

Exponents and Radicals

Definition of an Exponent

Define $a^n = a \cdot a \cdot a \cdot a \cdots a$ or a multiplied out n times

Example 1 Simplify the following exponents

a) $4^3 = 4 \cdot 4 \cdot 4 = 64$

b) $5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 15625$

c) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

d) $-3^4 = -81$

Properties for Exponents

Rule 1 $a^0 = 1$ example $4^0 = 1$

Rule 2 $a^{-n} = \frac{1}{a^n}$ example $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Rule 3 $a^{\frac{1}{n}} = \sqrt[n]{a}$ example $5^{\frac{1}{3}} = \sqrt[3]{5}$

Rule 4 $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ example $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

Example 2

Simplify 5^3

Solution: $5^3 = 5 \cdot 5 \cdot 5 = 125$

Example 3

Simplify 8^0

Solution: $8^0 = 1$

Example 4

Simplify $16^{\frac{5}{2}}$

Solution: $16^{\frac{5}{2}} = (\sqrt[2]{16})^5 = 4^5 = 1024$

Operations with Exponents

Rule 5 $a^n a^m = a^{n+m}$ *example* $x^4 x^7 = x^{4+7} = x^{11}$

Rule 6 $\frac{a^n}{a^m} = a^{n-m}$ *example* $\frac{x^9}{x^4} = x^5$

Rule 7 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ *example* $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

Rule 8 $(ab)^n = a^n b^n$ *example* $(xy)^3 = x^3 y^3$

Rule 9 $(a^n)^m = a^{nm}$ *example* $(x^3)^4 = x^{12}$

Example 5

Simplify $(y^2)^6$

Solution: $(y^2)^6 = y^{12}$ (Rule 9)

Example 6

Simplify $(3m)^4$

Solution: $(3m)^4 = 3^4 m^4 = 81m^4$ (Rule 8)

Example 7

Simplify $(2x^5)^6$

Solution: $(2x^5)^6 = 2^6(x^5)^6 = 64x^{30}$ (Rules 8 and 9)

Example 8

Simplify $\frac{x^{12}}{x^5}$

Solution: $\frac{x^{12}}{x^5} = x^{12-5} = x^7$ (Rule 6)

Example 9

Simplify $\left(\frac{x}{y}\right)^3$

Solution: $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$ (Rule 7)

Example 10 (Practice Problems)**Simplify the following expressions**

1) Simplify $(4x^3)(5x^8)$

Solution: $(4x^3)(5x^8) = 20x^{3+8} = 20x^{11}$

2) Simplify $\frac{20s^{13}t^4}{10s^{10}t^2}$

Solution: $\frac{20s^{13}t^4}{10s^{10}t^2} = 2s^{13-10}t^{4-2} = 2s^3t^2$

3) Simplify $(3xy)^2$

Solution: $(3xy)^2 = 3^2x^2y^2 = 9x^2y^2$

4) Simplify $8^{\frac{4}{3}}$

Solution: $8^{\frac{4}{3}} = (\sqrt[3]{8})^4 = 2^4 = 16$

Examples from the book page 0-18

4) Evaluate $7x^{-2}$ where $x = 4$

Solution: $7x^{-2} = 7(4)^{-2} = \frac{7}{4^2} = \frac{7}{16}$

8) Evaluate $5(-x)^3$ where $x = 3$

Solution: $5(-x)^3 = 5(-3)^3 = 5(-27) = -135$

14) Evaluate $x^{-\frac{3}{4}}$ where $x = 16$

Solution: $x^{-\frac{3}{4}} = 16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$

Example 11

Simplify $(8x^4)(2x^3)$

$$(8x^4)(2x^3) = 16x^{4+3} = 16x^7$$

Example 12

Simplify $(4x^3)^2$

$$(4x^3)^2 = 4^2(x^3)^2 = 16x^6$$

Example 13

Simplify $\frac{r^4}{r^6}$

$$\frac{r^4}{r^6} = r^{4-6} = r^{-2} = \frac{1}{r^2}$$

Example 14

Simplify $(2x^2yz^5)^0$

$$(2x^2yz^5)^0 = 1$$
