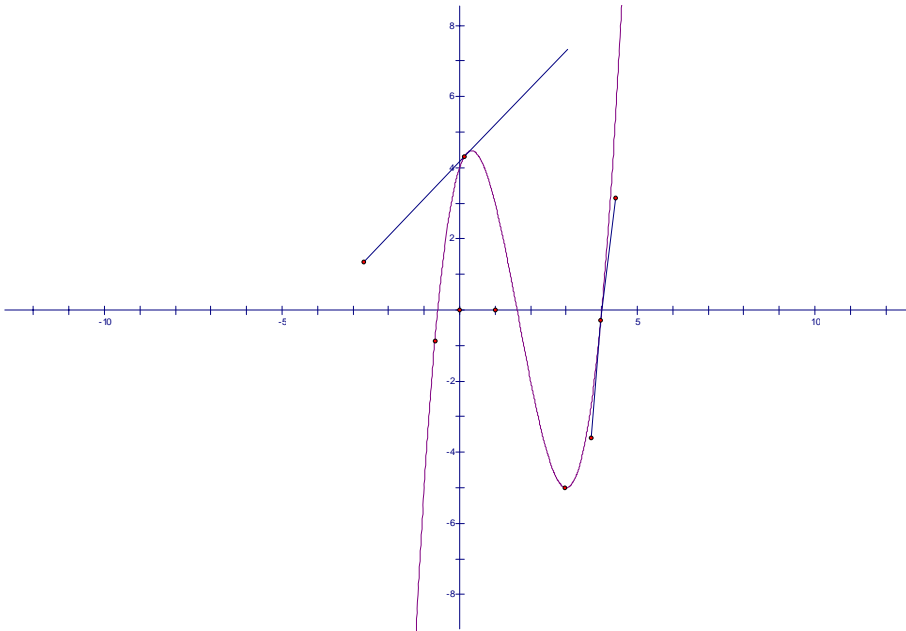


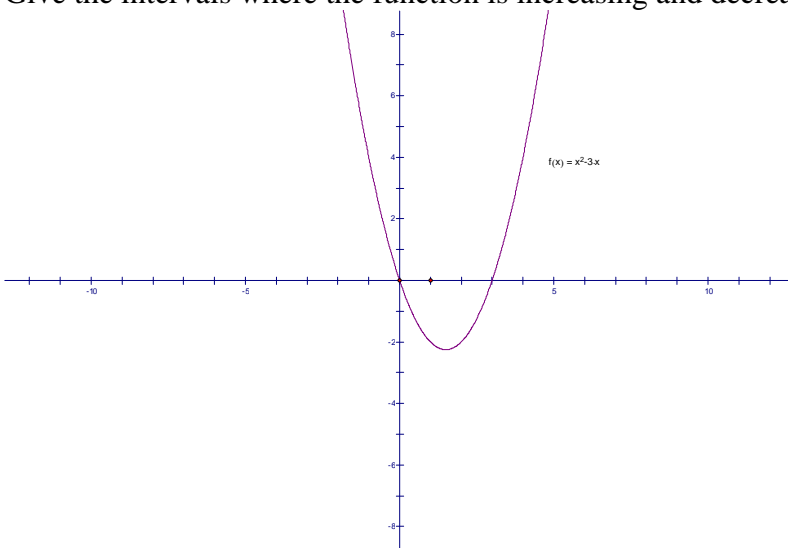
## Section 3.1

### Increasing and Decreasing Functions



#### Example 1

Give the intervals where the function is increasing and decreasing.



Decreasing:  $\left(-\infty, \frac{3}{2}\right)$

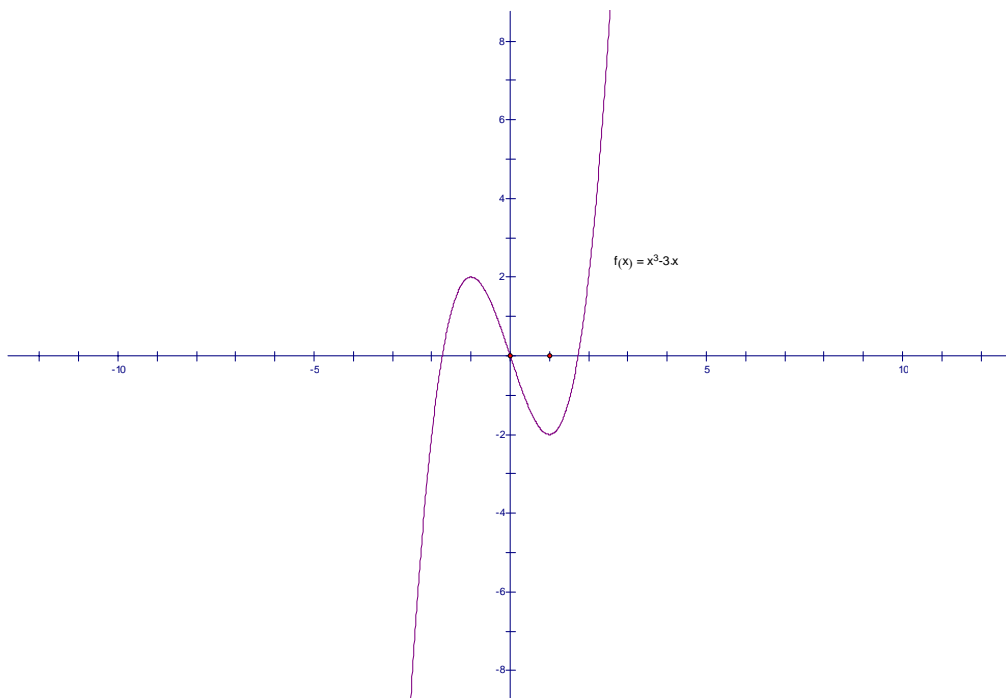
Increasing:  $\left(\frac{3}{2}, \infty\right)$

---

---

### Example 2

Give the intervals where the function increasing and decreasing



Increasing:  $(-\infty, -1) \cup (1, \infty)$  Decreasing:  $(-1, 1)$

---

### Test for increasing and decreasing functions

Let  $f$  be a differentiable function on the interval

- 1) If  $f'(x) > 0$  for all  $x$  in  $(a, b)$ , then  $f$  is increasing on  $(a, b)$
  - 2) If  $f'(x) < 0$  for all  $x$  in  $(a, b)$ , then  $f$  is decreasing on  $(a, b)$
  - 3) If  $f'(x) = 0$  for all  $x$  in  $(a, b)$ , then  $f$  is constant on  $(a, b)$
-

---

**Example 3**

Give the intervals where the function is increasing and decreasing

$$f(x) = x^2 - 4x$$

$$f'(x) = 2x - 4$$

$$2x - 4 = 0$$

$$2x - 4 + 4 = 0 + 4$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

$$f'(-1) = 2(-1) - 4 = -2 - 4 = -6$$

$$f'(3) = 2(3) - 4 = 2$$

Interval	$(-\infty, 2)$	$(2, \infty)$
Test Value	$x = -1$	$x = 3$
Sign of $f'(x)$	Negative	Positive
Conclusion	Decreasing	Increasing

---

---

**Example 4**

Find the intervals where the function is decreasing and increasing

$$f(x) = x^3 - 4$$

$$f'(x) = 3x^2$$

$$3x^2 = 0$$

$$\frac{3x^2}{3} = \frac{0}{3}$$

$$x^2 = 0$$

$$x = 0$$

$$f'(-1) = 3(-1)^2 = 3$$

$$f'(1) = 3(1)^2 = 3$$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Positive	Positive
Conclusion	Increasing	Increasing

---

---

**Example 5**

Find the intervals where the function is increasing or decreasing.

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$3x = 0 \text{ or } x - 2 = 0$$

$$\frac{3x}{3} = \frac{0}{3} \text{ or } x - 2 + 2 = 0 + 2$$

$$x = 0 \qquad x = 2$$

$$f'(-1) = 3(-1)^2 - 6(-1) = 3 + 6 = 9$$

$$f'(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$$

$$f'(3) = 3(3)^2 - 6(3) = 27 - 18 = 9$$

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Test Value	$x = -1$	$x = 1$	$x = 3$
Sign of $f'(x)$	Positive	Negative	Positive
Conclusion	Increasing	Decreasing	Increasing

---

---

**Example 6**

Find the intervals where the functions is increasing and decreasing

$$f(x) = x^2 - 4$$

$$f'(x) = 2x$$

$$2x = 0$$

$$\frac{2x}{2} = \frac{0}{2}$$

$$x = 0$$

$$f'(-1) = 2(-1) = -2$$

$$f'(1) = 2(1) = 2$$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Negative	Positive
Conclusion	Decreasing	Increasing

---

**Exercises 3.1**

Find the intervals where the functions is increasing and decreasing.

1)  $f(x) = -x^2 - 2x$

2)  $f(x) = x^3 + 3$

3)  $f(x) = x^2 - 6x + 5$

4)  $f(x) = x^3 - 6x^2$

5)  $f(x) = -2x^4$

6)  $f(x) = x^3 - x$