Math 126 Section 4.2
Natural Exponential Functions

The exponential function

Limit definition of e^x

$$\lim_{x \to 0}(1+x)^{\frac{1}{x}} = e$$

x	.1	.01	.001
$f(x)=(1+x)^{\frac{1}{x}}$	$f(.1)=(1+.1)^{\frac{1}{.1}}$ $f(.1)=(1.1)^{10}$ $f(.1)=2.59$	$f(.01)=(1+.01)^{\frac{1}{.01}}$ $f(.01)=(1.01)^{100}$ $f(.01)=2.70$	$f(.001)=(1+.001)^{\frac{1}{.001}}$ $f(.001)=(1.001)^{1000}$ $f(.001)=2.717$

x	-.1	-.01	-.001
$f(x)=(1+x)^{\frac{1}{x}}$	$f(-.1)=(1-.1)^{-\frac{1}{.1}}$ $f(-.1)=(.9)^{-10}$ $f(.1)=2.86$	$f(-.01)=(1-.01)^{-\frac{1}{.01}}$ $f(-.01)=(.99)^{-100}$ $f(-.01)=2.73$	$f(-.001)=(1-.001)^{-\frac{1}{.001}}$ $f(-.001)=(.999)^{-1000}$ $f(-.001)=2.719$

Actual approximation for $e \approx 2.718$

Example 1

Evaluate each expression

a) e^2

$e^2 \approx 2.718$

b) $e^2 e^4$

$e^2 e^4 = e^6$

c) $\frac{e^4}{e^3}$

$\frac{e^4}{e^3} = e$

d) $\frac{e^4}{e^{-3}}$

$$\frac{e^4}{e^3} = e^{4-(-3)} = e^7$$

Solving exponential equations

Example 2

Solve $e^{2x} = e^9$

$$e^{2x} = e^9$$
$$2x = 9$$
$$\frac{2x}{2} = \frac{9}{2}$$
$$x = \frac{9}{2}$$

Example 3

Solve $e^{\sqrt{x}} = e^2$

$$e^{\sqrt{x}} = e^2$$
$$\sqrt{x} = 2$$
$$\left(\sqrt{x}\right)^2 = 2^2$$
$$x = 4$$

Example 4

Solve $e^{x^2-2} = e^2$

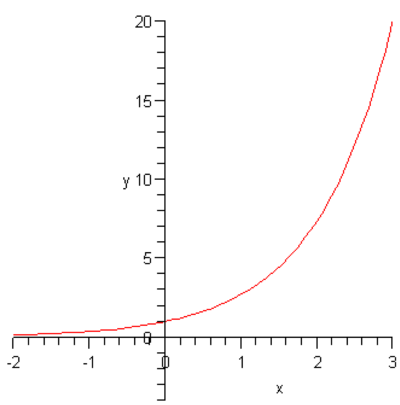
$$e^{x^2-2} = e^2$$
$$x^2 - 2 = 2$$
$$x^2 - 2 + 2 = 2 + 2$$
$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = \pm 2$$

Graphing the exponential function

Example 5

Graph $y = e^x$

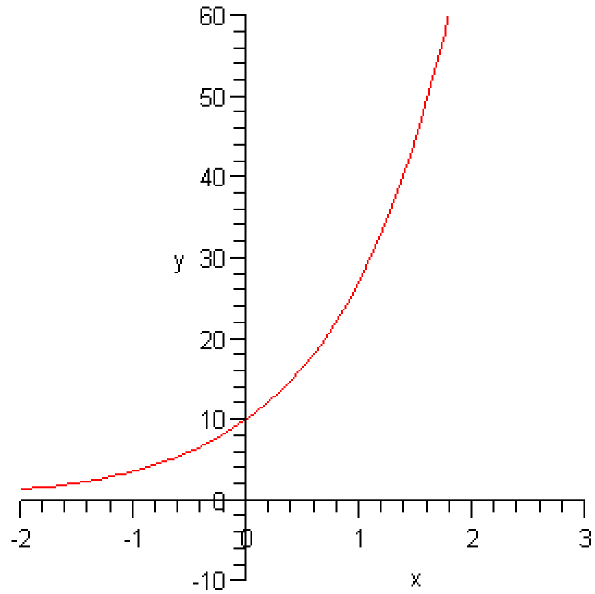
x	y
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^{o} = 1$
1	$y = e^{1} = 2.7$
2	$y = e^{2} = 7.4$



Example 6

Graph $y = 10e^{.2x}$

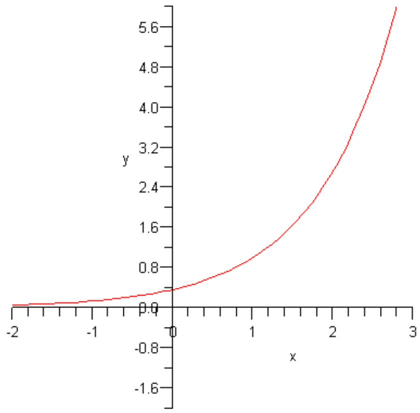
x	y
-2	$y = 10e^{.2(-2)} = 10e^{-.4} = 6.7$
-1	$y = 10e^{.2(-1)} = 10e^{-.2} = 8.2$
0	$y = 10e^{.2(0)} = 10e^{0} = 10$
1	$y = 10e^{.2(1)} = 10e^{.2} = 12.2$
2	$y = 10e^{.2(2)} = 10e^{.4} = 14.9$



Other graph of the exponential function

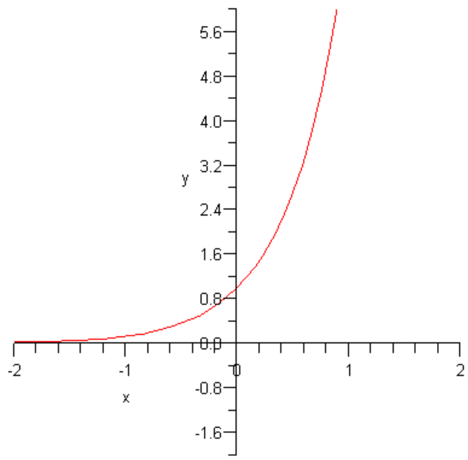
Example 7

Graph $y = e^{x-1}$



Example 8

Graph $y = e^{2x}$



Continuous interest

$A = accumulated\ Balance,\ P = principle\ ,\ r = rate,\ t = time$

$A = Pe^{rt}$

Example 9

Suppose you deposit $10,000 is a saving bond that compounds interest continuously at a rate of 5 % per month. Find out the balance after 10 years.

$$A = ?$$
$$P = \$10{,}000$$
$$r = .05$$
$$t = 10$$
$$A = 10{,}000e^{.05(10)} = 10{,}000e^{.5} = \$16487.21$$

Example 10

Suppose you deposit $5,000 is a saving bond that compounds interest continuously at a rate of 2 % per month. Find out the balance after 5 years.

$$A = ?$$
$$P = \$5{,}000$$
$$r = .02$$
$$t = 10$$
$$A = 5{,}000e^{.02(5)} = 5{,}000e^{.10} = \$5525.85$$

Compound Interest Formula

$$P = \text{ principle}$$
$$R = rate$$
$$T = time$$
$$n = number\ of\ times\ \text{int}\, erest\ is\ computed$$
$$A = Accumulated\ Balance$$
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example 11

Mike invests $5000 in a special saving bond that compound interest quarterly at a rate of 5% per years. How much money would Mike have in this saving bond after 10 years?

$P = \$5000,\ T = 10,\ R = .05,\ n = 4\ (quarterly)$

$$A = \$5000\left(1 + \frac{.05}{4}\right)^{4\cdot 10} = \$5000(1 + .0125)^{40} = \$5000(1.0125)^{40} = \$5000(1.6436) = \$8218.09$$

Present Value

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Derivation of the present value formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{P\left(1 + \frac{r}{n}\right)^{nt}}{\left(1 + \frac{r}{n}\right)^{nt}}$$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Example 12

How much should be deposited in an account paying 5% interest compound monthly in order to have a new balance of $22,000 in 5 years?

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{22000}{\left(1+\frac{.05}{12}\right)^{(12)5}} = \frac{22000}{(1.00416667)^{60}} = \frac{22000}{1.28336} = \$17{,}142.48$$

Example 13

The certain population of 2500 bacteria is modeled by the function $P = 2500(e)^{.09t}$ where t is time in months. Use the population to predict the population of bacteria in 36 months.

$$P = 2500e^{.09t}$$
$$P = 2500e^{.09(26)}$$
$$P = 2500e^{3.24}$$
$$P = 63834$$

Example 13

The population of Chicago is modeled by the function $P = 2{,}700{,}000e^{.01t}$ where t is the time in years. Use the model to predict the population of Chicago in 10 years.

$$P = 2{,}700{,}000e^{.01(10)}$$
$$P = 2{,}700{,}000e^{.1}$$
$$P = 2983961$$

Example 10 (See page 271 #32

n	1	2	4	12	356	Continuous Compounding
A						

$P = \$3500, \quad r = 5\%, \; t = 10 \; years$

$n = 1$

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{1}\right)^{1(10)} = 3500(1.05)^{10} = \$5701.13$

$n = 2$

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{2}\right)^{2(10)} = 3500(1.025)^{20} = \$5735.15$

$n = 4$

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{4}\right)^{4(10)} = 3500(1.0125)^{40} = \$5752.66$

$n = 12$

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{12}\right)^{12(10)} = 3500(1.00416667)^{120} = \$5764.53$

$n = 365$

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{365}\right)^{365(10)} = 3500(1.000137)^{3650} = \$5770.32$

$Continuous \; Interest$

$A = Pe^{rt} = \$3500e^{(.05)(10)} = \$3500e^{.5} = \$3770.52$