

Math 126

Section 1.3

Lines in a plane and slope

Slope

$$(\text{Slope}) m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

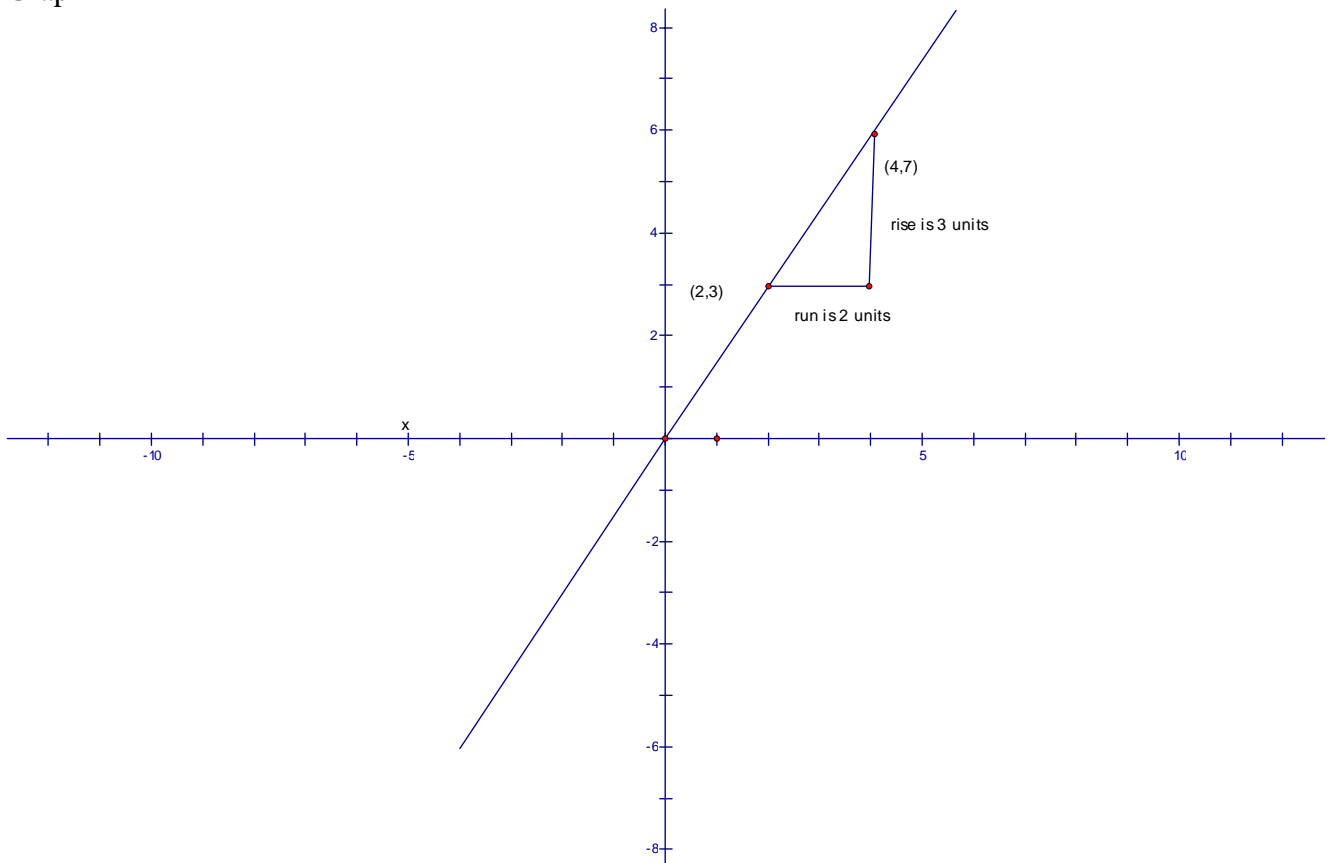
Example 1

Find the slope between the given points

- 1) (2,3) and (4,6)

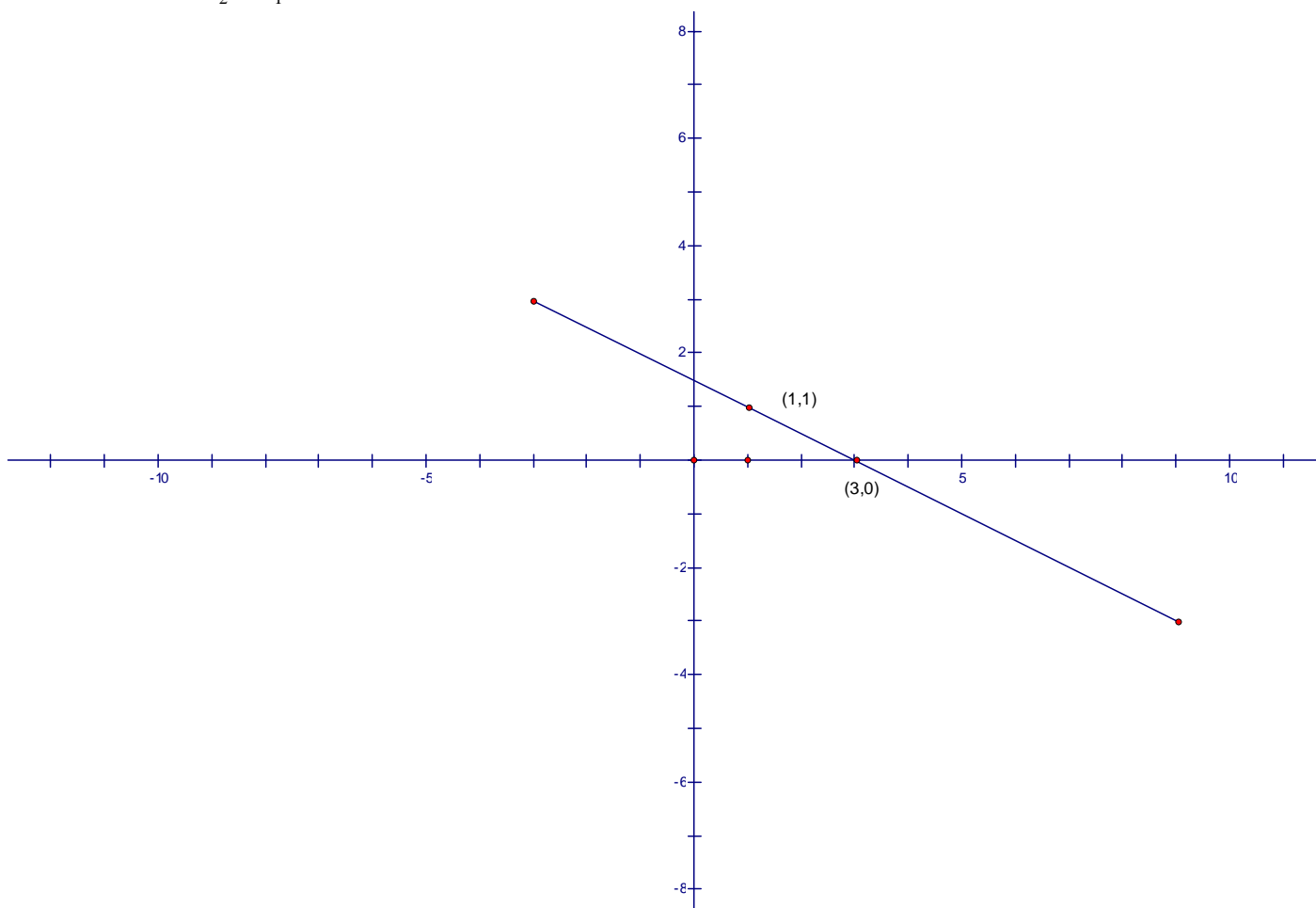
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{4 - 2} = \frac{3}{2}$$

Graph



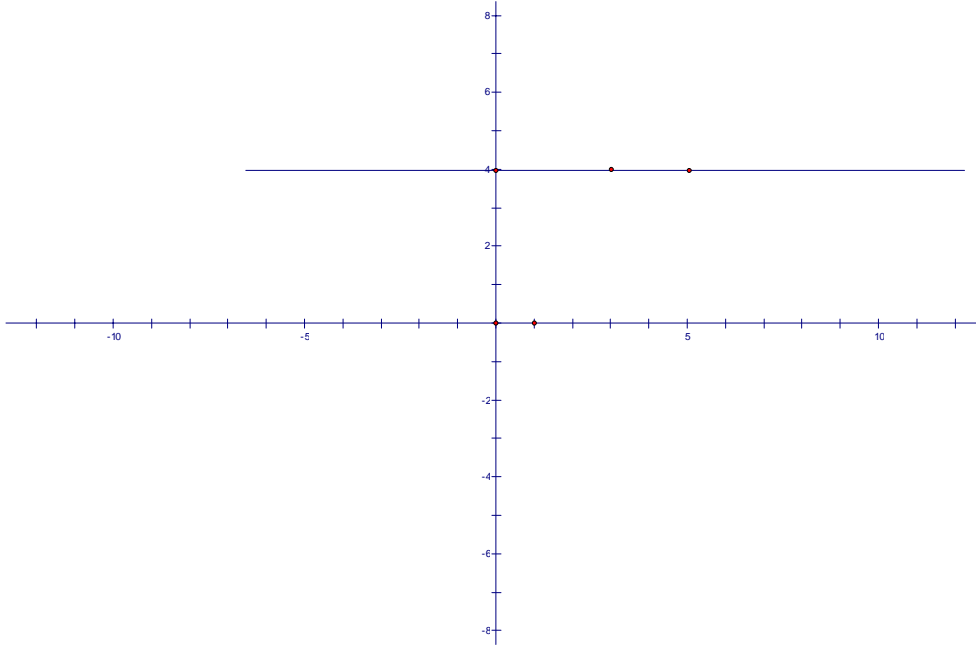
2) (1,1) and (3,0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{3 - 1} = -\frac{1}{2}$$



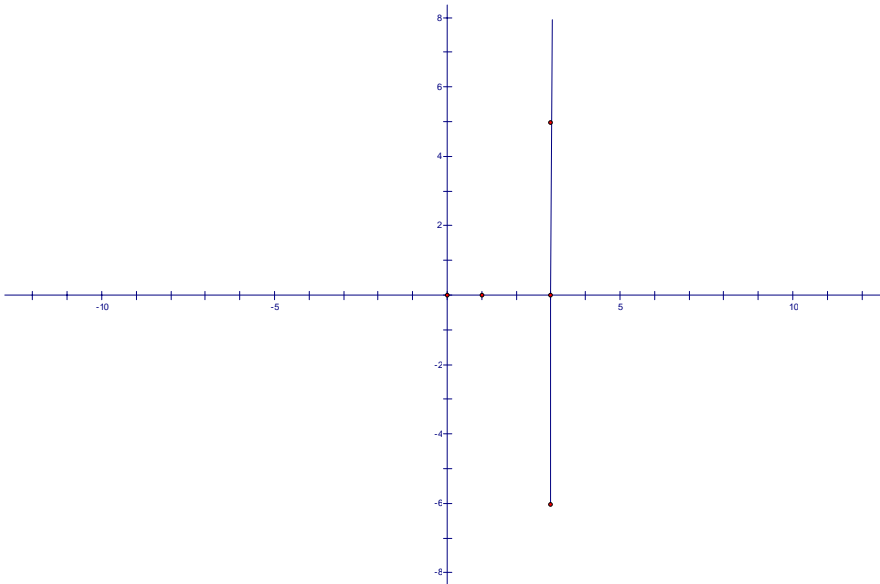
3) (3,4) and (5,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{5 - 3} = \frac{0}{2} = 0$$



c) (3,5) and (3,8)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{3 - 3} = \frac{3}{0}$$



Slope-intercept form of an equation

$$y = mx + b$$

$$m = \text{slope}$$

$$b = y\text{-intercept}$$

Example 2

Find the slope and y-intercept, given the equation of the line.

$$2x + y = 40$$

$$2x - 2x + y = -2x + 40$$

$$y = -2x + 40$$

$$m = -2, b = 40$$

Example 3

Find the slope and y-intercept, given the equation of the line.

$$6x - 5y = 15$$

$$6x - 6x - 5y = -6x + 15$$

$$-5y = -6x + 15$$

$$\frac{-5y}{-5} = \frac{-6x}{-5} + \frac{15}{-5}$$

$$y = \frac{6}{5}x - 3$$

$$m = \frac{6}{5}, b = -3$$

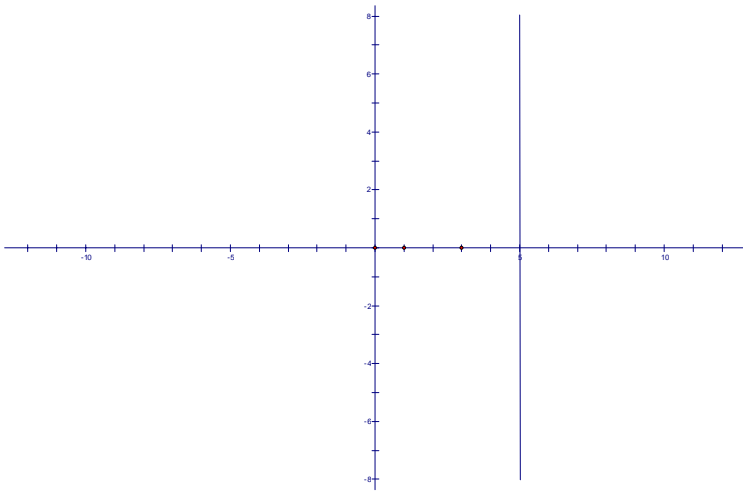
Example 4

Find the slope and y-intercept, given the equation of the line.

$$x + 5 = 0$$

This equation can not be expressed in slope-intercept form, so this line has no slope (undefined slope) and no y intercept

See graph below



Example 5

Find the slope and y-intercept, given the equation of the line.

$$y = -1$$

$$y = 0x - 1$$

$$m = 0, b = -1$$

Point slope formula: $y - y_1 = m(x - x_1)$

Example 6

Write the equation of the line that passes through the given points. (Use the equation to graph the line.): $(-3,-4)$ and $(1,4)$

Find the slope first: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-4)}{1 - (-3)} = \frac{8}{4} = 2$

Next, use the point slope formula and write answer in slope-intercept form with the either point $(-3,-4)$ and $(1,4)$. This example use the point $(1,4)$

$$y - y_1 = m(x - x_1)$$

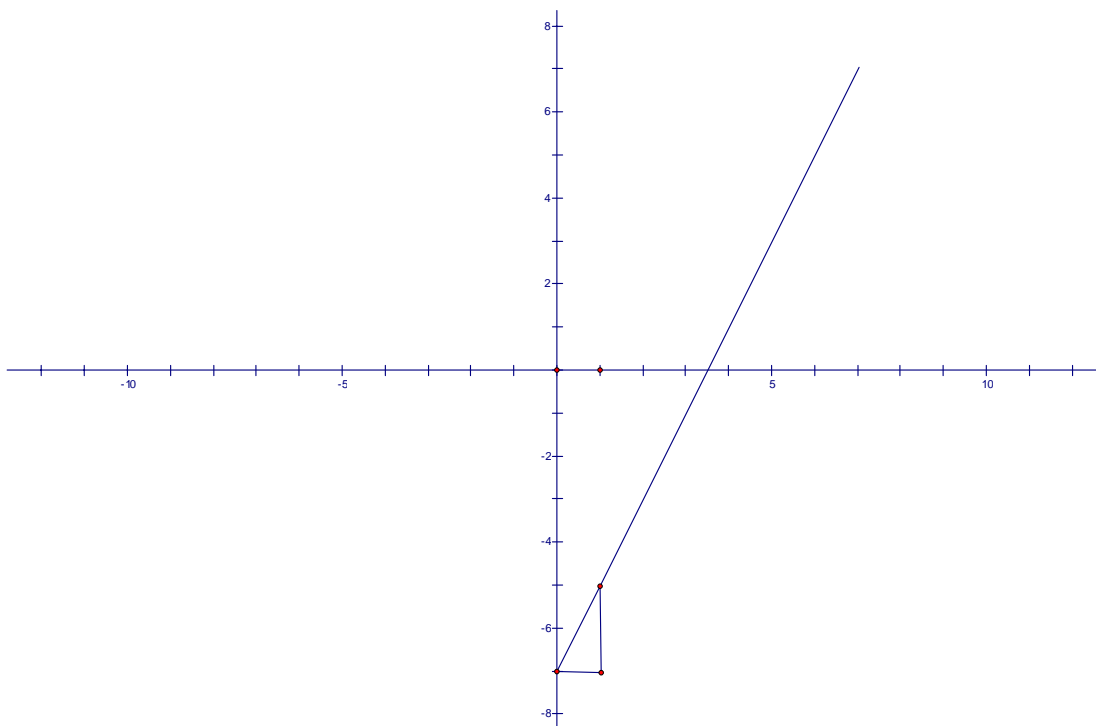
$$y - 4 = 2(x - 1)$$

$$y - 4 = 2x - 2$$

$$y - 4 + 4 = 2x - 2 + 4$$

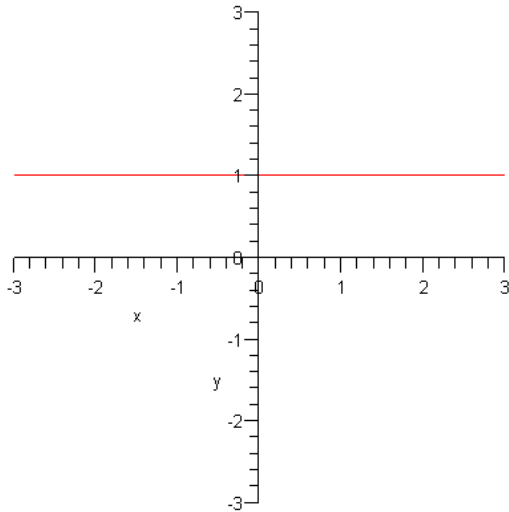
$$y = 2x + 2$$

$$m = 2, b = 2$$



Example 7

Write an equation of the line that passes through the given points. (Use the equation to graph the line.): (6,1) and (10,1)



Find the slope first: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{10 - 6} = \frac{0}{4} = 0$

Use point-slope formula

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 0(x - 1)$$

$$y - 10 = 0$$

$$y = 10$$

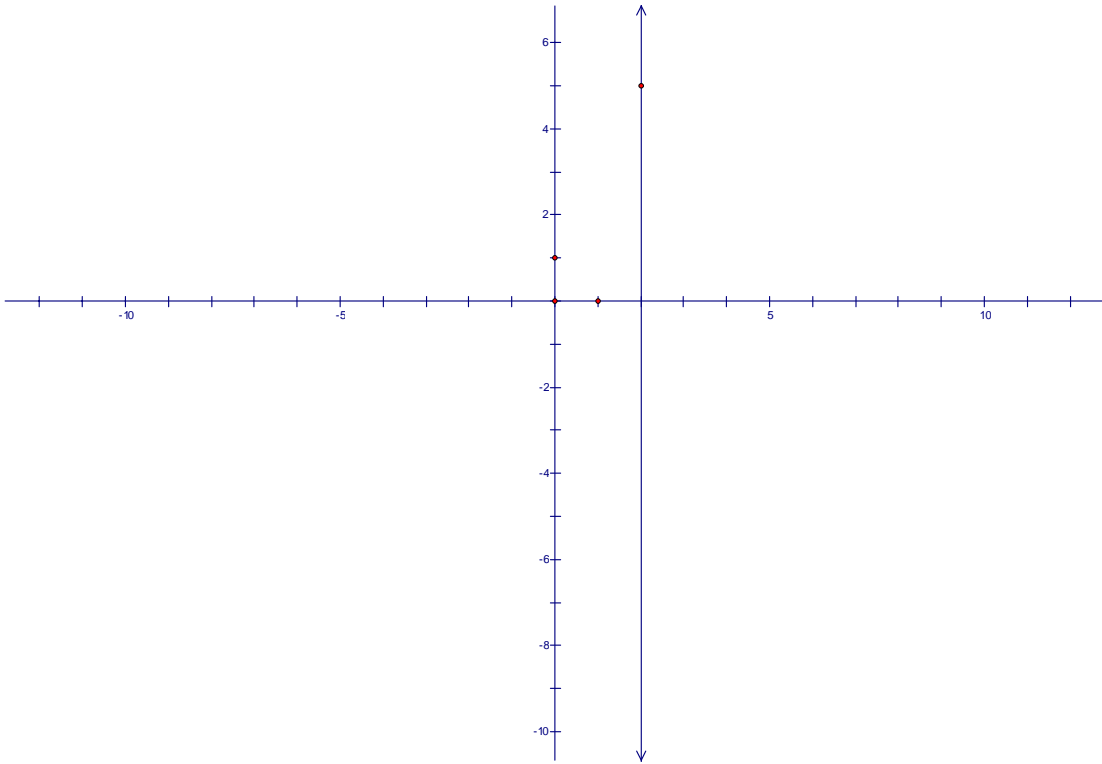
Example 8

Write the equation of the line that passes through the points (2,5) and (2,-10)

Find the slope first:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 5}{2 - 2} = \frac{-15}{0} \text{ (undefined slope)} \Rightarrow \text{the graph has no } y\text{-int or slope}$$

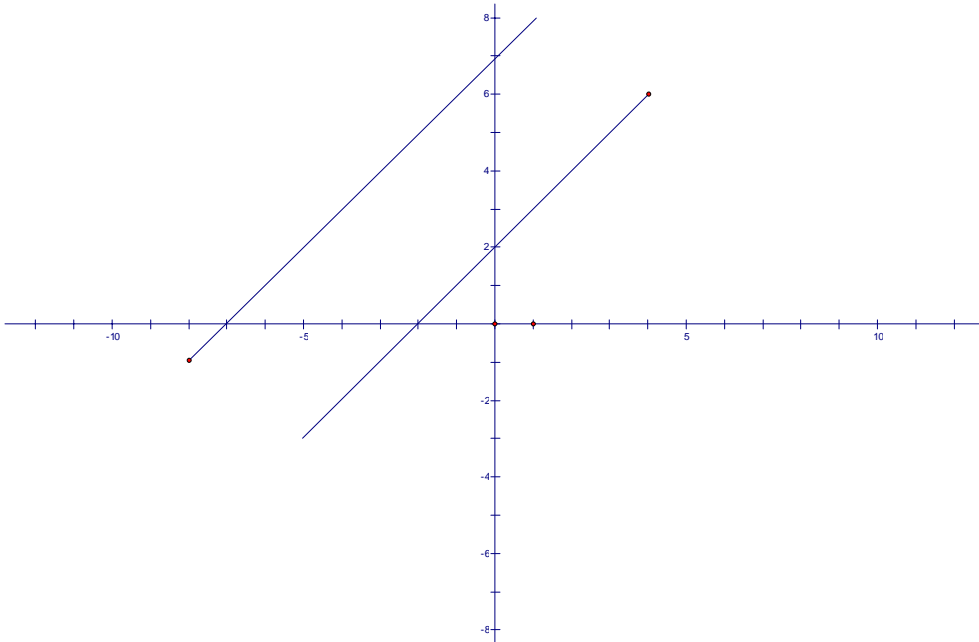
See graph below;



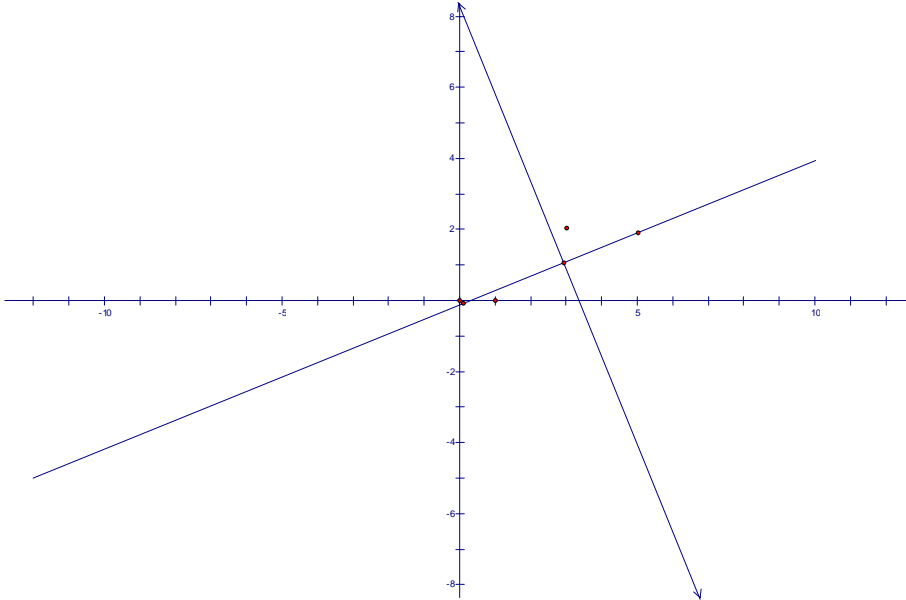
Equation: $x = 2$

Parallel and Perpendicular Lines

Parallel lines have the same slope.



Perpendicular lines have slope that are negative reciprocals of each other.



Example 9

Find the equation of a line that passes through the given point and is:

- parallel to the given line
- perpendicular to the given line

$$\text{line : } 4x - 2y = 3$$

Point on the line: (2,1)

Find the slope the line.

$$4x - 2y = 3$$

$$4x - 4x - 2y = -4x + 3$$

$$-2y = -4x + 3$$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{3}{2}$$

$$y = 2x + \frac{3}{2}$$

$$m = 2, b = \frac{3}{2}$$

Slope of parallel line is the same as the line $\Rightarrow m_{\parallel} = 2$

Slope of the perpendicular line would be $m_{\perp} = -\frac{1}{2}$

Find the equations

Parallel Line

Use $m = 2$ and the point $(2,1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 2)$$

$$y - 1 = 2x - 4$$

$$y = 2x - 3$$

Perpendicular Line

Use $m = -\frac{1}{2}$ and the point $(2,1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y - 1 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 2$$

