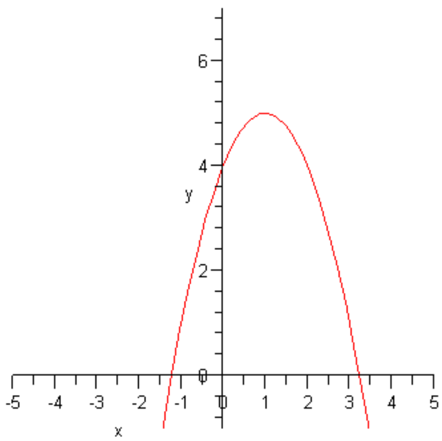
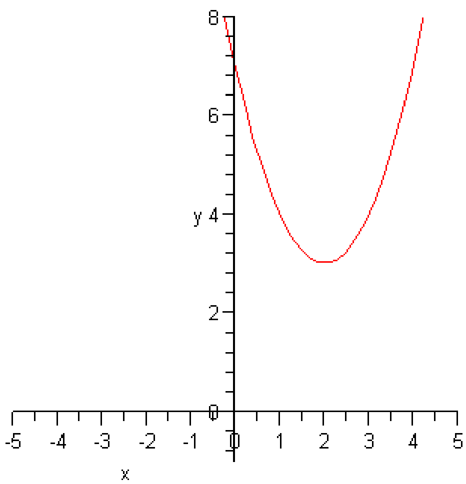
Section 3.5

Optimization (Business Applications)

Maximize Profit



Minimize Cost



Maximizing Revenue

Example 1

Find the number of units x that produces a maximum profit.

$R = 48x^2 - 0.02x^3$

$R = 96x - .06x^2$

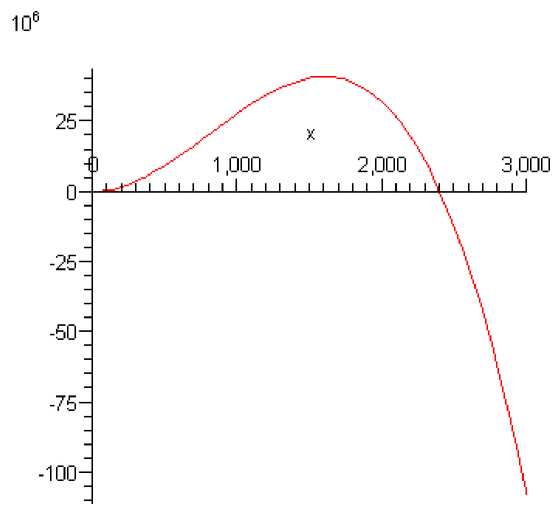
$96x - .06x^2 = 0$

$x(96 - .06x) = 0$

$x = 0 \ or \ \ 96 - .06x = 0$

$-.06x = -96$

$x = 1600$



Thus, 1600 units will produce a maximum profit.

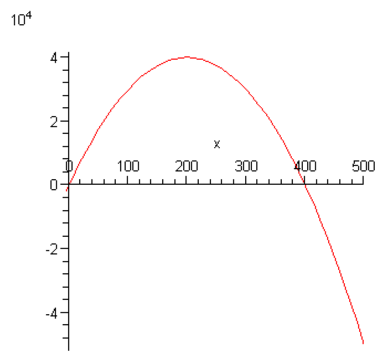
Example 2

Let $R = 400x - x^2$ represent the revenue earned by a local outfitter company that sells backpacks where x is the number of backpacks sold in a month.

$$R = 400x - x^2$$

$$R =' 400 - 2x$$

$$\begin{aligned} 400 - 2x &= 0 \\ 400 - 2x + 2x &= 0 + 2x \\ 400 &= 2x \\ x &= 200 \end{aligned}$$



Thus, selling 200 backpacks in one month would produce a maximum profit.

Example 3

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.   Hint: $R(x) = xp$

$$Demand\ \ Function:\ p = 6000 - 0.4x^2$$
$$Cost\ \ Function: C = 2400x + 5200$$

Solution: First find the revenue function using $R(x) = xp$, then find the profit function by subtracting the cost function from the revenue function. Once you have the profit function, you simply take the derivative of the profit function and set the result equal to zero.

$$Demand\ \ Function:\ p = 6000 - 0.4x^2$$
$$Cost\ \ Function:\ C = 2400x + 5200$$

$$R(x) = xp = x\left(6000 - 0.4x^2\right) = 6000x - 0.4x^2$$

$$P(x) = R(x) - C(x)$$
$$P(x) = 6000x - 0.4x^3 - \left(2400x + 5200\right)$$
$$P(x) = 6000x - 0.4x^3 - 2400x - 5200$$
$$P(x) = -0.4x^3 + 3600x + 5200$$

$$P'(x) = -1.2x^2 + 3600$$
$$-1.2x^2 + 3600 = 0$$
$$-1.2x^2 = -3600$$
$$\frac{-1.2x^2}{-1.2} = \frac{-3600}{-1.2}$$
$$x^2 = 3000$$
$$x = \sqrt{3000}$$
$$x \approx 54.8\ or\ 55\ \ units$$

Example 4

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.

$$Demand\ \ Function$$

$$p = 50 - .1\sqrt{x} = 50 - .1x^{\frac{1}{2}}$$

$$C = 35x + 500$$

$$R(x) = xp = x(50 - .1x^{\frac{1}{2}})$$

$$R(x) = 50x - ..1x^{\frac{3}{2}}$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 50x - x^{\frac{3}{2}} - (35x + 500)$$

$$P(x) = 50x + .1x^{\frac{3}{2}} - 35x - 500$$

$$P(x) = -.1x^{\frac{3}{2}} + 15x - 500$$

$$P'(x) = -.15x^{\frac{1}{2}} + 15$$

$$-.15\sqrt{x} + 15 = 0$$

$$-.15\sqrt{x} = -15$$

$$\frac{-.15\sqrt{x}}{-.15} = \frac{-15}{.15}$$

$$\sqrt{x} = 100$$

$$x = 10000$$

Example 6

Suppose that you have 320 square centimeters of aluminum to make a cylinder shaped coke can. Find the radius of the can that would produce a maximum volume.

First find formula for the volume of the coke can.

Surface area of cylinder: $S = 2\pi r + 2\pi rh$

Volume of a cylinder: $V = \pi r^2 h$

First, take the surface area formula and solve for h

$$S = 2\pi r + 2\pi rh$$
$$320 = 2\pi r + 2\pi rh$$
$$320 - 2\pi r = 2\pi rh$$
$$\Rightarrow h = \frac{320 - 2\pi r}{2\pi r}$$

Next, substitute $\frac{320 - 2\pi r}{2\pi r}$ **in for h in the volume formula.**

$$V = \pi r^2 h$$
$$V = \pi r^2 \left(\frac{320 - 2\pi r}{2\pi r}\right)$$
$$V = \frac{320\pi r}{2\pi r} - \frac{2\pi r^3}{2\pi r}$$
$$V = 160r - \pi r^3$$

Now, take the derivative of the volume and set the result equall to zero

$$V' = 160 - 3\pi r^2$$

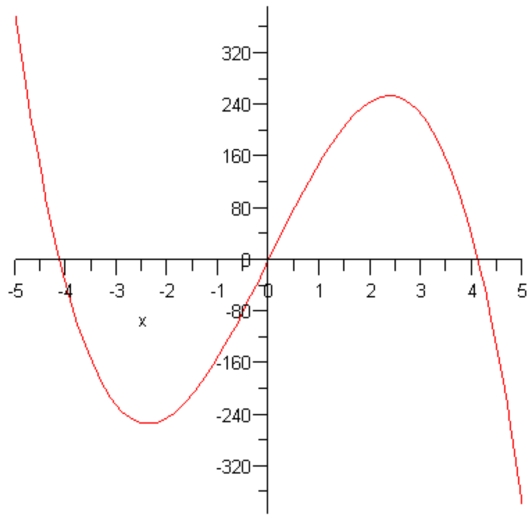
$$0 = 160 - 3\pi r^2$$
$$3\pi r^2 = 160$$
$$r^2 = \frac{160}{3\pi}$$
$$r^2 = 16.98 \Rightarrow r = \sqrt{16.98} \Rightarrow r = \pm 4.1\ cm$$

Eliminating the negative answer we get the radius of the can is 4.1 cm.

Now find the height use the radius is 4.1 cm

$$h = \frac{320 - 2\pi r}{2\pi r} = \frac{320 - 2(3.14)(4.1)}{2(3.14)(4.1)} = \frac{294.252}{25.748} = 11.4\ cm$$

The graph of $V = 160r - \pi r^3$



Definition of Price Elasticity

If $p = f(x)$ is a differentiable function, then price elasticity of demand is given by

$$\eta = \frac{\frac{p}{x}}{p'}$$

If $|\eta| > 1$ for a given price, then the demand is elastic.
If $|\eta| < 1$ for a given price, then the demand is inelastic.
If $|\eta| = 1$ for a given price, then the demand has unit elasticity.

Example 7

Find the price of elasticity of the demand for the demand function at the indicated x-value.

Demand Function: $p = 500 - 3x$ at $x = 20$

$$\frac{dp}{dx} = -3$$

$$\eta = \frac{\frac{p}{x}}{\frac{dp}{dx}} = \frac{\frac{500-3x}{x}}{-3}$$

$$at\ \ x = 20\ ;\ \eta = \frac{\frac{500-3(20)}{20}}{-3} = \frac{\frac{500-60}{20}}{-3} = \frac{\frac{440}{20}}{-3} = -\frac{440}{60} = -\frac{22}{3}$$

$$\left|-\frac{22}{3}\right| = \frac{22}{3} > 1 \Rightarrow the\ \ demand\ \ function\ \ is\ \ elastic$$

Example 8

Find the price of elasticity of the demand for the demand function at the indicated x-value.

Demand Function: $p = 400 - 6x$ at $x = 40$

$$\frac{dp}{dx} = -6$$

$$\eta = \frac{\frac{p}{x}}{\frac{dp}{dx}} = \frac{\frac{400-6x}{x}}{-6}\ at\ \ x = 20\ ;\ \eta = \frac{\frac{400-6(40)}{40}}{-6} = \frac{\frac{400-240}{40}}{-6} = \frac{\frac{160}{40}}{-6} = -\frac{160}{240} = -\frac{2}{3}$$

$$\left|-\frac{2}{3}\right| = \frac{2}{3} < 1 \Rightarrow the\ \ demand\ \ function\ \ is\ \ inelastic$$

Exercises 3.5

1) Let $R = 300x - 3x^2$ represent the revenue earned by a local outfitter company that sells backpacks where x is the number of backpacks sold in a month. Find the number of backpacks x that will produce a maximum revenue.

2) Let $R = 600x - 2x^2$ represent the revenue earned by a shoe company that sells hiking shoes where x is the number of hiking shoes sold in a month. Find the number of shoes x that will produce a maximum revenue.

3) Find the number of units x that will produce a minimum cost, given the cost function is $C(x) = 2.5x^2 + 15x + 90$

4) Find the number of units x that will produce a minimum cost, given the cost function is $C(x) = .002x^3 - 6x$

5) Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.

   $Demand\ \ Function: \ p = 100 - x$

   $Cost\ \ Function: C = 40x + 100$

6) Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.

   $Demand\ \ Function: \ p = 120 - 80x$

   $Cost\ \ Function: C = 150x + 600$

7) Suppose that you have 400 square centimeters of aluminum to make a cylinder shaped coke can. Find the radius of the can that would produce a maximum volume using the following formula for the volume of the can.

   $V = 200r - \pi r^3$