Math 121

Section 0.4

Factoring Polynomials

Types of Factoring

1) Factor by taking out a Greatest Common Factor (GFC)
2) Factor a trinomial as two binomials.
3) Factor a binomial as a difference of two squares
4) Factor a binomial as a difference of cubes or a sum of cubes
5) Factor by grouping

<u>Factoring out a Greatest Common Factor</u>

Taking out a greatest common factor is essentially the same as working the distributive property backwards.

Review of distributive property

$3(x+5)=3x+3\cdot 5=3x+15$

Example 1

Factor out a greatest common factor $4x+16$

Solution: $4x+16=4(x+4)$

Example 2

Factor out a greatest common factor $5x-35$

Solution: $5x-35=5(x-7)$

Example 3

Factor out a greatest common factor $2x^3-8x^2+16x$

Solution: $2x^3-8x^2+16x=2x(x^2-4x+8)$

Example 4

Factor out a greatest common factor $12x^3y + 24x^2y^2 + 48xy$

Solution: $12x^3y + 24x^2y^2 + 48xy = 12xy(x^2 + 2xy + 4)$

Factoring a trinomial as two binomials

Factoring a trinomial is the same as working the **FOIL** process

So, here is a short review of **FOIL**

$$(x+3)(x+4)$$
$$x \cdot x + 4x + 3x + 3 \cdot 4$$
$$x^2 + 3x + 4x = 12$$
$$x^2 + 7x + 12$$

Example of factoring a trinomial as two binomials

Factor $x^2 + 10x + 25$

In this example you want to find two numbers that multiply to get 25 and add to get 10.

By using x as the first entry in each binomial you get:

$$x^2 + 10x + 25 = (x+5)(x+5)$$

Here are some similar examples

Example 5

Factor $x^2 - 10x + 24$

Answer: $(x-6)(x-4)$ Hint: Find two numbers that multiply to get 24 and add to get -10

Example 6

Factor $x^2 - 2x - 35$

Answer: $(x-7)(x+5)$ Hint: Find two integers that multiply two get -35 and add to get -2 which is the factors -7 and 5

Example 7

Factor $x^2 + 4x - 12$

Answer: $(x-2)(x+6)$

Example 8

Factor $3x^2 + 7x + 2$

Answer: $(3x+1)(x+2)$

Factor a binomial as a difference of squares

Factor $x^2 - 4$

Answer: $(x-2)(x+2)$ Hint: Basically use the **FOIL** process backwards again and find two integers that multiply to get -4 and add to get zero. This process will cancel out the x-terms.

Check:

$(x-2)(x+2)$
$x^2 + 2x - 2x - 4$
$x^2 - 4$

Other similar examples

Example 9

Factor $m^2 - 36$

Answer: $(x-6)(x+6)$

Example 10

Factor $x^2 - 144$

Answer: $(x+12)(x-12)$

Example 11

Factor $9x^2 - 25y^2$

Answer:

$9x^2 - 25y^2$

$(3x)^2 - (5y)^2$

$(3x+5y)(3x-5y)$

General Form of a Difference of Squares: $A^2 - B^2 = (A+B)(A-B)$

<u>Difference of Cubes and sum of Cubes</u>

Main formulas

Difference of Two Cubes

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Sum of Two Cubes

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

Example 12

Factor $x^3 - 27$

Answer:

$$x^3 - 3^3$$
$$(x-3)(x^2 + 3x + 3^2)$$
$$(x-3)(x^2 + 3x + 9)$$

Example 13

Factor $x^3 + 64$

Answer:

$$x^3 + 64$$
$$x^3 + 4^3$$
$$(x+4)(x^2 - 4x + 4^2)$$
$$(x+4)(x^2 - 4x + 16)$$

Example 14

Factor $x^3 + 8$

Answer;

$$x^3 + 8$$
$$x^3 + 2^3$$
$$(x+2)(x^2 - 2x + 2^2)$$
$$(x+2)(x^2 - 2x + 4)$$

Example 15

Factor $8x^3 - 27y^3$

Answer;

$$8x^3 - 27y^3$$
$$(2x)^3 - (3y)^3$$
$$(2x - 3y)\left((2x)^2 - (2x)(3y) + (3y)^2\right)$$
$$(2x - 3)\left(4x^2 - 6xy + 9y^2\right)$$

Factoring by Grouping

Example

Factor $x^3 - 2x^2 - 4x + 8$

Solution:

$$x^3 - 2x^2 - 4x + 8$$
$$x^2(x-2) - 4(x-2)$$
$$\left(x^2 - 4\right)(x-2)$$
$$(x-2)(x+2)(x-2)$$
$$(x-2)^2(x+2)$$

Factor out a x^2 from the first two terms and -4 from the last two terms

Factor out (x – 2) form both terms

Similar types of examples

Example 16

Factor $m^3 - 3m^2 - 12m + 36$

Solution:

$$m^3 - 3m^2 - 12m + 36$$
$$m^2(m-3) - 12(x-3)$$
$$\left(m^2 - 12\right)(m-3)$$

Definition: The **zero of a polynomial** $P(x)$ is a value x such that $P(x) = 0$ when x is substituted into the polynomial $P(x)$.

Example 17

Factor $c^3 - 4c^2 - 16c + 64$

Solution:

$$c^3 - 4c^2 - 16c + 64$$
$$c^2(c-4) - 16(c-4)$$
$$(c^2 - 16)(c-4)$$
$$(c-4)(c+4)(c-4)$$
$$or\ (c-4)^2(c+4)$$

Don't forget that $(c^2 - 16)$ **is a difference of squares**

Applications of factoring: Finding the zeros of a polynomial

Example 18

Find all zeros of the polynomial: $x^2 - 7x + 12$

Solve $x^2 - 7x + 12 = 0$

$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$
$$x - 3 = 0 \quad or \quad x - 4 = 0$$
$$x - 3 + 3 = 0 + 3 \quad x - 4 + 4 = 0 + 4$$
$$x = 3 \qquad\qquad x = 4$$

Example 19

Find all zeros of the polynomial: $x^2 - x - 20$

Solution: Solve $x^2 - x - 20 = 0$

$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$x - 5 = 0 \quad or \quad x + 4 = 0$$
$$x - 5 + 5 = 0 + 5 \quad x + 4 - 4 = 0 - 4$$
$$x = 5 \qquad\qquad x = -4$$

Example 20

Find all zero of the polynomial: $m^2 - 81$

Solution: Solve $m^2 - 81 = 0$

$$m^2 - 81 = 0$$
$$(m+9)(m-9) = 0$$
$$m + 9 = 0 \quad or \quad m - 9 = 0$$
$$m + 9 - 9 = 0 - 9 \quad m - 9 + 9 = 0 + 9$$
$$x = -9 \qquad\qquad x = 9$$

Example 21

Find all zeros of the polynomial: $x^2 - 5x = 0$

Solution: Solve $x^2 - 5x = 0$

$$x^2 - 5x = 0$$
$$x(x-5) = 0$$
$$x = 0 \quad or \quad x - 5 = 0$$
$$x = 5$$

Example 22

Find all zeros of the polynomial: $x^2 - 3x^2 - 4x + 12$

Solution: Solve $x^2 - 3x^2 - 4x + 12 = 0$

$$x^3 - 3x^2 - 4x + 12 = 0$$
$$x^2(x-3) - 4(x-3) = 0$$
$$(x^2 - 4)(x-3) = 0$$
$$(x-2)(x+2)(x-3) = 0$$
$$x - 2 = 0 \ \ or \ \ x + 2 = 0 \ \ or \ \ x - 3 = 0$$
$$Thus\ the\ solutions\ are: x = 2, x = -2, x = 3$$

Example 23

Find all zeros of the polynomial: $m^4 - 16$

Solution: Solve $m^4 - 16 = 0$

$$m^4 - 16 = 0$$
$$(m^2 + 4)(m^2 - 4) = 0$$
$$(m^2 + 4)(m-2)(m+2) = 0$$
$$m^2 + 4 = 0 \Rightarrow No\ Solution$$
$$m - 2 = 0 \Rightarrow m = 2$$
$$m + 2 = 0 \Rightarrow m = -2$$
$$Thus\ the\ solutions\ are: m = -2, m = 2$$
