

Math 126
Section 11.1
Derivatives of logarithms

The derivative of the natural logarithm

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Example 1

Find the derivative of $f(x) = \ln(x)$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

Example 2

Find the derivative of $y = 4\ln(x)$

$$y = 4\ln(x)$$

$$y' = 4 \cdot \frac{1}{x} = \frac{4}{x}$$

The Chain Rule for the Natural Logarithm

$$y = \ln u$$

$$y' = \frac{1}{u} \cdot du$$

$$y' = \frac{du}{u}$$

Example 3

Find the derivative of $f(x) = \ln(x^3)$

$$f(x) = \ln(x^3)$$

$$f(x) = \ln u \text{ where } u = x^3 \Rightarrow du = 3x^2$$

$$f'(x) = \frac{du}{u}$$

$$f'(x) = \frac{3x^2}{x^3} = \frac{3}{x}$$

Example 4

Find the derivative of $f(x) = \ln(2x^2 + 5x)$

$$f(x) = \ln(2x^2 + 5x)$$

$$\text{Let } f(x) = \ln u \text{ where } u = 2x^2 + 5x \Rightarrow du = 4x + 5$$

$$f'(x) = \frac{du}{u}$$

$$f'(x) = \frac{4x + 5}{2x^2 + 5x}$$

Example 5

Find the derivative of $f(x) = \ln(4x^2)$

$$f(x) = \ln(4x^2)$$

Let $f(x) = \ln u$ where $u = 4x^2 \Rightarrow du = 8x$

$$f'(x) = \frac{du}{u}$$

$$f'(x) = \frac{8x}{4x^2} = \frac{2}{x}$$

Other Derivatives**Example 6**

Find the derivative of $y = x^2 \ln(x)$

$$y = x^2 \ln(x)$$

$$y' = (x^2)' \ln(x) + (\ln(x))' (x^2)$$

$$y' = 2x \ln x + \frac{1}{x} \cdot x^2$$

$$y' = 2x \ln x + x$$

Example 7

Find the derivative of $y = (3x^2 + 4x)\ln(x)$

$$y = (3x^2 + 4x)\ln(x)$$

$$y' = (3x^2 + 4x)' \ln(x) + (\ln(x))' (3x^2 + 4x)$$

$$y' = (6x + 4)\ln x + \frac{1}{x} \cdot (3x^2 + 4x)$$

$$y' = (6x + 4)\ln(x) + 3x + 4$$

Example 8

Find the derivative of $y = 2x^3 \ln(3x)$

$$y = 2x^3 \ln(3x)$$

$$y' = (2x^3)' \ln(3x) + (\ln(3x))' (2x^3)$$

$$y' = 2x^3 \ln(3x) + \frac{3}{3x} \cdot 2x^2$$

$$y' = 2x^3 \ln(3x) + 2x$$

Example 9

Find the Equation to a tangent to the function $y = \ln(2x)$ at $\left(\frac{1}{2}, 0\right)$

$y = \ln(2x)$ and let $y = \ln u$ $u = 2x$ where $du = 2$

$$y' = \frac{du}{u} = \frac{2}{2x} = \frac{1}{x}$$

$$m = \frac{1}{\frac{1}{2}} = 2$$

The equation of the tangent line using the point slope formula

$$y - 0 = 2\left(x - \frac{1}{2}\right)$$

$$y = 2x - 1$$

Example 10

Find the derivative of $y = (\ln(x))^3$

$$y = (\ln(x))^3$$

$$y = u^3 \text{ where } u = \ln(x) \Rightarrow du = \frac{1}{x}$$

$$y' = 3u^2 du$$

$$y' = 3(\ln(x))^2 \left(\frac{1}{x} \right)$$

$$y' = \frac{3}{x} (\ln(x))^2$$