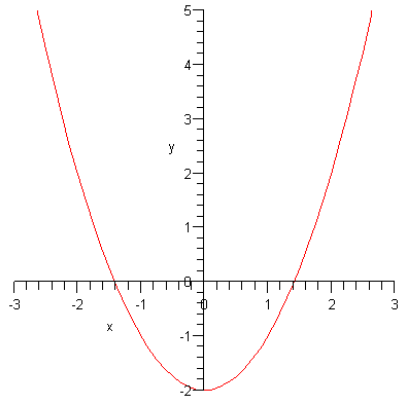


# Math 121

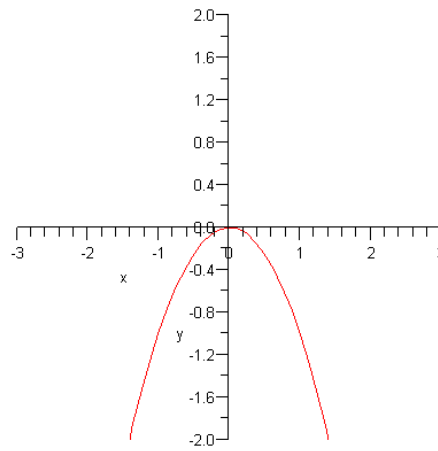
## Section 3.2

### Introduction to Extrema Points

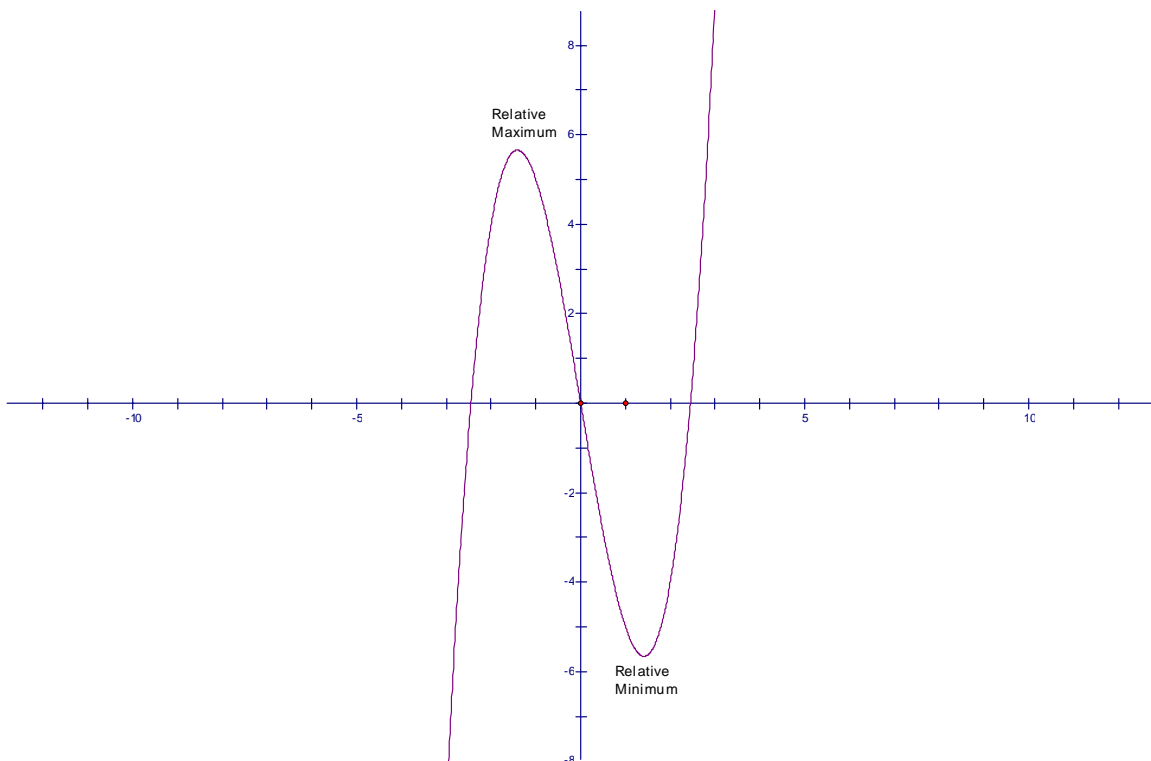
Examples of extrema points



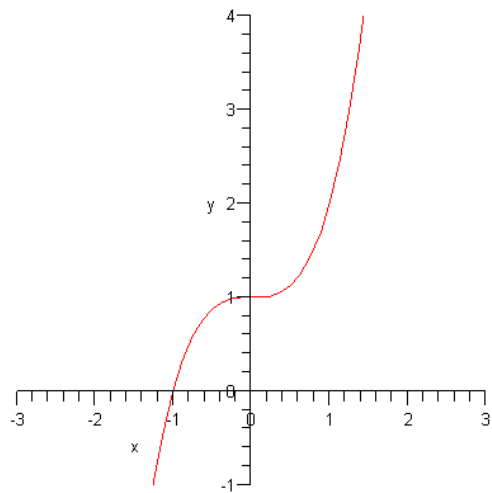
Absolute Minimum



Absolute Maximum



## Relative Minimum and Relative Maximum



## Inflection Point

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### **Types of Critical Points**

- 1) Relative Maximum
  - 2) Relative Minimum
  - 3) Absolute Maximum
  - 4) Absolute Minimum
-

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## Testing for extrema points over all real numbers

### Example 1

Find all extrema points of the function.

$$f(x) = x^2 + 2x$$

$$f'(x) = 2x + 2$$

$$2x + 2 = 0$$

$$2x + 2 - 2 = 0 - 2$$

$$2x = -2$$

$$\frac{2x}{2} = \frac{-2}{2}$$

$$x = -1$$

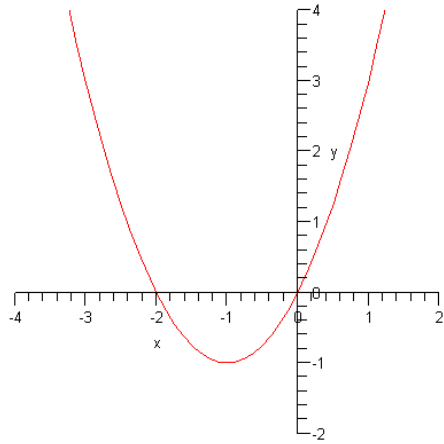
$$f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$$

*Critical pt.*  $(-1, -1)$

$$f'(-2) = 2(-2) + 2 = -2$$

$$f'(1) = 2(1) + 2 = 4$$

|                 |                 |                |
|-----------------|-----------------|----------------|
| Interval        | $(-\infty, -1)$ | $(-1, \infty)$ |
| Test Value      | $x = -2$        | $x = 1$        |
| Sign of $f'(x)$ | Negative        | Positive       |
| Conclusion      | Decreasing      | Increasing     |



The function is increasing when  $x$  is less than  $-1$  and increasing when  $x$  is greater than  $-1$ .  
 $\Rightarrow$  At  $(-1, -1)$ ,  $f$  has an absolute min.

### Example 2

Find all extrema points of the function.  $f(x) = x^3 - 1$

$$f(x) = x^3 - 1$$

$$f'(x) = 3x^2$$

$$3x^2 = 0$$

$$\frac{3x^2}{3} = \frac{0}{3}$$

$$x^2 = 0$$

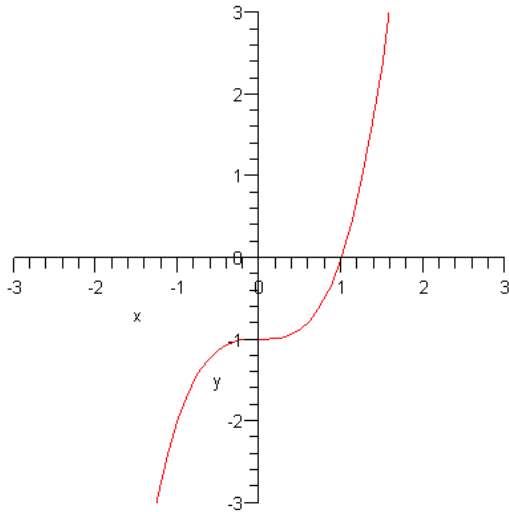
$$x = 0$$

$$\Rightarrow f(0) = 0^3 - 1 = -1 \Rightarrow \text{critical pt. is } (0, -1)$$

$$f'(-1) = 3(-1)^2 = 3$$

$$f'(1) = 3(1)^2 = 3$$

|                 |                |               |
|-----------------|----------------|---------------|
| Interval        | $(-\infty, 0)$ | $(0, \infty)$ |
| Test Value      | $x = -1$       | $x = 1$       |
| Sign of $f'(x)$ | Positive       | Positive      |
| Conclusion      | Increasing     | Increasing    |



The function is increasing when  $x$  is less than zero and greater than zero, so the function has an inflection point at  $(0, -1)$ . (See graph above)

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### Example 3

Find all extrema points of the function.  $f(x) = 2x^3 - 3x^2$

$$f(x) = 2x^3 - 3x^2$$

$$f'(x) = 6x^2 - 6x$$

$$6x^2 - 6x = 0$$

$$6x(x - 1) = 0$$

$$6x = 0 \text{ or } x - 1 = 0$$

$$\frac{6x}{6} = \frac{0}{6} \quad x = 1$$

$$x = 0$$

$$f(0) = 2(0)^3 - 3(0)^2 = 0 - 0 = 0$$

$$f(1) = 2(1)^3 - 3(1)^2 = 2 - 3 = -1$$

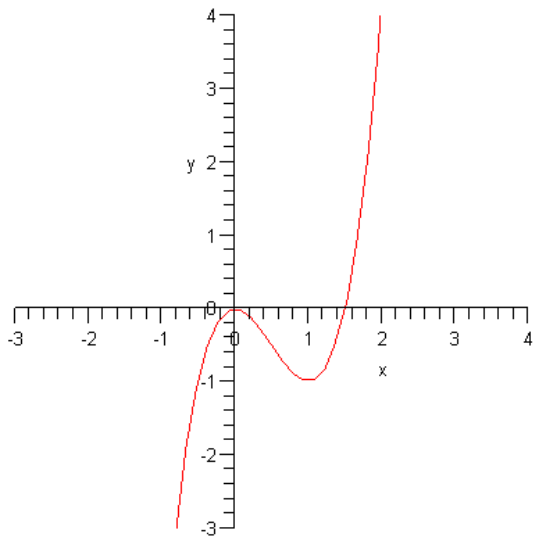
*Testing the derivative*

$$f'(-1) = 6(-1)^2 - 6(-1) = 6 + 6 = 12$$

$$f'\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) = \frac{3}{2} - 3 = -\frac{3}{2}$$

$$f'(2) = 6(2)^2 - 6(2) = 24 - 12 = 12$$

|                 |                |                   |               |
|-----------------|----------------|-------------------|---------------|
| Interval        | $(-\infty, 0)$ | $(0, 1)$          | $(1, \infty)$ |
| Test Value      | $x = -1$       | $x = \frac{1}{2}$ | $x = 2$       |
| Sign of $f'(x)$ | Positive       | Negative          | Positive      |
| Conclusion      | Increasing     | Decreasing        | Increasing    |



The function has a relative maximum at  $x = 0$  and a relative minimum at  $(0, 0)$  and  $(1, -1)$   
(See diagram above)

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**Example 4**

Find all extrema points of the function.  $f(x) = -x^2$

$$f(x) = -x^2$$

$$f'(x) = -2x$$

$$-2x = 0$$

$$\frac{-2x}{-2} = \frac{0}{-2}$$

$$x = 0$$

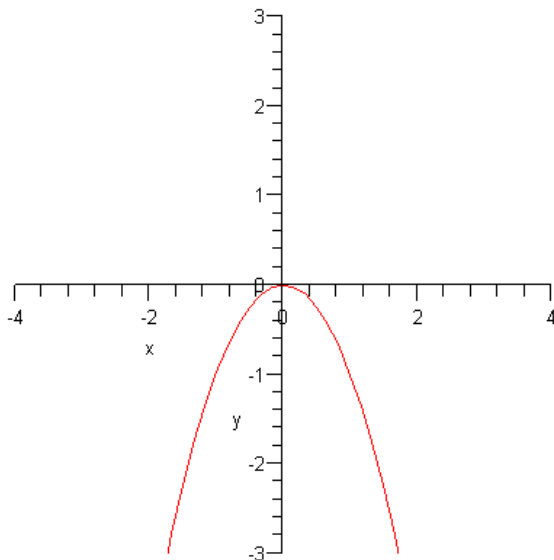
$$f'(-1) = -2(-1) = 2$$

$$f'(1) = -2(1) = -2$$

|                 |                |               |
|-----------------|----------------|---------------|
| Interval        | $(-\infty, 0)$ | $(0, \infty)$ |
| Test Value      | $x = -1$       | $x = 1$       |
| Sign of $f'(x)$ | Positive       | Negative      |
| Conclusion      | Increasing     | Decreasing    |

The function has an absolute maximum at  $x = 0$

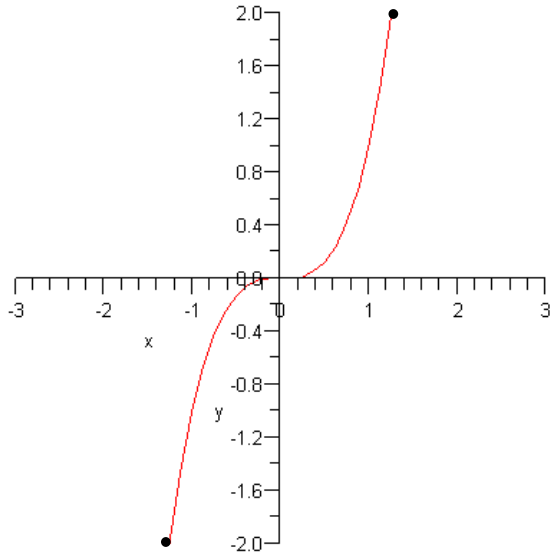
See Graph



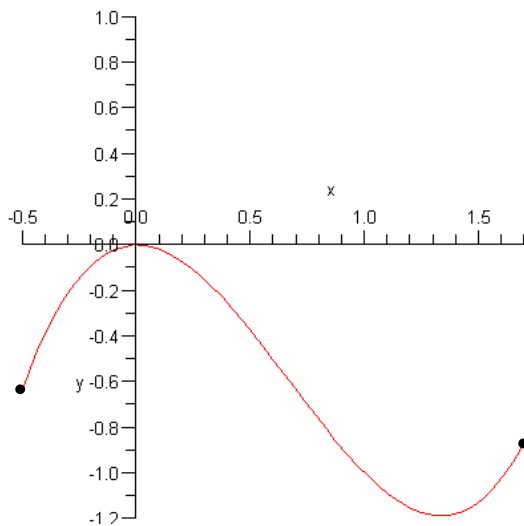
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**Critical points on a closed interval.** The endpoints of the closed interval must be considered as critical points. It turns out that the endpoints of a closed interval can either be relative extrema or absolute extrema.

**Here is an example of a graph where one endpoint is an absolute maximum and the other endpoint is an absolute minimum.**



**This is an example of a graph where one endpoint is a relative maximum and the other endpoint is a relative minimum.**



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**Example 5**

Find all extreme points of the function  $f(x) = x^2 - 4x$  on the interval  $[0,4]$

$$f(x) = x^2 - 4x$$

$$f'(x) = 2x - 4$$

$$2x - 4 = 0$$

$$2x - 4 + 4 = 0 + 4$$

$$2x = 4$$

$$x = 2$$

Test the points at  $x = 0, x = 2, x = 4$

Find the y-coordinates of these points

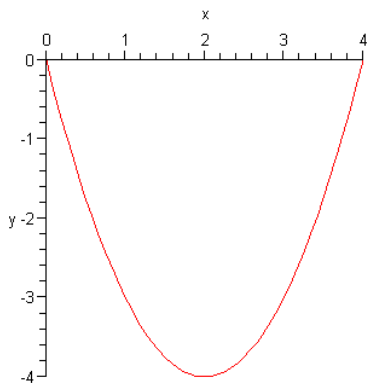
$$f(0) = 0^2 - 4(0) - 0 = 0$$

$$f(2) = 2^2 - 4(2) = 4 - 8 = -4$$

$$f(4) = 4^2 - 4(4) = 16 - 16 = 0$$

The critical points are at  $(0,0), (2,-4), (4,0)$

Make a sketch of the graph using these points.



Looking at the graph above there is an absolute minimum at  $(2,-4)$  and relative maximums at  $(0,0)$  and  $(4,0)$ .

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### Example 6

Find all extreme points of the function  $f(x) = x^3 + 1$  on the interval  $[-2,2]$

$$f(x) = x^3 + 1$$

$$f'(x) = 3x^2$$

$$3x^2 = 0$$

$$\frac{3x^2}{3} = \frac{0}{3}$$

$$x^2 = 0$$

$$\sqrt{x^2} = \sqrt{0}$$

$$x = 0$$

Test the points at  $x = 0, x = -2, x = 2$

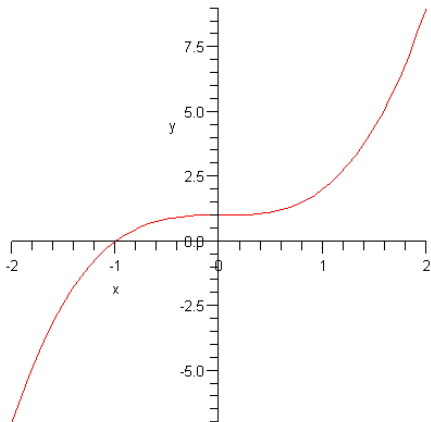
$$f(0) = 0^3 + 1 = 0 + 1 = 1$$

$$f(-2) = (-2)^3 + 1 = -8 + 1 = -7$$

$$f(2) = 2^3 + 1 = 9$$

Critical Points are  $(-2,-7)$ ,  $(0,1)$ , and  $(2,9)$

Sketch a graph using these points



Critical Points:

$(-2,-7) \Rightarrow$  *Absolute Minimum*

$(0,1) \Rightarrow$  *Inflection Pt*

$(2,9) \Rightarrow$  *Absolute Maximum*

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### Exercises Section 3.2

Find all extrema points of the given functions.

1)  $f(x) = 2x^2 + 8x$

2)  $f(x) = 4x^3 - 4$

3)  $f(x) = x^3 - 3x$

4)  $f(x) = 4x^3 + 6x^2$

For each functions listed below:

- Find the intervals where the function is increasing and the function is decreasing
- Find all extrema points
- Make a sketch of the graph

5)  $f(x) = x^2 - 8x$

6)  $f(x) = x^3 + 6x^2$

Find all extrema points of the given function on the given interval

7)  $f(x) = x^2 + 1; [-2,2]$

8)  $f(x) = x^3 - 2; [-1,2]$

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