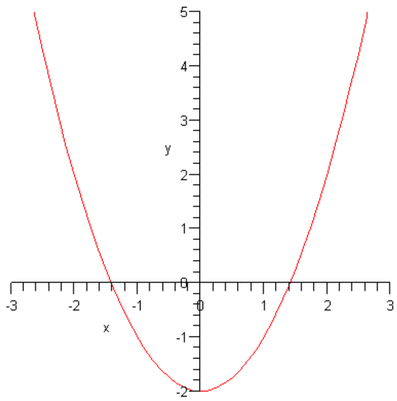
Math 126

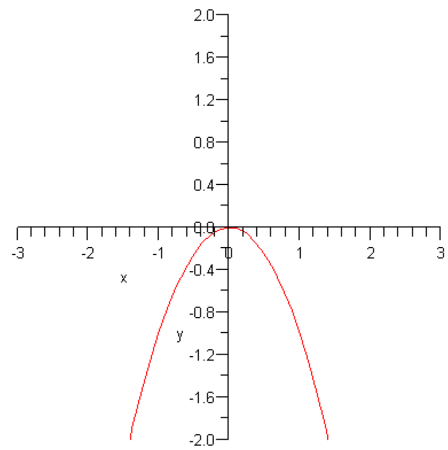
Section 10.1

Introduction to Extrema Points

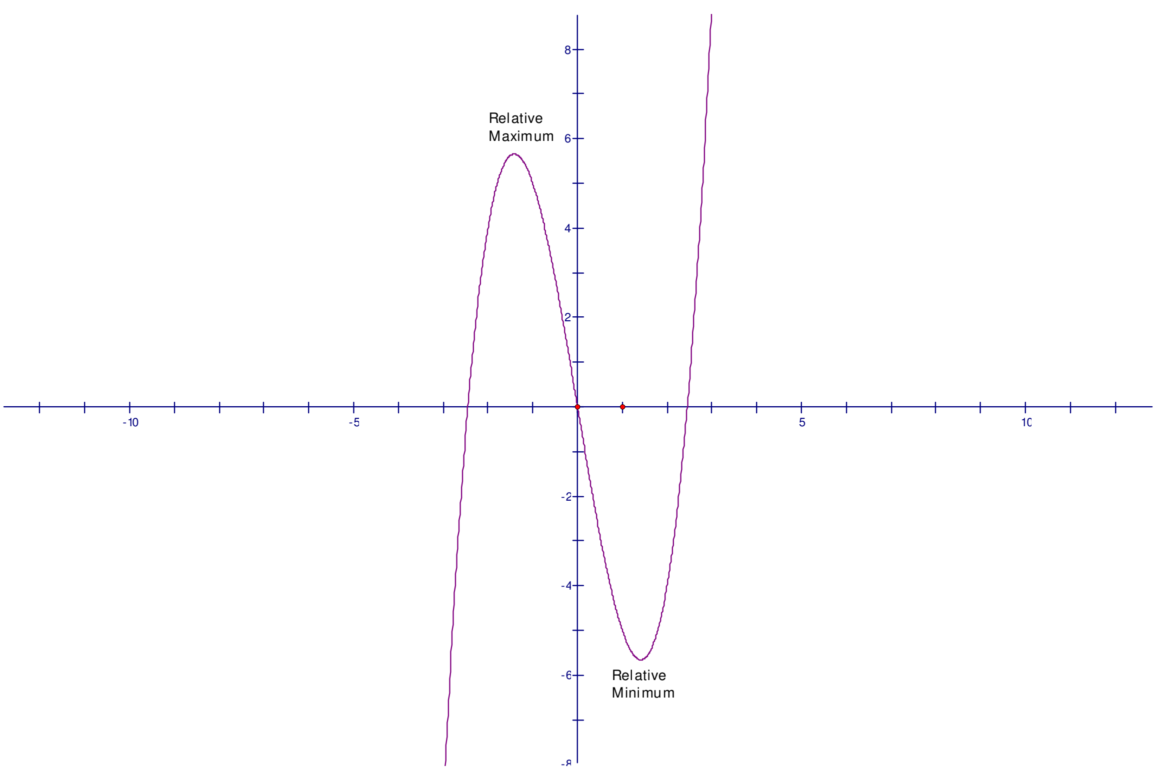
Examples of extrema points



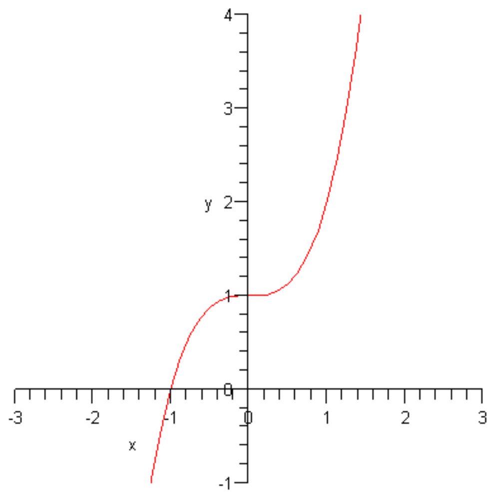
Absolute Minimum



Absolute Maximum



Relative Minimum and Relative Maximum



Inflection Point

Types of Critical Points

1) Relative Maximum
2) Relative Minimum
3) Absolute Maximum
4) Absolute Minimum

Testing for extrema points over all real numbers

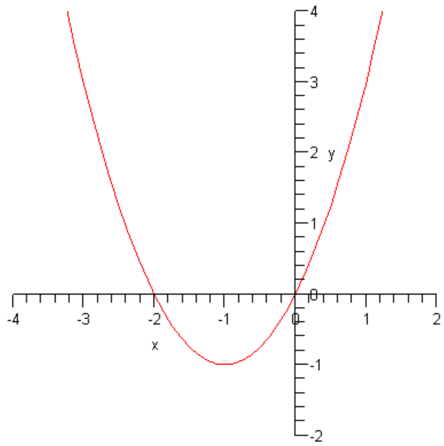
Example 1

Find all extrema points of the function.

$$f(x) = x^2 + 2x$$
$$f'(x) = 2x + 2$$
$$2x + 2 = 0$$
$$2x + 2 - 2 = 0 - 2$$
$$2x = -2$$
$$\frac{2x}{2} = \frac{-2}{2}$$
$$x = -1$$
$$f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$$
$$Critical\ \ pt.\ (-1,-1)$$

$$f'(-2) = 2(-2) + 2 = -2$$
$$f'(1) = 2(1) + 2 = 4$$

Interval	$(-\infty,-1)$	$(-1,\infty)$
Test Value	$x = -2$	$x = 1$
Sign of $f'(x)$	Negative	Positive
Conclusion	Decreasing	Increasing



The function is increasing when x is less than -1 and increasing when x is greater than -1.
$\Rightarrow At\ (-1,-1),\ f\ \ has\ an\ absolute\ \min.$

Example 2

Find all extrema points of the function. $f(x) = x^3 - 1$

$f(x) = x^3 - 1$

$f'(x) = 3x^2$

$3x^2 = 0$

$\frac{3x^2}{3} = \frac{0}{3}$

$x^2 = 0$

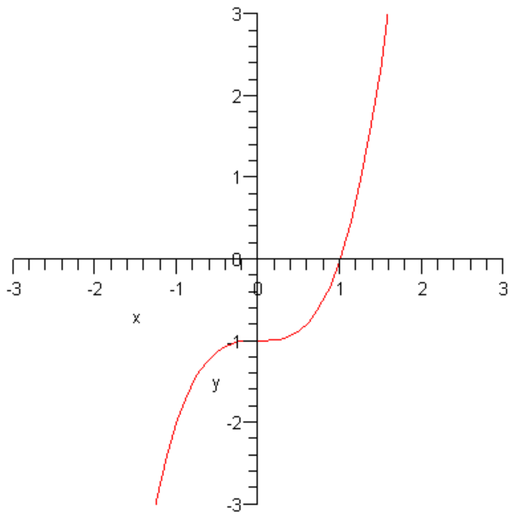
$x = 0$

$\Rightarrow f(0) = 0^3 - 1 = -1 \Rightarrow critical\ \ pt.\ is\ (0,-1)$

$f'(-1) = 3(-1)^2 = 3$

$f'(1) = 3(1)^2 = 3$

Interval	$(-\infty,0)$	$(0,\infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Positive	Positive
Conclusion	Increasing	Increasing



The function is increasing when x is less than zero and greater than zero, so the function has an inflection point at $(0,-1)$. (See graph above)

Example 3

Find all extrema points of the function. $f(x) = 2x^3 - 3x^2$

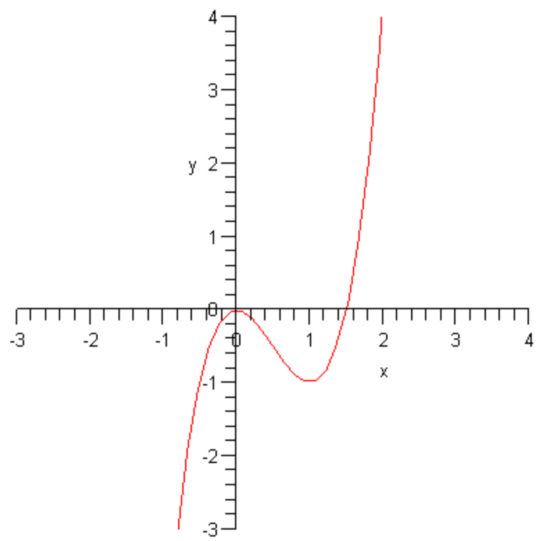
$$f(x) = 2x^3 - 3x^2$$
$$f(x) = 6x^2 - 6x$$
$$6x^2 - 6x = 0$$
$$6x(x-1) = 0$$
$$6x = 0 \text{ or } x - 1 = 0$$
$$\frac{6x}{6} = \frac{0}{6} \qquad x = 1$$
$$x = 0$$

$$f(0) = 2(0)^3 - 3(0)^2 = 0 - 0 = 0$$
$$f(1) = 2(1)^3 - 3(1)^2 = 2 - 3 = -1$$

Testing the derivative

$$f'(-1) = 6(-1)^2 - 6(-1) = 6 + 6 = 12$$
$$f'\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) = \frac{3}{2} - 3 = -\frac{3}{2}$$
$$f'(2) = 6(2)^2 - 6(2) = 24 - 12 = 12$$

Interval	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
Test Value	$x=-1$	$x=\frac{1}{2}$	$x=2$
Sign of $f'(x)$	Positive	Negative	Positive
Conclusion	Increasing	Decreasing	Increasing



The function has a relative maximum at x = 0 and a relative minimum at $(0,0)$ and $(1,-1)$ (See diagram above)

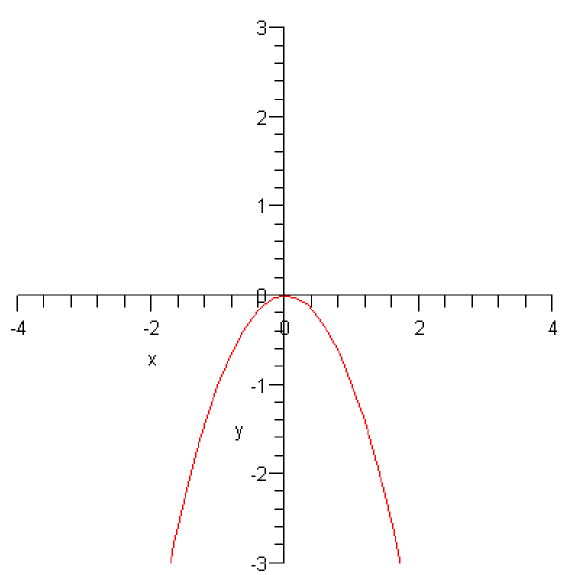
Example 4

Find all extrema points of the function. $f(x) = -x^2$

$$f(x) = -x^2$$
$$f'(x) = -2x$$
$$-2x = 0$$
$$\frac{-2x}{-2} = \frac{0}{-2}$$
$$x = 0$$
$$f'(-1) = -2(-1) = 2$$
$$f'(1) = -2(1) = -2$$

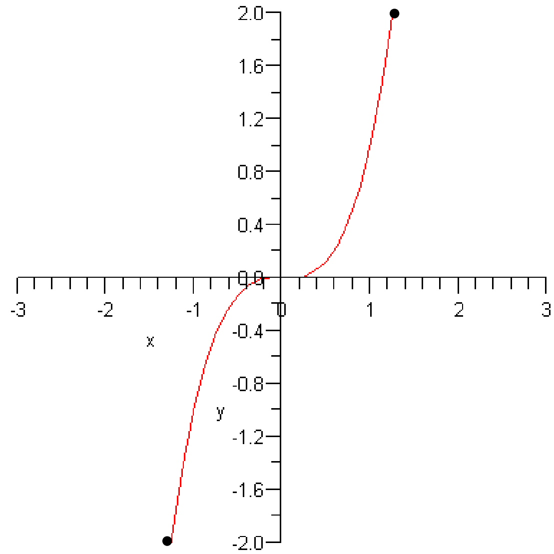
Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Positive	Negative
Conclusion	Increasing	Decreasing

The function has an absolute maximum at x = 0
See Graph

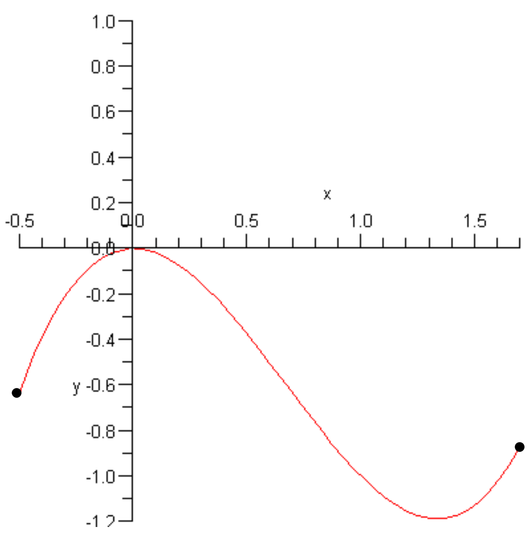


Critical points on a closed interval. The endpoints of the closed interval must be considered as critical points. It turns out that the endpoints of a closed interval can either be relative extrema or absolute extrema.

Here is an example of a graph where one endpoint is an absolute maximum and the other endpoint is an absolute minimum.



This is an example of a graph where one endpoint is a relative maximum and the other endpoint is a relative minimum.



Example 5

Find all extreme points of the function $f(x) = x^2 - 4x$ on the interval [0,4]

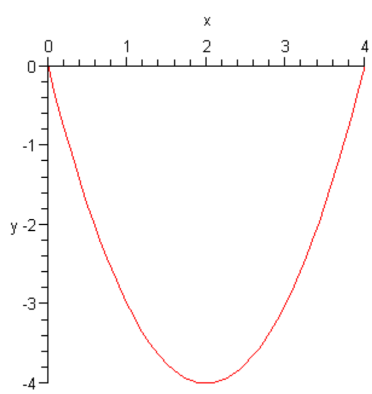
$$f(x) = x^2 - 4x$$
$$f'(x) = 2x - 4$$
$$2x - 4 = 0$$
$$2x - 4 + 4 = 0 + 4$$
$$2x = 4$$
$$x = 2$$

Test the points at $x = 0, x = 2, x = 4$
Find the y-coordinates of these points

$$f(0) = 0^2 - 4(0) - 0 = 0$$
$$f(2) = 2^2 - 4(2) = 4 - 8 = -4$$
$$f(4) = 4^2 - 4(4) = 16 - 16 = 0$$

The critical points are at (0,0),(2,-4),(4,0)

Make a sketch of the graph using these points.



Looking at the graph above there is an absolute minimum at (2,-4) and relative maximums at (0,0) and (4,0).

Example 6

Find all extreme points of the function $f(x) = x^3 + 1$ on the interval [-2,2]

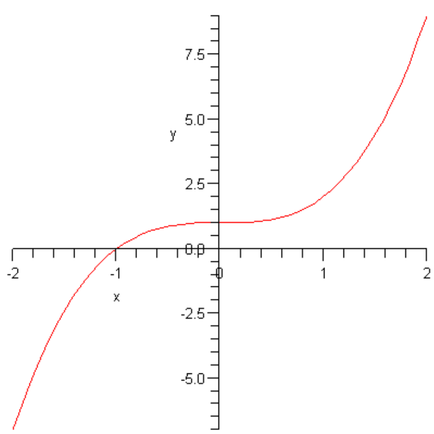
$$f(x) = x^3 + 1$$
$$f'(x) = 3x^2$$
$$3x^2 = 0$$
$$\frac{3x^2}{3} = \frac{0}{3}$$
$$x^2 = 0$$
$$\sqrt{x^2} = \sqrt{0}$$
$$x = 0$$

Test the points at $x = 0, x = -2, x = 2$

$$f(0) = 0^3 + 1 = 0 + 1 = 1$$
$$f(-2) = (-2)^3 + 1 = -8 + 1 = -7$$
$$f(2) = 2^3 + 1 = 9$$

Critical Points are (-2,-7),(0,1), and (2,9)

Sketch a graph using these points



Critical Points:

$(-2,-7) \Rightarrow$ *Absolute Minimum*

$(0,1) \Rightarrow$ *Inflection Pt*

$(2,9) \Rightarrow$ *Absolute Maximum*

Exercises Section 3.2

Find all extrema points of the given functions.

1) $f(x) = 2x^2 + 8x$
2) $f(x) = 4x^3 - 4$
3) $f(x) = x^3 - 3x$
4) $f(x) = 4x^3 + 6x^2$

For each functions listed below:

- Find the intervals where the function is increasing and the function is decreasing
- Find all extrema points
- Make a sketch of the graph

5) $f(x) = x^2 - 8x$
6) $f(x) = x^3 + 6x^2$

Find all extrema points of the given function on the given interval

7) $f(x) = x^2 + 1$; [-2,2]
8) $f(x) = x^3 - 2$; [-1,2]