

## Math 121

### Section 2.5 Chain Rule

---

#### Example 1

Find the derivative of the function:  $f(x) = (3x^2 + 5)^5$

$$f(x) = (3x^2 + 5)^5$$

Let  $f(x) = u^5$  where  $u = 3x^2 + 5 \Rightarrow du = 6x$

$$f'(x) = 5u^4 du$$

$$f'(x) = 5(3x^2 + 5)^4 (6x)$$

$$f'(x) = (30x)(3x^2 + 5)^4$$

---

#### Example 2

Find the derivative of the function:  $f(x) = (x^2 + 3x)^7$

$$f(x) = (x^2 + 3x)^7$$

Let  $f(x) = u^7$  where  $u = x^2 + 3x \Rightarrow du = 2x + 3$

$$f'(x) = 7u^{7-1} du$$

$$f'(x) = 7u^6 du$$

$$f'(x) = 7(x^2 + 3x)^6 (2x + 3)$$

$$f'(x) = (14x + 21)(x^2 + 3x)^6$$

---

**Example 3**

Find the derivative of  $f(x) = (4x^3 + 5x^2 + 4x)^4$

$$f(x) = (4x^3 + 5x^2 + 4x)^4$$

$$\text{Let } f(x) = u^4 \text{ where } u = 4x^3 + 5x^2 + 4x \Rightarrow du = 12x^2 + 10x + 4$$

$$f'(x) = 4u^{4-1} du$$

$$f'(x) = 4u^3 du$$

$$f'(x) = 4(4x^3 + 5x^2 + 4x)^3(12x^2 + 10x + 4)$$

---

**Example 4**

Find the derivative of  $f(x) = \sqrt{x^2 - 6x}$

$$f(x) = \sqrt{x^2 - 6x}$$

$$f(x) = (x^2 - 6x)^{\frac{1}{2}}$$

$$\text{Let } f(x) = u^{\frac{1}{2}} \text{ where } u = x^2 - 6x \Rightarrow du = 2x - 6$$

$$f'(x) = \frac{1}{2} u^{\frac{1}{2}-1} du$$

$$f'(x) = \frac{1}{2} u^{-\frac{1}{2}} du$$

$$f'(x) = \frac{1}{2} (x^2 - 6x)^{-\frac{1}{2}} (2x - 6)$$

$$f'(x) = \frac{2x - 6}{2(x^2 - 6x)^{\frac{1}{2}}}$$

$$f'(x) = \frac{2x - 6}{2\sqrt{x^2 - 6x}}$$

---

---

**Example 5**

Find the derivative of  $f(x) = \sqrt{x^3 - 3x}$

$$f(x) = \sqrt{x^3 - 3x}$$

$$f(x) = (x^3 - 3x)^{\frac{1}{2}}$$

Let  $f(x) = u^{\frac{1}{2}}$  where  $u = x^3 - 3x \Rightarrow du = 3x^2 - 3$

$$f'(x) = \frac{1}{2} u^{\frac{1}{2}-1} du$$

$$f'(x) = \frac{1}{2} u^{-\frac{1}{2}} du$$

$$f'(x) = \frac{1}{2} (x^3 - 3x)^{-\frac{1}{2}} (3x^2 - 3)$$

$$f'(x) = \frac{3x^2 - 3}{2(x^3 - 3x)^{\frac{1}{2}}}$$

$$f'(x) = \frac{3x^2 - 3}{2\sqrt{x^3 - 3x}}$$

---

**Example 6**

Find the derivative of  $f(x) = \sqrt[3]{x^2 + 4}$

$$f(x) = \sqrt[3]{x^2 + 4}$$

$$f(x) = (x^2 + 4)^{\frac{1}{3}}$$

Let  $f(x) = u^{\frac{1}{3}}$  where  $u = x^2 + 4 \Rightarrow du = 2x$

$$f'(x) = \frac{1}{3} u^{\frac{1}{3}-1} du = \frac{1}{3} u^{-\frac{2}{3}} du = \frac{du}{3u^{\frac{2}{3}}} = \frac{2x}{3(x^2 + 4)^{\frac{2}{3}}} = \frac{2x}{3\sqrt[3]{(x^2 + 4)^2}}$$

---

---

**Example 7**

Find the slope of tangent to the function  $f(x) = (x^2 + 2x)^3$  at the point (1,27)

$$f(x) = (x^2 + 2x)^3$$

$$\text{Let } f(x) = u^3 \text{ where } u = x^2 + 2x \Rightarrow du = 2x + 2$$

$$f'(x) = 3u^{3-1} du$$

$$f'(x) = 3u^2 du$$

$$f'(x) = 3(x^2 + 2x)^2 (2x + 2)$$

$$f'(x) = (6x + 6)(x^2 + 2x)^2$$

Slope

$$f'(1) = (6(1) + 6)(1^2 + 2(1))^2 = (6 + 6)(1 + 2)^2 = 12(3)^2 = 12(9) = 108$$

$$m = 108$$

---

---

**Example 8**

Find the equation of tangent line to the function  $f(x) = (x^2 + 1)^4$  at the point  $(0,1)$

$$f(x) = (x^2 + 1)^4$$

$$\text{Let } f(x) = u^4 \text{ where } u = x^2 + 1 \Rightarrow du = 2x$$

$$f'(x) = 4u^{4-1} du$$

$$f'(x) = 4u^3 du$$

$$f'(x) = 4(x^2 + 1)^3 (2x)$$

$$f'(x) = (8x)(x^2 + 1)^3$$

**Slope**

$$f'(0) = (8(0))(0^2 + 1)^3 = 0(1^3) = 0 \cdot 1 = 0$$

$$m = 0$$

**Equation**

$$y - 1 = 0(x - 0)$$

$$y - 1 = 0$$

$$y = 1$$

**Exercises****Find the derivative**

1)  $f(x) = (x^2 + 5x)^6$

2)  $f(x) = (2x^2 + 3x)^4$

3)  $f(x) = (x^3 - 4x)^3$

4)  $f(x) = \sqrt{x^2 + 11x}$

**Find the slope of tangent line to the given function at the given point.**

5)  $f(x) = (x^2 - 1)^5$  at  $(0, -1)$