Section 1.4 Functions

Key Terms

Relation: A set of ordered pairs

Domain: The set of all x-values or first coordinates in a relation.

Range: The set of all y-values or second coordinates in a relation.

Example 1

Find the domain and range of the relation {(1,2),(2,4),(3,6),(6,7),(8,9)}

Domain: {1,2,3,6,8}

Range: {2,4,6,7,9}

Function: A relation where every element in the domain is paired with exactly one element in the range.

Example 2

Give the domain and range of each relation and determine if the relation is a function.

{(1,2),(3,4),(5,9),(6,5),(9.9)}

Answer: Domain: {1,3,5,6,9}
Range: {2,4,5,9}
This is a function

Example 3

Give the domain and range of each relation and determine if the relation is a function.

{(1,1),(2,3),(2,5),(4,6),(6,8)}

Answer: Domain: {1,2,4,6}
Range: {1,3,5,6,8}
This is not a function because 2 is paired with two values 3 and 5

Example 4 Find the domain and relation of the given relation, and determine if the relation is a function

$$Domain \Rightarrow Range$$
$$Greg \Rightarrow Jane$$
$$John \Rightarrow Jill$$
$$Bob \Rightarrow Beth$$
$$Mike \Rightarrow Carol$$
$$Mike \Rightarrow Molly$$

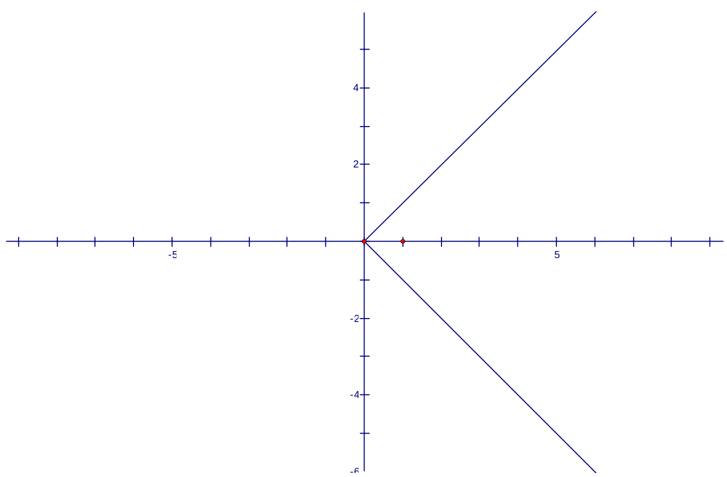
Domain: {Greg, John, Bob, Mike}, Range={Jane, Jill, Beth, Carol, Molly}
This is not a function. (Mike is paired with Carol and Molly)

Vertical Line Test

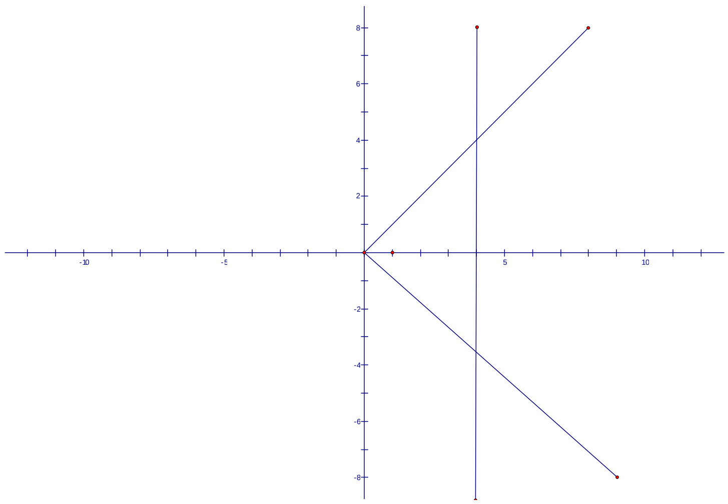
If a vertical line can be drawn so that it intersects the graph of a relation 2 or more times, then the relation is not a function.

Examples 5

Use the graph to determine if the relation is a function, and give the domain and range.



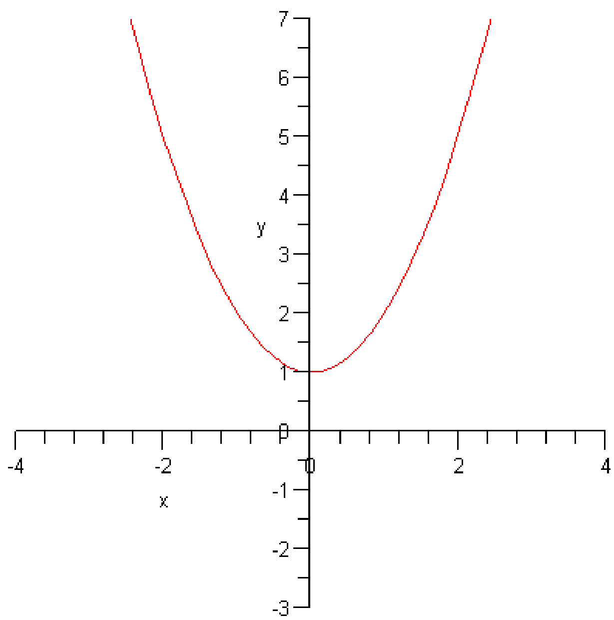
Solution:



This is not a function, since the relation fails the vertical line test.
Domain: $(0, \infty)$, **Range:** $(-\infty, \infty)$

Example 6

Use the graph to determine if the relation is a function, and give the domain and range.



$f(x) = x^2+1$

This is a function since every vertical line on the graph would intersect the relation only once. Domain: $(-\infty, \infty)$: Range: $[1, \infty)$

Function Notation

Example 7

$f(x) = x^2 + 1$

$g(x) = 5x + 4$

$h(x) = 4x^3$

Given the three functions above, find the following values.

1) Find $f(3)$

$f(3) = 3^2 + 1 = 9 + 1$

2) Find $g(-4)$

$g(-4) = 5(-4) + 4 = -20 + 4 = -16$

3) Find $h(-4)$

$g(-4) = (-4)^3 = -64$

4) Find $g(x-2)$

$g(x-2) = 5(x-2) + 4 = 5x - 10 + 4 = 5x - 6$

5) Find $f(x+1) = (x+1)^2 + 1 = (x+1)(x+1) + 1 = x^2 + 2x + 1 + 1 = x^2 + 2x + 2$

6) Find $f(x+h)$

$f(x+h) = (x+h)^2 + 1 = (x+h)(x+h) + 1 = x^2 + 2xh + h^2 + 1$

Composition of Two Functions

$f(g(x))$ *and* $g(f(x))$ *or* $f \circ g(x)$ *and* $g \circ f(x)$

Example 8

Given $f(x)=x^2-2 \; and \; g(x)=2x+3$, find $f(g(2)) \; and \; g(f(2))$

$Since \; g(2)=2(2)+3=4+3=7,$
$f(g(2))= f(7)=7^2-2=49-2=47$

$Since \; f(2)=2^2-2=4-2=2,$
$f(g(2))= f(2)=2(2)+3=4+3=7$

Example 9

Using the functions from **Example 8**, find $f(g(4)) \; and \; g(f(-3))$

$Since \; g(-4)=2(-4)+3=-8+3=-5,$
$f(g(-4))= f(-5)=(-5)^2-2=25-2=23$

$Since \; f(-3)=(-3)^2-2=9-2=7,$
$f(g(7))= f(7)=2(7)+3=14+3=17$

Example 10

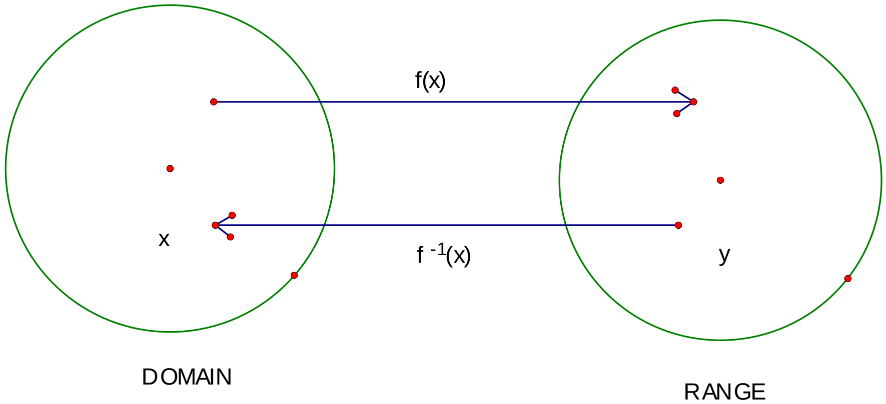
Using the functions from **Example 8**, find $f(g(x)) \; and \; g(f(x))$

$f(g(x))= f(2x+3)=(2x+3)^2-2=(2x+3)(2x+3)-2=4x^2+12x+9-2=4x^2+12x+7$

$g(f(x))=g(x^2-2)=3(x^2-2)+3=3x^2-6+3=3x^2+3$

Inverse Functions

An **inverse function** $f^{-1}(x)$ of a function $f(x)$ is function that maps each point range back to the domain where $f^{-1}(f(x))=x$ and $f(f^{-1}(x))=x$



Find the inverse function of the function $f = \{(1,3),(2,5),(3,2),(4,4)\}$

Solution: Just switch the x and y components.

Thus, the inverse function would be $f^{-1} = \{(3,1),(5,2),(2,3),(4,4)\}$

Example 11 Find the inverse function of $f(x) = 3x - 4$

Solution: Replace $f(x)$ with y, switch x and y, and then solve for y:

$$f(x) = 3x - 4 \;\Rightarrow\; y = 3x - 4$$
$$Switch\ x\ and\ y \Rightarrow x = 3y - 4$$
$$Solve\ for\ y$$
$$x = 3y - 4$$
$$x + 4 = 3y - 4 + 4$$
$$x + 4 = 3y$$
$$\frac{x+4}{3} = \frac{3y}{3}$$
$$\frac{x+4}{3} = y \Rightarrow f^{-1}(x) = \frac{x+4}{3}$$

Example 11

Find the inverse function of $f(x) = \frac{x}{2} + 5$

Solution: Replace $f(x)$ with y, switch x and y, and then solve for y:

$$f(x) = \frac{x}{2} + 5 \;\Rightarrow y = \frac{x}{2} + 5$$

$$Switch\ x\ and\ y \Rightarrow x = \frac{y}{2} + 5$$

$$Solve\ for\ y$$

$$x = \frac{y}{2} + 5$$

$$x - 5 = \frac{y}{2} + 5 - 5$$

$$x - 5 = \frac{y}{2}$$

$$2(x-5) = 2\left(\frac{y}{2}\right)$$

$$2x - 10 = y \Rightarrow f^{-1}(x) = 2x = 10$$

Example 12

Find the inverse function of $f(x) = x^3 - 1$and then sketch a graph of $f(x)$ and $f^{-1}(x)$

$$f(x) = x^3 - 1 \;\Rightarrow y = x^3 - 1$$

$$Switch\ x\ and\ y \Rightarrow x = y^3 - 1$$

$$Solve\ for\ y$$

$$x = y^3 - 1$$

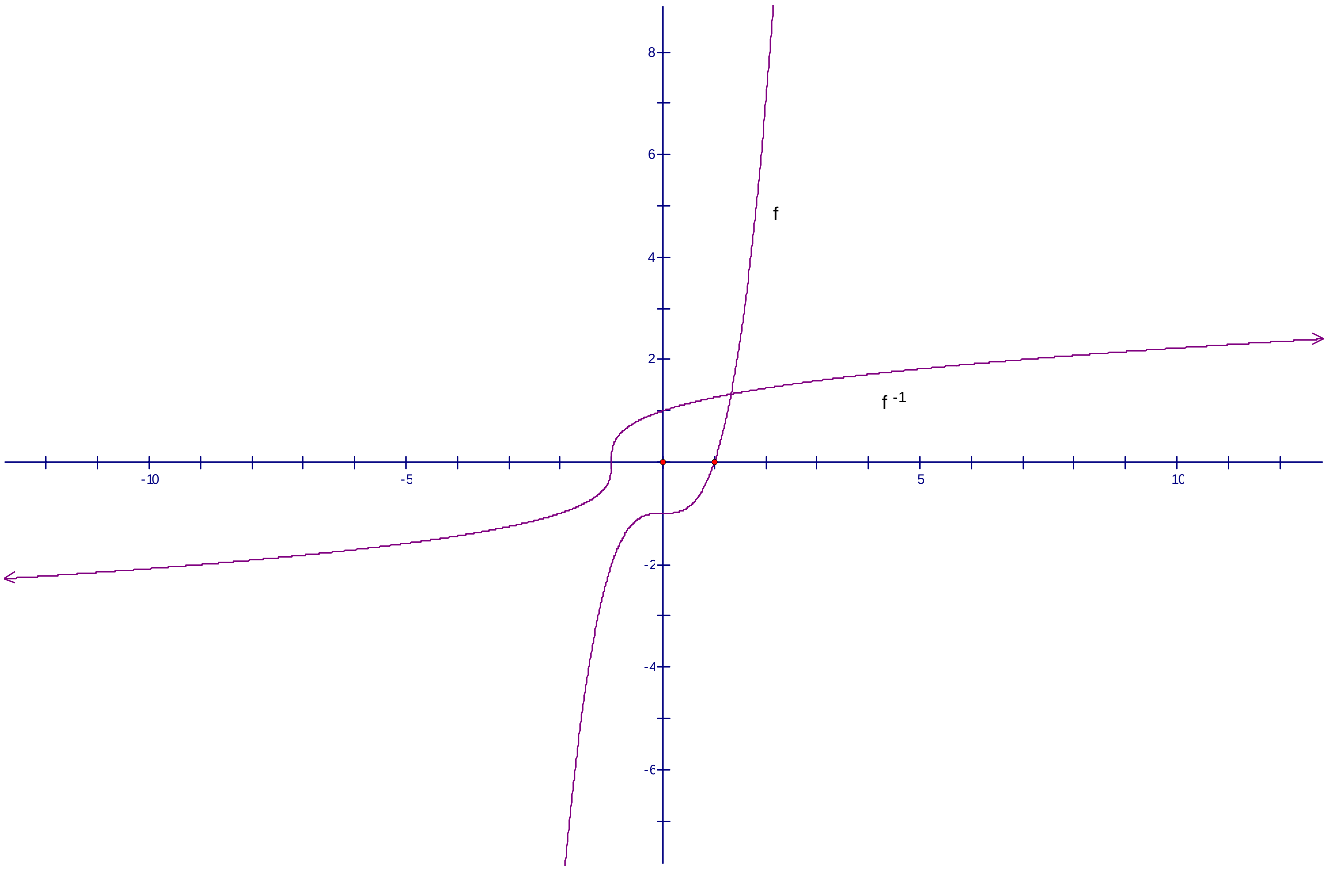
$$x + 1 = y^3 - 1 + 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x+1} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+1} = y \Rightarrow f^{-1}(x) = \sqrt[3]{x+1}$$

Graph of $f(x)$ and $f^{-1}(x)$



Example 13

Find the inverse function of $f(x) = x^2 + 3$

$$f(x) = x^2 + 3 \;\Rightarrow y = x^2 + 3$$
$$Switch\ x\ and\ y \Rightarrow x = y^2 + 3$$
$$Solve\ for\ y$$
$$x = y^2 + 3$$
$$x - 3 = y^2 + 3 - 3$$
$$x - 3 = y^2$$
$$\sqrt{x-3} = \sqrt{y^2}$$
$$\Rightarrow \pm\sqrt[2]{x+1} = y \quad However,\ \pm\sqrt{x-3}\ is\ not\ a\ function$$

Therefore, f does not have an inverse function
