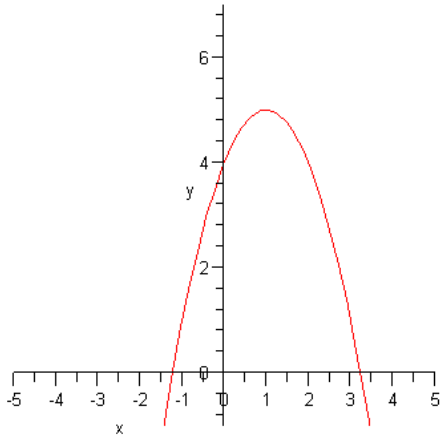


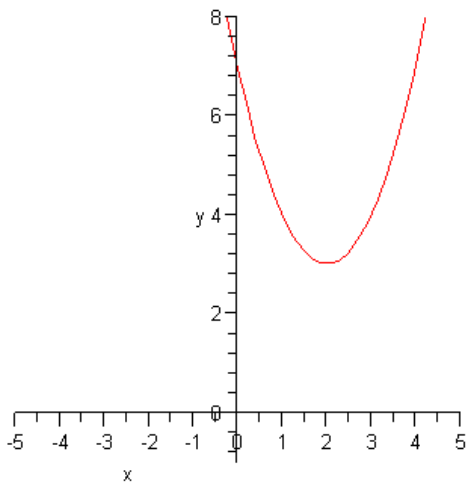
Section 10.3

Optimization (Business Applications)

Maximize Profit



Minimize Average Cost



Average Cost: $\overline{C(x)} = \frac{C(x)}{x}$

Maximizing Revenue

Example 1

Find the number of units x that produces a maximum profit.

$$P = 48x^2 - 0.02x^3$$

$$P = 48x^2 - 0.02x^3$$

$$P' = 96x - .06x^2$$

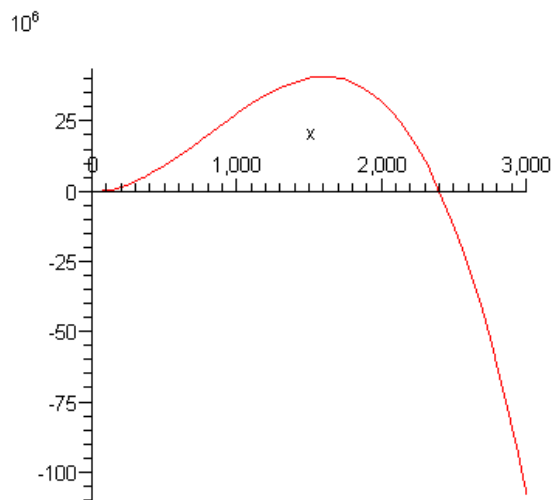
$$96x - .06x^2 = 0$$

$$x(96 - .06x) = 0$$

$$x = 0 \text{ or } 96 - .06x = 0$$

$$-.06x = -96$$

$$x = 1600$$



Thus, 1600 units will produce a maximum profit.

Example 2

Let $R = 400x - x^2$ represent the revenue earned by a local outfitter company that sells backpacks where x is the number of backpacks sold in a month.

$$R = 400x - x^2$$

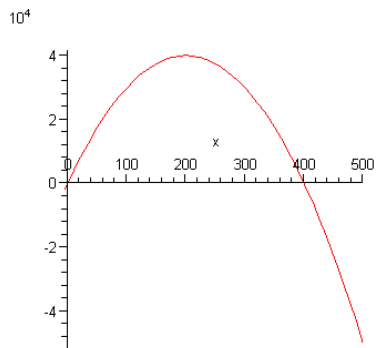
$$R' = 400 - 2x$$

$$400 - 2x = 0$$

$$400 - 2x + 2x = 0 + 2x$$

$$400 = 2x$$

$$x = 200$$



Thus, selling 200 backpacks in one month would produce a maximum profit.

Example 3

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit. Hint: $R(x) = xp$

$$\text{Demand Function : } p = 6000 - 0.4x^2$$

$$\text{Cost Function : } C = 2400x + 5200$$

Solution: First find the revenue function using $R(x) = xp$, then find the profit function by subtracting the cost function from the revenue function. Once you have the profit function, you simply take the derivative of the profit function and set the result equal to zero.

$$\text{Demand Function : } p = 6000 - 0.4x^2$$

$$\text{Cost Function : } C = 2400x + 5200$$

$$R(x) = xp = x(6000 - 0.4x^2) = 6000x - 0.4x^3$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 6000x - 0.4x^3 - (2400x + 5200)$$

$$P(x) = 6000x - 0.4x^3 - 2400x - 5200$$

$$P(x) = -0.4x^3 + 3600x - 5200$$

$$P'(x) = -1.2x^2 + 3600$$

$$-1.2x^2 + 3600 = 0$$

$$-1.2x^2 = -3600$$

$$\frac{-1.2x^2}{-1.2} = \frac{-3600}{-1.2}$$

$$x^2 = 3000$$

$$x = \sqrt{3000}$$

$$x \approx 54.8 \text{ or } 55 \text{ units}$$

Example 4

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.

Demand Function

$$p = 50 - .1\sqrt{x} = 50 - .1x^{\frac{1}{2}}$$

$$C = 35x + 500$$

$$R(x) = xp = x(50 - .1x^{\frac{1}{2}})$$

$$R(x) = 50x - .1x^{\frac{3}{2}}$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 50x - x^{\frac{3}{2}} - (35x + 500)$$

$$P(x) = 50x + .1x^{\frac{3}{2}} - 35x - 500$$

$$P(x) = -.1x^{\frac{3}{2}} + 15x - 500$$

$$P'(x) = -.15x^{\frac{1}{2}} + 15$$

$$-.15\sqrt{x} + 15 = 0$$

$$-.15\sqrt{x} = -15$$

$$\frac{-.15\sqrt{x}}{-.15} = \frac{-15}{.15}$$

$$\sqrt{x} = 100$$

$$x = 10000$$

Example 6

Suppose that you have 320 square centimeters of aluminum to make a cylinder shaped coke can. Find the radius of the can that would produce a maximum volume.

First find formula for the volume of the coke can.

$$\text{Surface area of cylinder: } S = 2\pi r + 2\pi rh$$

$$\text{Volume of a cylinder: } V = \pi r^2 h$$

First, take the surface area formula and solve for h

$$S = 2\pi r + 2\pi rh$$

$$320 = 2\pi r + 2\pi rh$$

$$320 - 2\pi r = 2\pi rh$$

$$\Rightarrow h = \frac{320 - 2\pi r}{2\pi r}$$

Next, substitute $\frac{320 - 2\pi r}{2\pi r}$ in for h in the volume formula.

$$V = \pi r^2 h$$

$$V = \pi r^2 \left(\frac{320 - 2\pi r}{2\pi r} \right)$$

$$V = \frac{320\pi r}{2\pi r} - \frac{2\pi r^3}{2\pi r}$$

$$V = 160r - \pi r^3$$

Now, take the derivative of the volume and set the result equal to zero

$$V' = 160 - 3\pi r^2$$

$$0 = 160 - 3\pi r^2$$

$$3\pi r^2 = 160$$

$$r^2 = \frac{160}{3\pi}$$

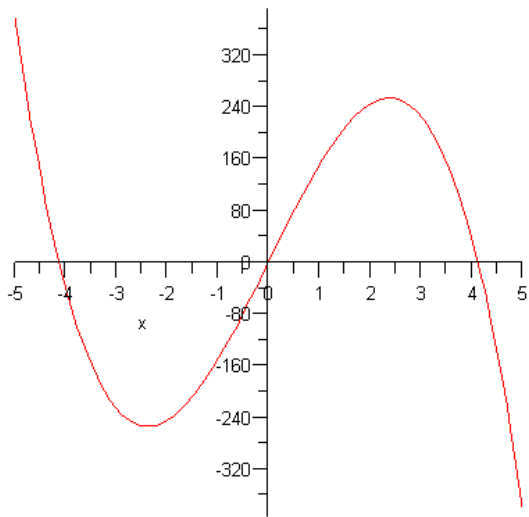
$$r^2 = 16.98 \Rightarrow r = \sqrt{16.98} \Rightarrow r = \pm 4.1 \text{ cm}$$

Eliminating the negative answer we get the radius of the can is 4.1 cm.

Now find the height use the radius is 4.1 cm

$$h = \frac{320 - 2\pi r}{2\pi r} = \frac{320 - 2(3.14)(4.1)}{2(3.14)(4.1)} = \frac{294.252}{25.748} = 11.4 \text{ cm}$$

The graph of $V = 160r - \pi r^3$



Price Elasticity of Demand

Economist can measure the response of consumers to the change in price of a product with the price elasticity of demand. For example, a drop in the price of tomatoes could cause a greater demand for tomatoes. The price elasticity of the demand in this case would be **elastic**. On the other hand, a drop in the price of dairy products may not have an effect the demand for dairy products. . The price elasticity of the demand in this case would be **inelastic**. The formula for elasticity of demand is provided below:

$$\eta = \frac{p}{p'} \frac{x}{x'}$$

For a given price and level of production x , the elasticity can be evaluated as follows:

$|\eta| > 1$: The demand is elastic.

$|\eta| < 1$: The demand is inelastic.

$|\eta| = 1$: The demand is unit elastic

Example 7

Find the price of elasticity of the demand for the demand function at the indicated x -value.

Demand Function: $p = 500 - 3x$ at $x = 20$

$$\frac{dp}{dx} = -3$$

$$\eta = \frac{p}{\frac{dp}{dx}} = \frac{500 - 3x}{-3}$$

$$\text{at } x = 20 ; \eta = \frac{500 - 3(20)}{-3} = \frac{500 - 60}{-3} = \frac{440}{-3} = -\frac{440}{3} = -\frac{22}{3}$$

$$\left| -\frac{22}{3} \right| = \frac{22}{3} > 1 \Rightarrow \text{the demand function is elastic}$$

Example 8

Find the price of elasticity of the demand for the demand function at the indicated x-value.

Demand Function: $p = 400 - 6x$ at $x = 40$

$$\frac{dp}{dx} = -6$$

$$\eta = \frac{\frac{p}{x}}{\frac{dp}{dx}} = \frac{\frac{400 - 6x}{x}}{-6} = \frac{400 - 6x}{x} \cdot \frac{1}{-6} = \frac{400 - 6x}{-6x}$$

$$\text{at } x = 40 ; \eta = \frac{400 - 6(40)}{-6(40)} = \frac{160}{-240} = -\frac{2}{3}$$

$$\left| -\frac{2}{3} \right| = \frac{2}{3} < 1 \Rightarrow \text{the demand function is inelastic}$$

Example 9

Find the price of elasticity of the demand for the demand function at the indicated x-value.

Demand Function: $p = 400 - 5x$ at $x = 40$

$$\frac{dp}{dx} = -5$$

$$\eta = \frac{\frac{p}{x}}{\frac{dp}{dx}} = \frac{\frac{400 - 5x}{x}}{-5} = \frac{400 - 5x}{x} \cdot \frac{1}{-5} = \frac{400 - 5x}{-5x}$$

$$\text{at } x = 40 ; \eta = \frac{400 - 5(40)}{-5(40)} = \frac{400 - 200}{-200} = \frac{200}{-200} = -1$$

$$|-1| = 1 \Rightarrow \text{the demand function is unit elastic}$$

Example 9

Given the cost function for producing car batteries is $C(x) = 400 + x^2$ where x is the number of car batteries manufactured, find the value of x that will minimize average cost.

$$C(x) = 400 + x^2$$

$$\overline{C(x)} = \frac{C(x)}{x} = \frac{400 + x^2}{x} = \frac{400}{x} + \frac{x^2}{x} = \frac{400}{x} + x = 400x^{-1} + x$$

$$\overline{C'(x)} = -400x^{-1-1} + 1$$

$$\overline{C'(x)} = -400x^{-2} + 1$$

$$\overline{C'(x)} = -\frac{400}{x^2} + 1$$

$$-\frac{400}{x^2} + 1 = 0$$

$$-\frac{400}{x^2} = -1$$

$$\frac{400}{x^2}(x^2) = 1(x^2)$$

$$400 = x^2$$

$$\sqrt{400} = \sqrt{x^2}$$

$$x = 20$$

Solution: 20 units (Car batteries)

Exercises 10.3

- 1) Let $R = 300x - 3x^2$ represent the revenue earned by a local outfitter company that sells backpacks where x is the number of backpacks sold in a month. Find the number of backpacks x that will produce a maximum revenue.
- 2) Let $R = 600x - 2x^2$ represent the revenue earned by a shoe company that sells hiking shoes where x is the number of hiking shoes sold in a month. Find the number of shoes x that will produce a maximum revenue.
- 3) Find the number of units x that will produce a minimum cost, given the cost function is $C(x) = 200 + x^2$
- 4) Find the number of units x that will produce a minimum cost, given the cost function is $C(x) = 144 + x^2$
- 5) Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.
Demand Function : $p = 100 - x$
Cost Function : $C = 40x + 100$
- 6) Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.
Demand Function : $70 - x$
Cost Function : $C = 50x + 600$
- 7) Find the price of elasticity of the demand for the demand function at the indicated x -value. Demand Function: $p = 400 - 6x$ at $x = 30$
- 8) Find the price of elasticity of the demand for the demand function at the indicated x -value. Demand Function: $p = 500 - 6x$ at $x = 40$
- 9) Find the price of elasticity of the demand for the demand function at the indicated x -value. Demand Function: $p = 500 - 5x$ at $x = 97$
- 10) Suppose that you have 400 square centimeters of aluminum to make a cylinder shaped coke can. Find the radius of the can that would produce a maximum volume using the following formula for the volume of the can.

$$V = 200r - \pi r^3$$