Math 126

Section 1.4 Functions

Key Terms

Relation: A set of ordered pairs

Domain: The set of all x-values or first coordinates in a relation.

Range: The set of all y-values or second coordinates in a relation.

Example 1

Find the domain and range of the relation {(1,2),(2,4),(3,6),(6,7),(8,9)}

Domain: {1,2,3,6,8}

Range: {2,4,6,7,9}

Function: A relation where every element in the domain is paired with exactly one element in the range.

Example 2

Give the domain and range of each relation and determine if the relation is a function.

{(1,2),(3,4),(5,9),(6,5),(9.9)}

Answer: Domain: {1,3,5,6,9}
Range: {2,4,5,9}
This is a function

Example 3

Give the domain and range of each relation and determine if the relation is a function.

{(1,1),(2,3),(2,5),(4,6),(6,8)}

Answer: Domain: {1,2,4,6}
Range: {1,3,5,6,8}
This is not a function because 2 is paired with two values 3 and 5

Example 4 Find the domain and relation of the given relation, and determine if the relation is a function

$Domain \Rightarrow Range$

$Greg \Rightarrow Jane$

$John \Rightarrow Jill$

$Bob \Rightarrow Beth$

$Mike \Rightarrow Carol$

$Mike \Rightarrow Molly$

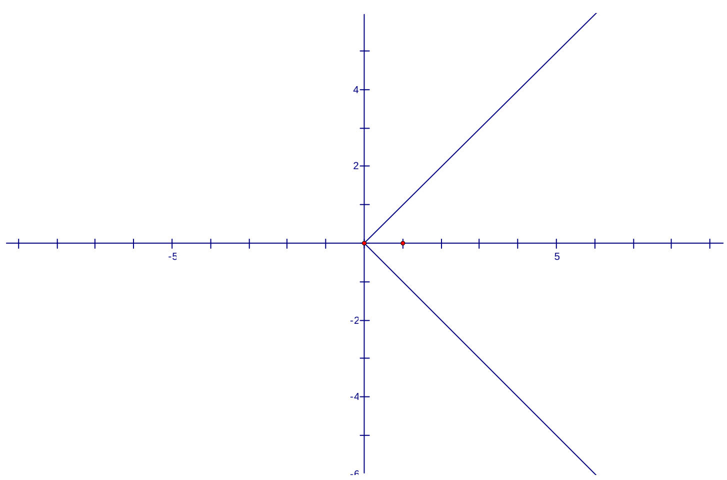
Domain: {Greg, John, Bob, Mike}, Range = {Jane, Jill, Beth, Carol, Molly}
This is not a function. (Mike is paired with Carol and Molly)

Vertical Line Test

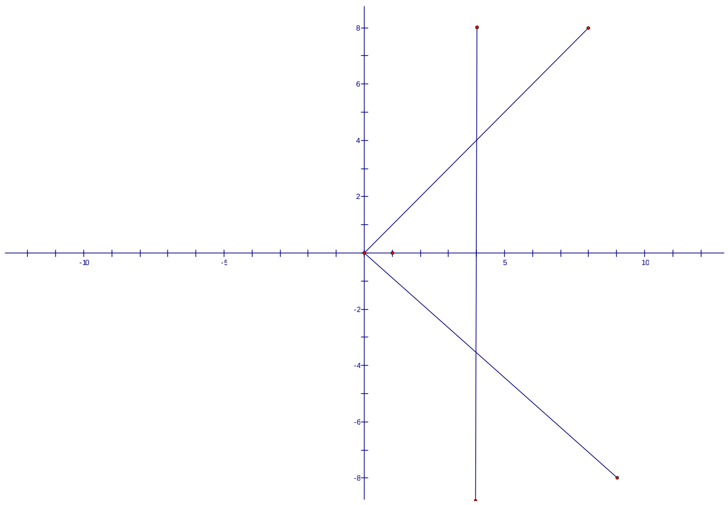
If a vertical line can be drawn so that it intersects the graph of a relation 2 or more times, then the relation is not a function.

Examples 5

Use the graph to determine if the relation is a function, and give the domain and range.



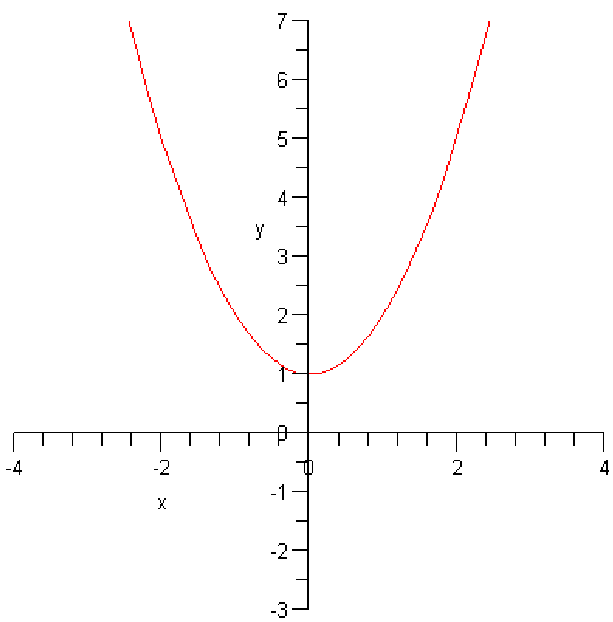
Solution:



This is not a function, since the relation fails the vertical line test.
Domain: $(0,\infty)$, **Range:** $(-\infty,\infty)$

Example 6

Use the graph to determine if the relation is a function, and give the domain and range.



$f(x) = x^2+1$

This is a function since every vertical line on the graph would intersect the relation only once. Domain: $\left(-\infty,\infty\right)$: Range: $[1,\infty)$

Function Notation

Example 7

$$f(x) = x^2 + 1$$
$$g(x) = 5x + 4$$
$$h(x) = 4x^3$$

Given the three functions above, find the following values.

1) Find $f(3)$
$f(3) = 3^2 + 1 = 9 + 1$

2) Find $g(-4)$
$g(-4) = 5(-4) + 4 = -20 + 4 = -16$

3) Find $h(-4)$
$g(-4) = (-4)^3 = -64$

4) Find $g(x-2)$
$g(x-2) = 5(x-2) + 4 = 5x - 10 + 4 = 5x - 6$

5) Find $f(x+1) = (x+1)^2 + 1 = (x+1)(x+1) + 1 = x^2 + 2x + 1 + 1 = x^2 + 2x + 2$

6) Find $f(x+h)$

$f(x+h) = (x+h)^2 + 1 = (x+h)(x+h) + 1 = x^2 + 2xh + h^2 + 1$

Composition of Two Functions

$f(g(x))$ *and* $g(f(x))$ *or* $f \circ g(x)$ *and* $g \circ f(x)$

Example 8

Given $f(x) = x^2 - 2$ *and* $g(x) = 2x + 3$, find $f(g(2))$ *and* $g(f(2))$

$$\text{Since } g(2) = 2(2) + 3 = 4 + 3 = 7,$$
$$f(g(2)) = f(7) = 7^2 - 2 = 49 - 2 = 47$$

$$\text{Since } f(2) = 2^2 - 2 = 4 - 2 = 2,$$
$$f(g(2)) = f(2) = 2(2) + 3 = 4 + 3 = 7$$

Example 9

Using the functions from **Example 8**, find $f(g(4))$ *and* $g(f(-3))$

$$\text{Since } g(-4) = 2(-4) + 3 = -8 + 3 = -5,$$
$$f(g(-4)) = f(-5) = (-5)^2 - 2 = 25 - 2 = 23$$

$$\text{Since } f(-3) = (-3)^2 - 2 = 9 - 2 = 7,$$
$$f(g(7)) = f(7) = 2(7) + 3 = 14 + 3 = 17$$

Example 10

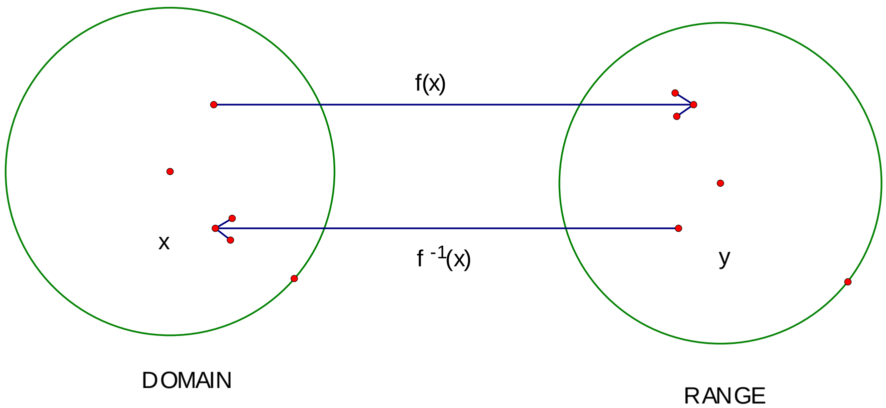
Using the functions from **Example 8**, find $f(g(x))$ *and* $g(f(x))$

$$f(g(x)) = f(2x+3) = (2x+3)^2 - 2 = (2x+3)(2x+3) - 2 = 4x^2 + 12x + 9 - 2 = 4x^2 + 12x + 7$$

$$g(f(x)) = g(x^2 - 2) = 3(x^2 - 2) + 3 = 3x^2 - 6 + 3 = 3x^2 + 3$$

Inverse Functions

An **inverse function** $f^{-1}(x)$ of a function $f(x)$ is function that maps each point range back to the domain where $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$



Find the inverse function of the function $f = \{(1,3),(2,5),(3,2),(4,4)\}$

Solution: Just switch the x and y components.

Thus, the inverse function would be $f^{-1} = \{(3,1),(5,2),(2,3),(4,4)\}$

Example 11 Find the inverse function of $f(x) = 3x - 4$

Solution: Replace $f(x)$ with y, switch x and y, and then solve for y:

$$f(x) = 3x - 4 \quad \Rightarrow y = 3x - 4$$
$$Switch\ x\ and\ y \Rightarrow x = 3y - 4$$
$$Solve\ \ for\ \ y$$
$$x = 3y - 4$$
$$x + 4 = 3y - 4 + 4$$
$$x + 4 = 3y$$
$$\frac{x+4}{3} = \frac{3y}{3}$$
$$\frac{x+4}{3} = y \Rightarrow f^{-1}(x) = \frac{x+4}{3}$$

Example 11

Find the inverse function of $f(x) = \frac{x}{2} + 5$

Solution: Replace $f(x)$ with y, switch x and y, and then solve for y:

$$f(x) = \frac{x}{2} + 5 \Rightarrow y = \frac{x}{2} + 5$$

$$\text{Switch } x \text{ and } y \Rightarrow x = \frac{y}{2} + 5$$

$$\text{Solve for } y$$

$$x = \frac{y}{2} + 5$$

$$x - 5 = \frac{y}{2} + 5 - 5$$

$$x - 5 = \frac{y}{2}$$

$$2(x-5) = 2\left(\frac{y}{2}\right)$$

$$2x - 10 = y \Rightarrow f^{-1}(x) = 2x = 10$$

Example 12

Find the inverse function of $f(x) = x^3 - 1$ and then sketch a graph of $f(x)$ and $f^{-1}(x)$

$$f(x) = x^3 - 1 \Rightarrow y = x^3 - 1$$

$$\text{Switch } x \text{ and } y \Rightarrow x = y^3 - 1$$

$$\text{Solve for } y$$

$$x = y^3 - 1$$

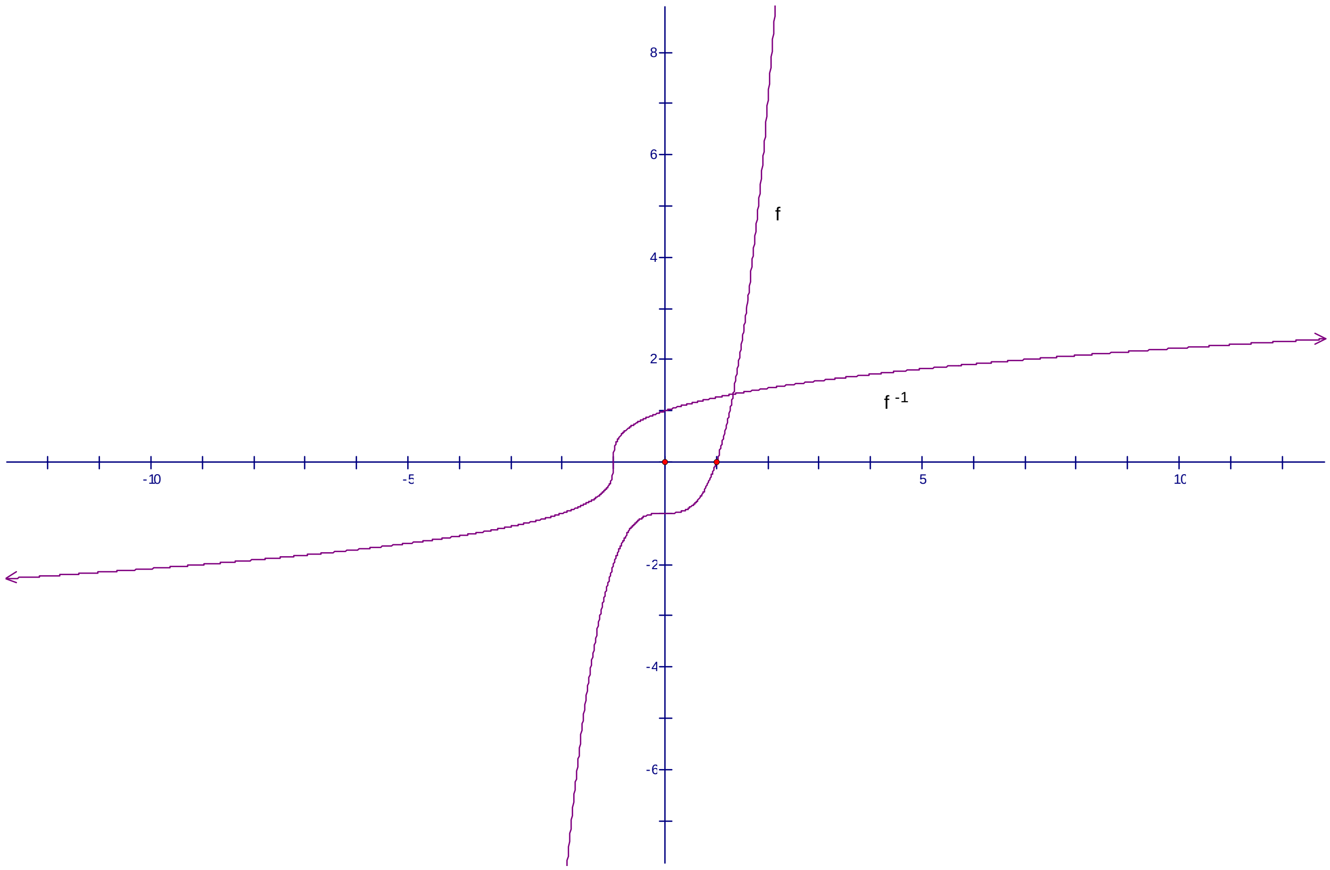
$$x + 1 = y^3 - 1 + 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x+1} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+1} = y \Rightarrow f^{-1}(x) = \sqrt[3]{x+1}$$

Graph of $f(x)$ and $f^{-1}(x)$



Example 13

Find the inverse function of $f(x) = x^2 + 3$

$$f(x) = x^2 + 3 \;\Rightarrow y = x^2 + 3$$

$$Switch\ x\ and\ y \Rightarrow x = y^2 + 3$$

$$Solve\ for\ y$$

$$x = y^2 + 3$$

$$x - 3 = y^2 + 3 - 3$$

$$x - 3 = y^2$$

$$\sqrt{x-3} = \sqrt{y^2}$$

$$\Rightarrow \pm\sqrt[2]{x+1} = y \quad However,\ \pm\sqrt{x-3}\ is\ not\ a\ function$$

Therefore, f does not have an inverse function

Functions

Given $f(x) = 2x^2 + 4,\ g(x) = 5x - 6,\ \textit{and}\ h(x) = x^3 - x$**, answer questions 1-10**

1) Find $f(2)$
2) Find $f(-3)$
3) Find $f(x+2)$
4) Find $h(2)$
5) Find $g(-1)$
6) Find $g(f(3))$
7) Find $f(g(4))$
8) Find $f(g(x))$

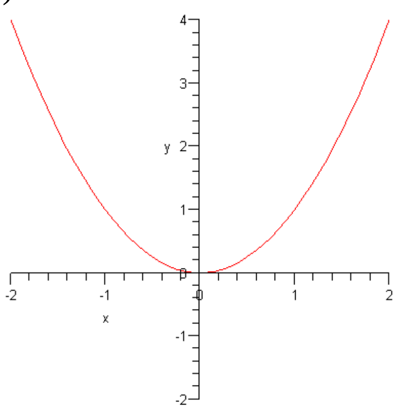
Find the inverse function, and then graph $f(x)$ **and** $f^{-1}(x)$

1) $f(x) = 3x - 4$
2) $f(x) = x^3$
3) $f(x) = \dfrac{x+2}{5}$

Find the domain and range of each relation, and then determine if the relation is a function.

1) $\{(1,3),(2,4),(4,2),(4,8)\}$

2)



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