

Math 121

Section 1.2 Graphing Equations

Sketching Graphs

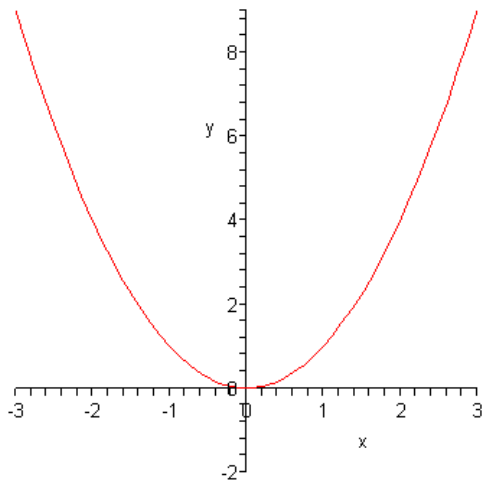
Basic Families of Graphs

Example 1 (The Standard Parabola)

Graph $y = x^2$

x	y
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$

Plot the following values $(-2,4),(-1,1),(0,0),(1,1),(2,4)$ from the table will give the following graph.

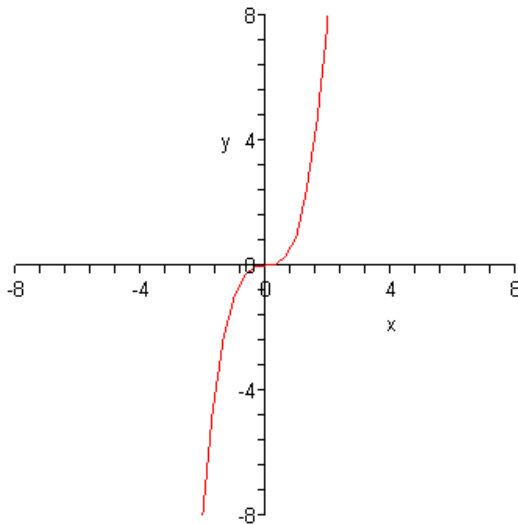


Example 2 (The standard “cubic” graph)

Graph $y = x^3$

x	$y = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$

Plot the values from the table will result in the following graph

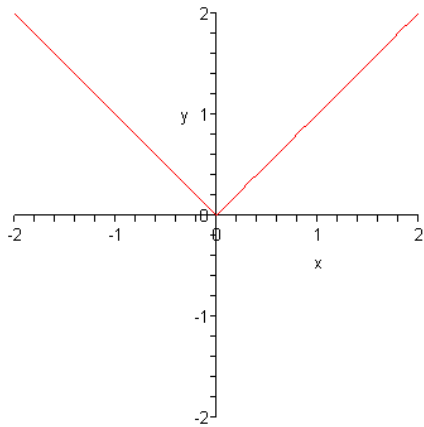


Example 3 (The Standard Absolute Value Graph)

Graph $y = |x|$

x	$y = x $
-2	$ -2 = 2$
-1	$ -1 = 1$
0	$ 0 = 0$
1	$ 1 = 1$
2	$ 2 = 2$

Plot the values from the table will give you a v-shaped graph



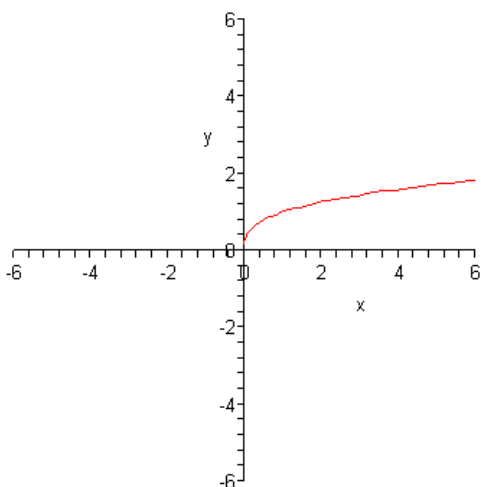
Example 4 (Standard Square Root Graph)

Graph $y = \sqrt{x}$

Again, use a table of values to make a graph of the equation

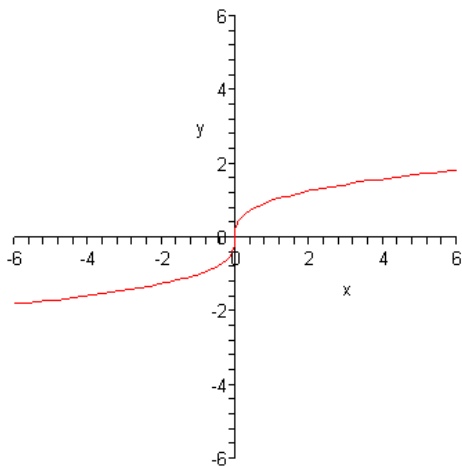
x	$y = \sqrt{x}$
0	$\sqrt{0} = 0$
1	$\sqrt{1} = 1$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$

Resulting Graph



Example 5

The graph of $y = \sqrt[3]{x}$



Horizontal and Vertical Translations (Shifts)

Horizontal Translation: An operation that moves the graph of an equation to the left or right while at the same time preserves the shape of the graph.

Vertical Translation: An operation that moves the graph of an equation to the left or right while at the same time preserves the shape of the graph.

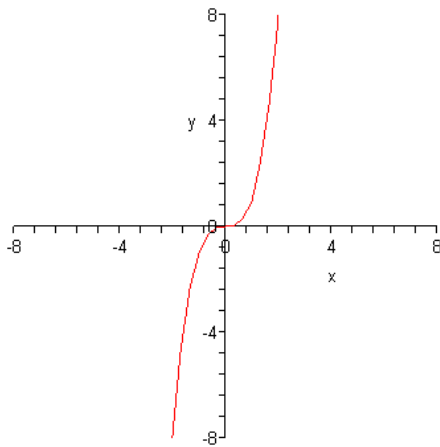
Example 6

Example of a Vertical Translation

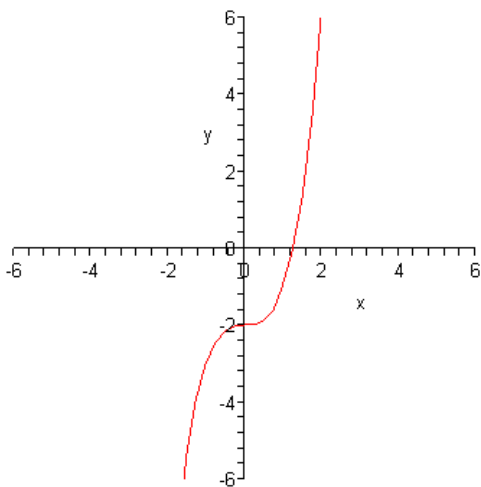
$$y = x^3 - 2$$

Since the -2 lies outside the x^3 term, the value -2 indicates a vertical translation of 2 units. The negative sign in value of -2 indicates that the translation will move the graph of $y = x^3$ down two units as shown below:

The graph of $y = x^3 - 2$



The graph of $y = x^3 - 2$ shifted down to units



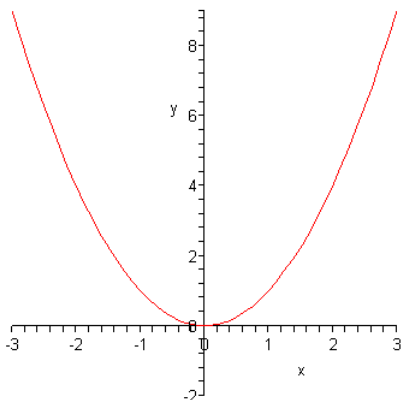
Example 7

Example of a Horizontal Translation

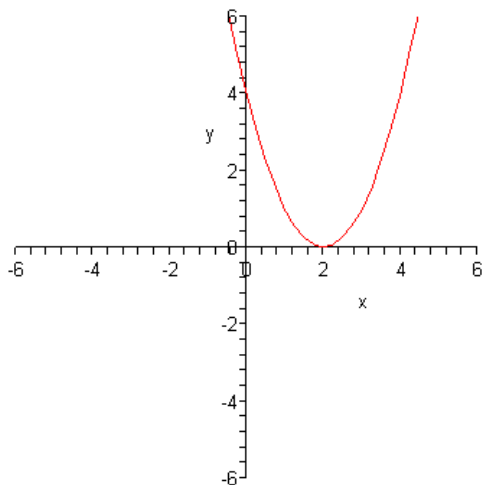
$$y = (x - 2)^2$$

In this example, the -2 inside the parentheses indicates that there is a horizontal translation of two units to the right. A negative sign inside the parentheses will always result in a shift to the right.

The original graph of $y = x^2$

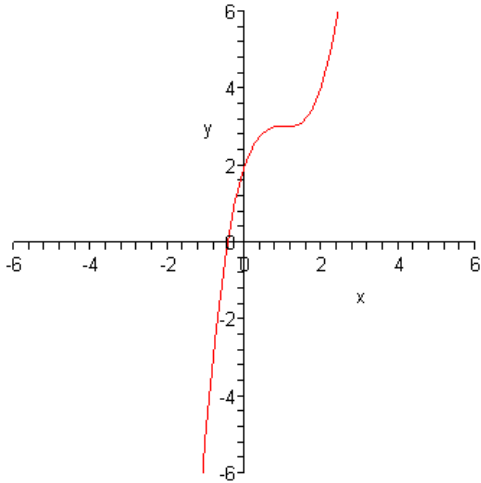


The graph of $y = x^2$ after a horizontal translation of 2 units to the right



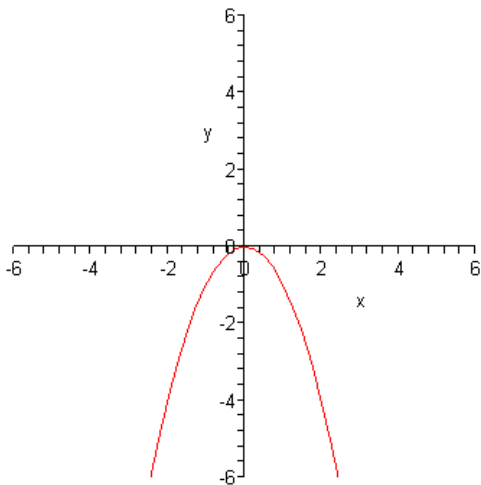
Example 8

The graph of $y = (x-1)^3 + 3$



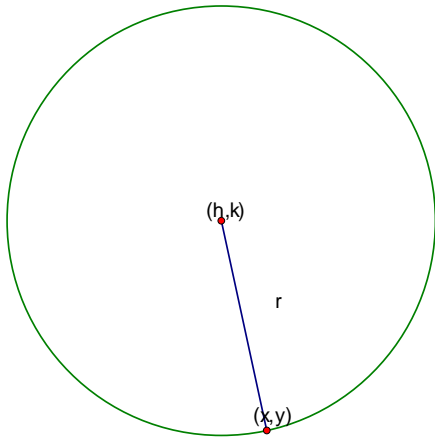
Example 9 The graph of $y = -x^2$

The graph of $y = -x^2$ is the inverted graph of $y = x^2$. The negative sign in front of the x^2 term simply turns the graph of $y = x^2$ upside down.



Circles

Given a circle with center (h,k) and radius r , use the distance formula to find the equation of the circle



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

$$r^2 = \left(\sqrt{(x - h)^2 + (y - k)^2} \right)^2$$

$$r^2 = (x - h)^2 + (y - k)^2$$

Thus, the equation a circle in standard form with radius r and center (h,k) is given by

$$r^2 = (x - h)^2 + (y - k)^2$$

General Form of an Equation of a circle

$$x^2 + y^2 + ax + by + c = 0$$

Example 10

Write the equation of a circle in standard form given the radius is 5 and the center is (0,0).

$$(x-0)^2 + (y-0)^2 = 5^2$$

$$x^2 + y^2 = 25$$

$$x^2 + y^2 - 25 = 25 - 25$$

$$x^2 + y^2 - 25 = 0$$

Example 11

Write the equation of a circle in standard form given the radius is 4 and the center is (-4,5).

$$(x - (-4))^2 + (y - 5)^2 = 4^2$$

$$(x + 4)^2 + (y - 5)^2 = 16$$

$$(x + 4)(x + 4) + (y - 5)(y - 5) = 16$$

$$x^2 + 4x + 4x + 16 + y^2 - 5y - 5y + 25 = 16$$

$$x^2 + y^2 + 8x - 10y + 41 = 16$$

$$x^2 + y^2 + 8x - 10y + 41 - 16 = 16 - 16$$

$$x^2 + y^2 + 8x - 10y + 25 = 0$$

Example 12

Find the center and radius of a circle with the given equation

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

$$x^2 + y^2 + 4x - 6y - 3 + 3 = 0 + 3 \text{ Add 3 to both sides}$$

$$x^2 + 4x + y^2 - 6y = 3$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 3 + 4 + 9 \text{ Complete the square for the 4x and 6y terms}$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$(x + 2)(x + 2) + (y - 3)(y - 3) = 16$$

$$(x + 2)^2 + (y - 3)^2 = 4^2$$

$$\text{center} = (-2, 3), \text{radius} = 4$$

Note: To complete the square for the 4x term take half of 4 which is 2 and find

$$2^2 = 4$$

To complete the square for the 6y term take half of 6 which is 3 and find $3^2 = 9$

Example 13

Find the center and radius of a circle with the given equation

$$x^2 + y^2 - 8x - 2y - 8 = 0$$

$$x^2 + y^2 - 8x - 2y - 8 + 8 = 0 + 8 \text{ Add 8 to both sides}$$

$$x^2 - 8x + y^2 - 2y = 8$$

$$x^2 - 8x + 16 + y^2 - 2y + 1 = 8 + 16 + 1 \text{ Complete the square for the 8x and } -2y \text{ terms}$$

$$x^2 - 8x + 16 + y^2 - 2y + 1 = 25$$

$$(x - 4)(x - 4) + (y - 1)(y - 1) = 25$$

$$(x - 4)^2 + (y - 1)^2 = 5^2$$

$$\text{center} = (4, 1)$$

$$\text{radius} = 5$$

Note: To complete the square for the -8x term take half of 8 which is 4 and find

$$4^2 = 16$$

To complete the square for the -2y term take half of 2 which is 1 and find $1^2 = 1$

Example 14

Find the center and radius of a circle with the given equation

$$x^2 + y^2 - 9 = 0$$

$$x^2 + y^2 - 9 + 9 = 0 + 9 \text{ Add 9 to both sides}$$

$$x^2 + y^2 = 9$$

$$(x-0)^2 + (y-0)^2 = 3^2$$

$$\text{center} = (0,0)$$

$$\text{radius} = 3$$
