b) Find $\lim_{x\to -3} f(x)$ **Solution:** $\lim_{x\to -3} f(x) = 4$

c) Find $\lim_{x\to 0^+} f(x)$

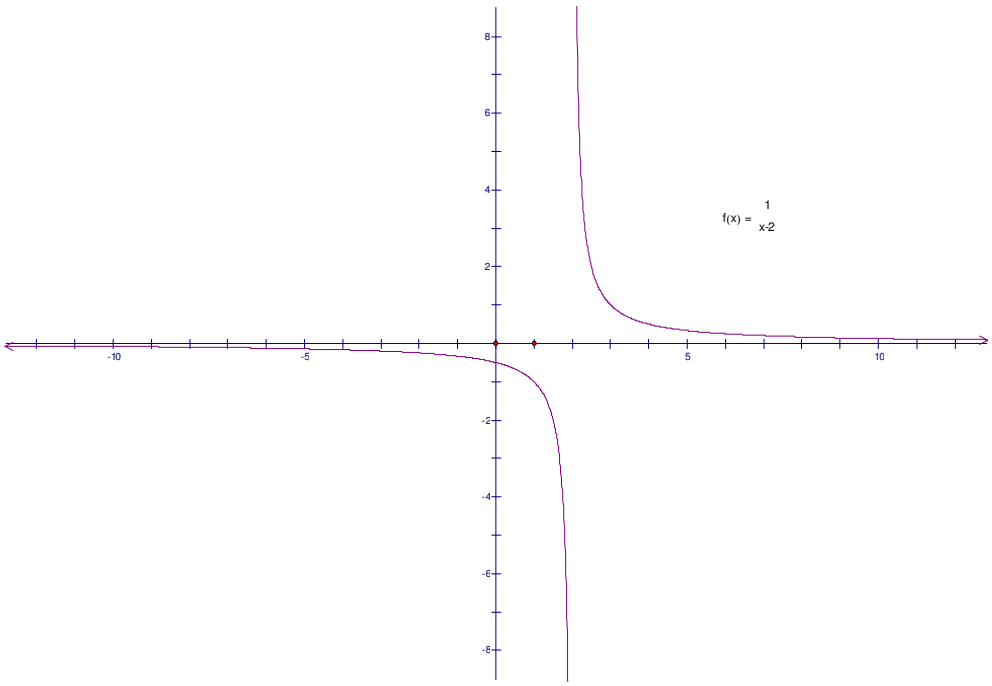
**Solution** $\lim_{x\to 0^+} f(x) = 1$ **Left hand limit (The y value x approaches from the left)**

d) Find $\lim_{x\to 0^-} f(x)$

**Solution:** $\lim_{x\to 0^-} f(x) = 4$ **Right hand limit (The y value x approaches from the right)**

e) Find $\lim_{x\to 0} f(x)$: *Does Not Exist* **The limit doesn't exist since the right hand and the left hand limits are different.**

**Example 10**




a) Find $\lim_{x\to 2} f(x)$:

**Solution:** No Limit (Limit does not exist)

b) Find $\lim_{x\to 0} f(x)$:

**Solution:** $\lim_{x\to 0} f(x) = \frac{1}{2}$

c) Find $\lim_{x\to 3} f(x)$

**Solution:** $\lim_{x\to 3} f(x) = 1$