

Math 121

Section 2.6

Higher-Order Derivatives

First Derivative

Second Derivative

Third Derivative

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2$$

$$f'''(x) = 24x$$

1 st derivative	$f'(x)$	$\frac{dy}{dx}$
2 nd derivative	$f''(x)$	$\frac{d^2 y}{dx^2}$
3 rd derivative	$f'''(x)$	$\frac{d^3 y}{dx^3}$
4 th derivative	$f^{(4)}(x)$	$\frac{d^4 y}{dx^4}$
n th derivative	$f^n(x)$	$\frac{d^n y}{dx^n}$

Example 1

Given $f(x) = 4x^5 - 3x^2 + 8$, find $f''(x)$

$$f'(x) = 20x^4 - 6x$$

$$f''(x) = 80x^3 - 6$$

Example 2

Given $f(x) = \frac{3}{x^2}$, find $f'''(x)$

$$f(x) = 3x^{-2}$$

$$f'(x) = -2 \cdot 3x^{-2-1} = -6x^{-3}$$

$$f''(x) = 18x^{-3-1}$$

$$f''(x) = 18x^{-4}$$

$$f''(x) = \frac{18}{x^4}$$

Example 3

Given $f(x) = 3x^2 + 4x$, find $f''(x)$

$$f(x) = 3x^2 + 4x$$

$$f'(x) = 6x + 4$$

$$f''(x) = 6$$

Example 4

Given $f(x) = x^5 - 3x^4$, find $f'''(x)$

$$f(x) = x^5 - 3x^4$$

$$f'(x) = 5x^4 - 12x^3$$

$$f''(x) = 20x^3 - 36x^2$$

$$f'''(x) = 60x^2 - 72x$$

Example 5 Given $f(x) = 9 - x^2$, find $f''(\sqrt{5})$

$$f(x) = 9 - x^2$$

$$f'(x) = -2x$$

$$f''(x) = -2$$

$$f''(\sqrt{5}) = -2$$

Example 6

Given $f(x) = 3x^3 - 9x + 1$, find the second derivative and solve for $f''(x) = 0$

Solve for $f''(x) = 0$

$$f(x) = 3x^3 - 9x + 1$$

$$f'(x) = 9x^2 - 9$$

$$f''(x) = 18x$$

$$18x = 0$$

$$\frac{18x}{18} = \frac{0}{18}$$

$$x = 0$$

Example 7

Given $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 2$, find the second derivative and solve for $f''(x) = 0$

Solve for $f''(x) = 0$

$$f(x) = x^4 - 8x^3 + 18x^2 - 16x + 2$$

$$f'(x) = 4x^3 - 24x^2 + 36x - 16$$

$$f''(x) = 12x^2 - 48x + 36$$

$$12x^2 - 48x + 36 = 0$$

$$12(x^2 - 4x + 3) = 0$$

$$12(x-3)(x-1) = 0$$

$$x-3=0 \text{ or } x-1=0$$

$$x=3 \quad x=1$$
