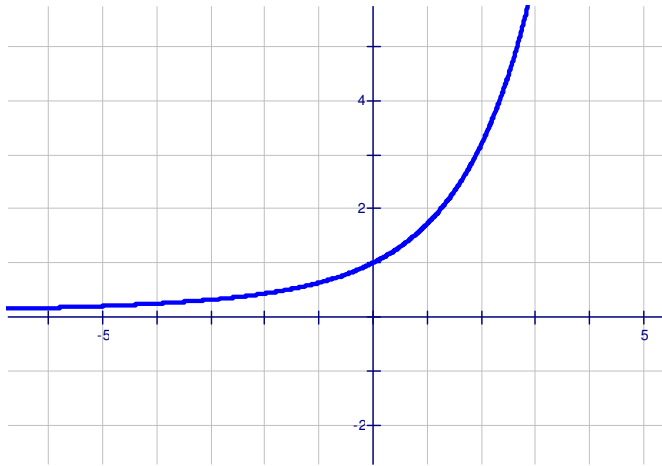


**Math 126**  
**Section 11.2**  
**Derivatives of Exponential Functions**

**The exponential function**

**The definition of the number  $e$**

$e$  is the number such that  $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$



**The derivative of the exponential function**

$$f(x) = e^x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x e^h - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h} = \lim_{h \rightarrow 0} e^x \left( \frac{e^h - 1}{h} \right) = e^x \left[ \lim_{h \rightarrow 0} \left( \frac{e^h - 1}{h} \right) \right] = e^x (1) = e^x \end{aligned}$$

**Definition: Derivative of  $e^x$**

$$\frac{d}{dx}(e^x) = e^x$$

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**Example 1**

Find the derivative of  $f(x) = e^x$

$$f(x) = e^x$$

$$f'(x) = e^x$$

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**Example 2**

Find the derivative of  $y = 5e^x$

$$y = 5e^x$$

$$y' = 5e^x$$

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**The Chain Rule for the Exponential Function**

$$\frac{d}{dx}(e^u) = e^u du = u'e^u$$

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**Example 3**

Find the derivative of  $f(x) = e^{x^2}$

$$f(x) = e^{x^2}$$

$$f(x) = e^u \text{ where } u = x^2 \Rightarrow du = 2x$$

$$f'(x) = e^u du$$

$$f'(x) = e^{x^2} (2x) \text{ or } f'(x) = 2xe^{x^2}$$

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**Example 4**

Find the derivative of  $f(x) = e^{2x^3}$

$$f(x) = e^{2x^3}$$

$$f(x) = e^u \text{ where } u = 2x^3 \Rightarrow du = 6x^2$$

$$f'(x) = e^u du$$

$$f'(x) = e^{3x^3} (6x^2) \text{ or } f'(x) = 6x^2 e^{3x^2}$$

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**Example 5**

Find the derivative of  $f(x) = x^2 e^{2x}$

$$f(x) = x^2 e^{2x}$$

$$f'(x) = (x^2)' e^{2x} + (e^{2x})' (x^2)$$

$$f'(x) = 2x e^{2x} + (2e^{2x}) x^2$$

$$f'(x) = 2x e^{2x} + 2x^2 e^{2x}$$

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**Example 6**

Find the derivative of  $f(x) = x^4 e^{2x^2}$

$$f(x) = x^4 e^{2x^2}$$

$$f'(x) = (x^4)' e^{2x^2} + (e^{2x^2})' (x^4)$$

$$f'(x) = 4x^3 e^{2x^2} + (4x e^{2x}) x^4$$

$$f'(x) = 4x^3 e^{2x^2} + 4x^5 e^{2x^2}$$

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**Example 7**

Find the derivative of  $f(x) = \frac{e^x}{x^2}$

$$f(x) = \frac{e^x}{x^2}$$

$$f'(x) = \frac{(x^2)(e^x)' - e^x(x^2)'}{(x^2)^2} = \frac{x^2 e^x - 2x e^x}{x^4}$$

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**Example 8**

Find the derivative of  $y = 5x^3 e^x$

$$y = 5x^3 e^x$$

$$y' = (5x^3)' e^x + (e^x)' (5x^3)$$

$$y' = 10x^2 e^x + 5x^3 e^x$$

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**Example 9**

Find the derivative of  $y = e^{2x} \ln(x)$

$$y = e^{2x} \ln(x)$$

$$y' = (e^{2x})' \ln(x) + (\ln(x))' (e^{2x})$$

$$y' = 2e^{2x} \ln(x) + \frac{1}{x} (e^{2x})$$

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