Section 2.4

Product and Quotient Rule

The Product Rule

If $f(x) = F(x)S(x)$, *then* $f'(x) = F'(x)S(x) + S'(x)F(x)$

Example 1

Given $f(x) = (x^2 - 4x)(x^2 - 3x)$, find $f'(x)$

$f'(x) = (2x-4)(x^2 - 3x) + (2x-3)(x^2 - 4x) = 2x^3 - 6x^2 - 4x^2 - 12x + 2x^3 - 8x^2 - 3x^2 + 12x$
$= 4x^3 - 21x^2 + 24x$

Example 2

Given $f(x) = (5x^3 + 4x^2 + 4x)(2x^2 + 6x)$, find $f'(x)$

$f'(x) = (15x^2 + 8x + 4)(2x^2 + 6x) + (4x+6)(5x^3 + 4x^2 + 4x)$

Example 3

Given $f(x) = (6x^3 - 5x + 2)(x^5 + x^4)$, find $f'(x)$

$f'(x) = (18x^2 - 5)(x^5 + x^4) + (5x^4 + 4x^3)(6x^3 - 5x + 2)$

Example 4

Given $g(x) = (3x^4 - 2x^3 + 10x^2 + 7x)(3x^6 + 2x^3)$, find $f'(x)$

$g'(x) = (12x^3 - 6x^2 + 20x + 7)(3x^6 + 2x^3) + (18x^5 + 6x^2)(3x^4 - 2x^3 + 10x^2 + 7x)$

Quotient Rule

$$If\ \ f(x)=\frac{T(x)}{B(x)},\ then\ \ f'(x)=\frac{B(x)T'(x)-T(x)B'(x)}{\left(B(x)\right)^2}$$

Example 5

Given $f(x)=\dfrac{x^2}{3x^3-4x+5}$, find $f'(x)$

$$f'(x)=\frac{(3x^3-4x+5)(2x)-(x^2)(9x^2-4)}{(3x^3-4x+5)^2}$$

$$f'(x)=\frac{6x^4-8x^2+10x-9x^4+4x^2}{(3x^3-4x+5)^2}$$

$$f(x)=\frac{-3x^4-4x^2+10x}{(3x^3-4x+5)^2}$$

Example 6

Given $f(x)=\dfrac{x^2-5x}{x^2-3x+3}$, then $f'(x)$

$$f'(x)=\frac{(x^2-3x+3)(2x-5)-(x^2-5x)(2x-3)}{(x^2-3x+3)^2}$$

Example 7

Given $h(x)=\dfrac{4x^2+5x}{x^4}$, find $h'(x)$

$$h'(x)=\frac{x^4(8x+5)-(4x^2+5x)(4x^3)}{(x^4)^2}$$

$$h'(x)=\frac{8x^5+5x^4-16x^5+20x^4}{x^8}$$

$$h'(x)=\frac{-8x^5+25x^4}{x^8}$$

Example 8

Find the slope of a tangent line to the curve given by the function $f(x) = (x^2 + 1)(2x + 5)$ through the point (-1,6)

First find the derivative of the function: $f'(x) = (2x)(2x + 5) + (2)(x^2 + 1)$

Next, find the value of the derivative at $x = -1$

$$f'(-1) = (2(-1))(2(-1) + 5) + 2((-1)^2 + 1) = (-2)(3) + 2(1 + 1) = -6 + 2(2) = -6 + 4 = -2$$

$m = -2$

Example 9

Find the slope of a tangent line to the curve given by the function $f(x) = \dfrac{x^2}{x+3}$ through the point (-1, $\dfrac{1}{2}$)

First find the derivative of the function: $f(x) = \dfrac{x^2}{x+3}$

$$f'(x) = \frac{(x+3)(2x) - (x^2)(1)}{(x+3)^2} = \frac{2x^2 + 6x - x^2}{(x+3)^2} = \frac{x^2 + 6x}{(x+3)^2}$$

Find the value of the derivative at $x = -1$

$$f'(-1) = \frac{(-1) + 6(-1)}{(-1+3)^2} = \frac{-1-6}{2^2} = -\frac{7}{4}$$

Example 10

Find $h'(t)$, given $h(t) = (t^5 - 1)(4t^2 - 7t - 3)$

$$h(t) = (t^5 - 1)(4t^2 - 7t - 3)$$
$$h'(t) = (5t^4)(4t^2 - 7t - 3) + (8t - 7)(t^5 - 1)$$

Example 11

Find $f'(t)$, given $f(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$

$$f(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$$
$$f'(x) = \frac{(x^3 + 3x + 2)(2x) - (x^2 - 1)(3x^2 + 3)}{(x^2 - 1)^2}$$