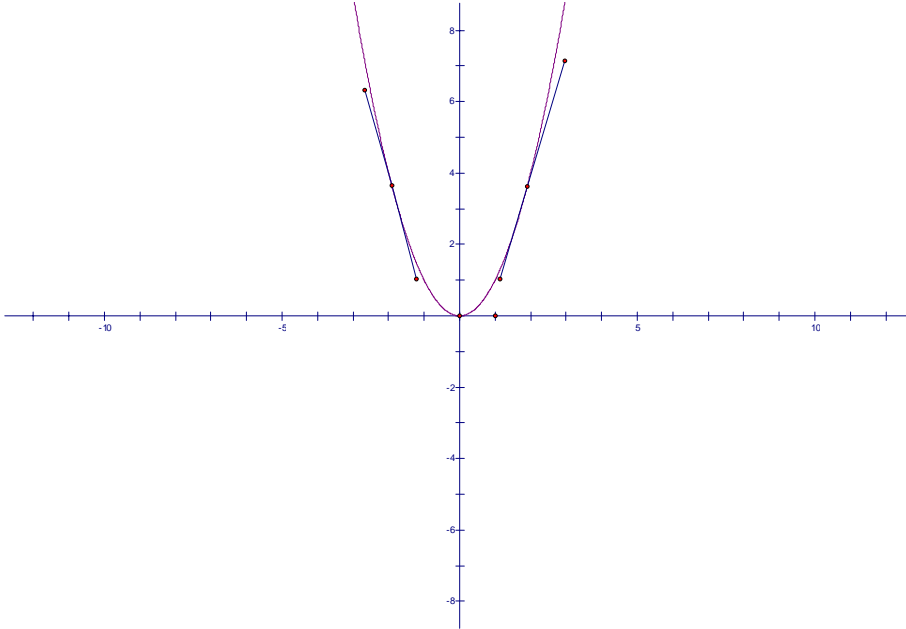
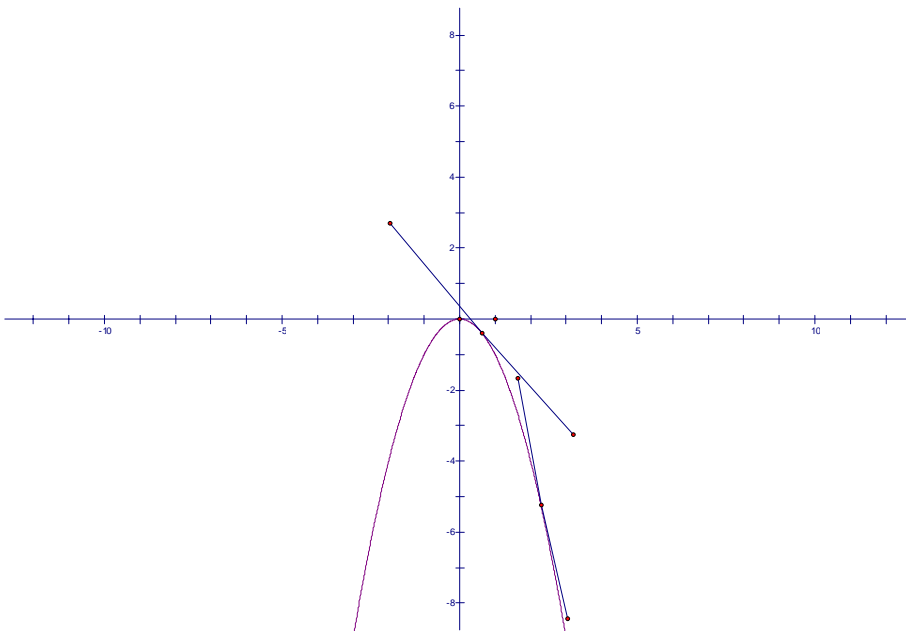


Section 3.3 Concavity

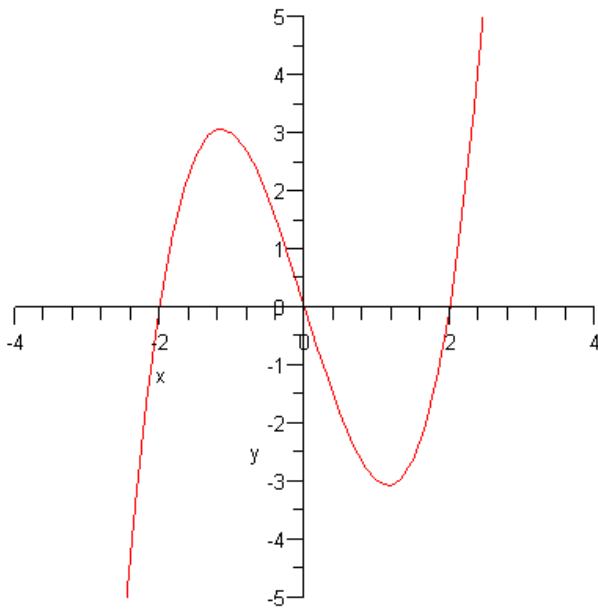
Concave Up: If a function $f(x)$ on I is concave up, then $f'(x)$ is increasing on I



Concave Down: If a function $f(x)$ on I is concave down, then $f'(x)$ is decreasing on I



Example of a graph that has both types of concavity



The graph above is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$

Test for Concavity

Let $f(x)$ be a function whose second derivative exist on I

- 1) If $f''(x) > 0$ for all x on I , then $f(x)$ is concave up.
 - 2) If $f''(x) < 0$ for all x on I , then $f(x)$ is concave down.
 - 3) If $f''(x) = 0$ for all x on I , then the test fails.
-

Example 1

Discuss the concavity of the function $f(x) = x^3 - 6$

$$f(x) = x^3 - 6$$

$$f'(x) = 3x^2$$

$$f''(x) = 6x$$

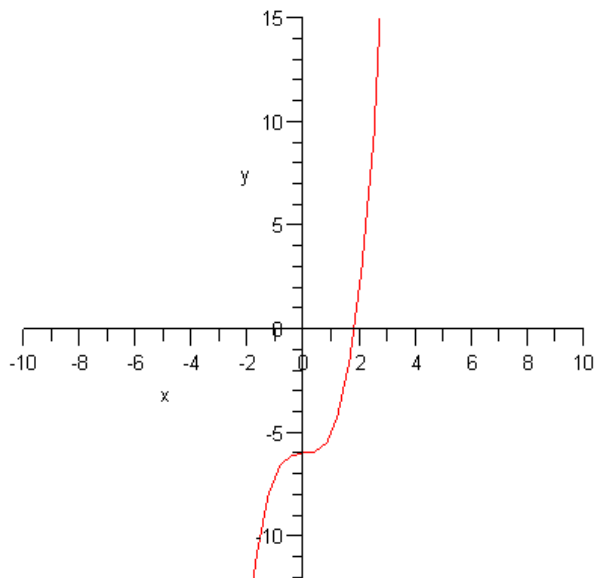
$$6x = 0$$

$$\frac{6x}{6} = \frac{0}{6}$$

$$x = 0$$

Test for Concavity

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f''(x)$	Negative	Positive
Conclusion	Concave Down	Concave Up



Example 2

Given the following function find all extrema points and test the concavity

$$f(x) = x^3 - 3x$$

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

$$3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$3(x-1)(x+1) = 0$$

$$(x-1)(x+1) = 0$$

$$x-1 = 0 \text{ or } x+1 = 0$$

$$x = 1 \quad x = -1$$

$$f'(2) = 3(2)^2 - 3 = 12 - 3 = 9$$

$$f'(0) = 3(0)^2 - 3 = 0 - 3 = -3$$

$$f'(-2) = 3(-2)^2 - 3 = 12 - 3 = 9$$

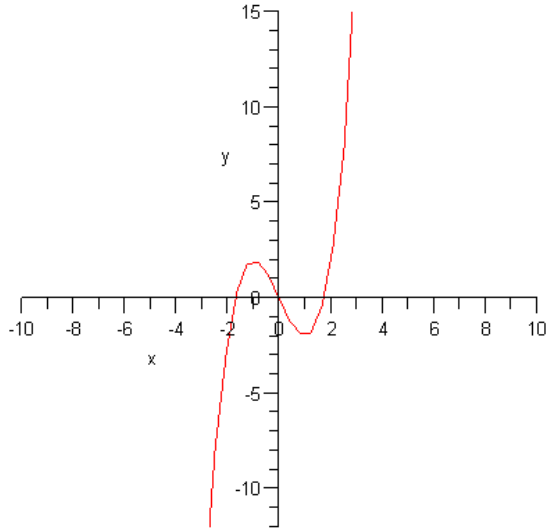
Test for Extrema points

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Test Value	$x = -2$	$x = 0$	$x = 2$
Sign of $f'(x)$	Positive	Negative	Positive
Conclusion	Increasing	Decreasing	Increasing

y coordinates of the critical points

$$f(1) = 1^3 - 3(1) = 1 - 3 = -2$$

$$f(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2$$



Thus, the function will have a relative maximum at $(-1, 2)$ and a relative minimum at $(1, -2)$

Concavity Test

$$f(x) = 6x$$

$$6x = 0$$

$$\frac{6x}{6} = \frac{0}{6}$$

$$x = 0$$

Test for Concavity

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f''(x)$	Negative	Positive
Conclusion	Concave Down	Concave Up

Example 3

Given the following function find all extrema points and test for concavity

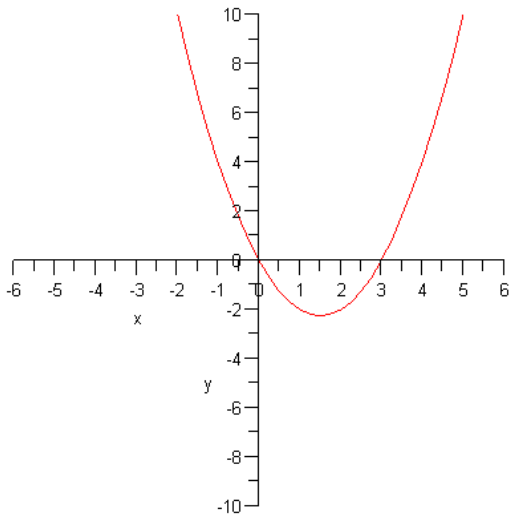
$$f(x) = x^2 - 2$$

$$f'(x) = 2x$$

$$2x = 0 \Rightarrow x = 0 \text{ Critical Point}$$

Test for Extrema Points

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Negative	Positive
Conclusion	Decreasing	Increasing



y coordinate of critical point $f(0) = 0^2 - 2 = -2$

Therefore, f has an absolute minimum at $(0, -2)$

Concavity Test

$$f''(x) = 2 > 0 \Rightarrow f \text{ is Concave Up}$$

Example 4

Find all critical points and describe the concavity

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

$$4x^3 = 0$$

$$\frac{4x^3}{4} = \frac{0}{4}$$

$$x^3 = 0$$

$$\sqrt[3]{x} = \sqrt[3]{0}$$

$$x = 0$$

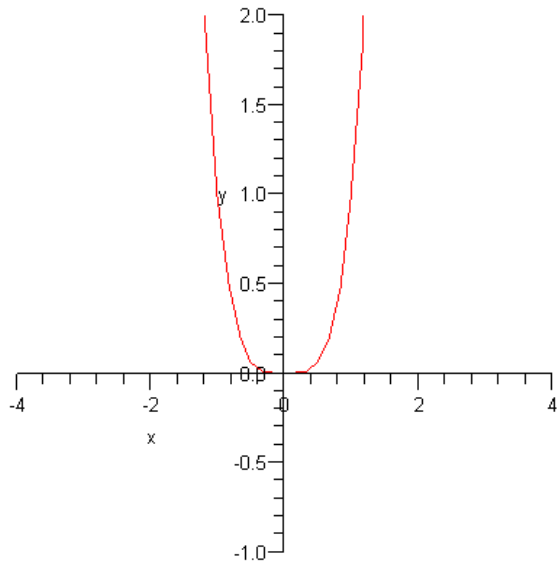
$$f'(-1) = (-1)^3 = -1$$

$$f'(1) = 1^3 = 1$$

y coordinate of critical point $f(0) = 0^4 = 0$

Test for Extrema Points

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	Negative	Positive
Conclusion	Decreasing	Increasing



f has a relative min at (0,0)

Concavity Test

$$f(x) = 12x^2$$

$$12x^2 = 0$$

$$x^2 = 0$$

$$x = 0$$

$$f'(-1) = 12(-1)^2 = 12$$

$$f'(1) = 12(1)^2 = 12$$

Interval	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f''(x)$	Positive	Positive
Conclusion	Concave Up	Concave Up

Exercises Section 3.3

Given the following functions, find all extrema points and test for concavity of each function.

1) $f(x) = 5x^3 - 5$

2) $f(x) = x^3 - 3x$

3) $f(x) = x^2 - 5x$

4) $f(x) = x^3 - 3x^2$

5) $f(x) = 3x + 2$