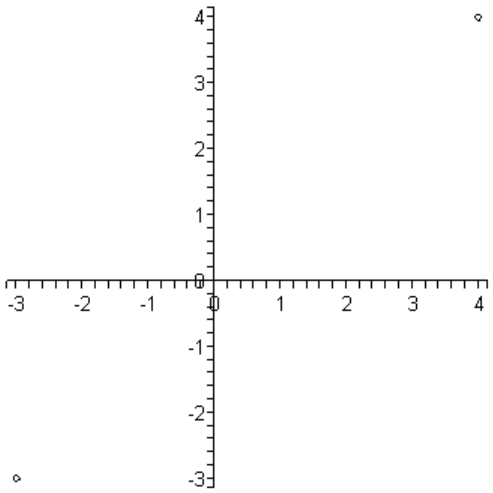
Math 121

Section 1.1

The Cartesian Plane and the Distance Formula

The Cartesian Plane

Quadrant I Quadrant II



Quadrant III Quadrant IV

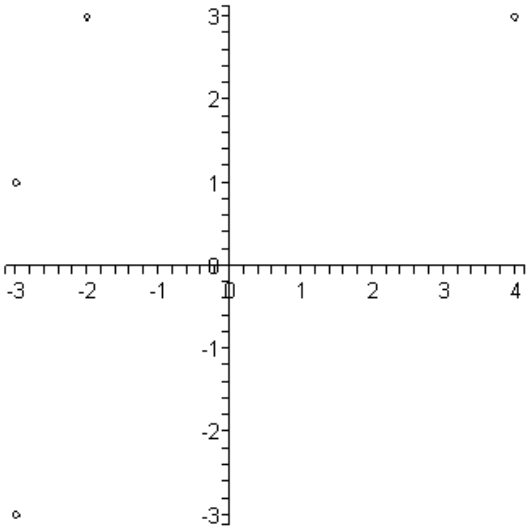
Independent axis is the x-axis
Dependent axis is the y-axis

Ordered pairs – (x,y)

x = x-coordinate

y = y-coordinate

Example 1 Graph the following ordered pairs: (-3,1), (-2,3), (-3,-4), (4,3)



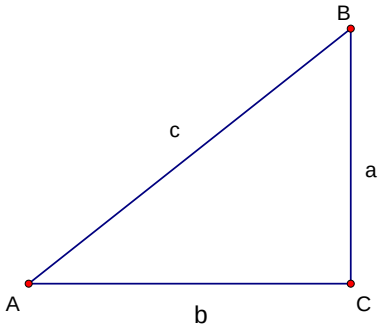
Distance formula

The distance between the points (x_1, y_1) and (x_2, y_2) is given by the formula:

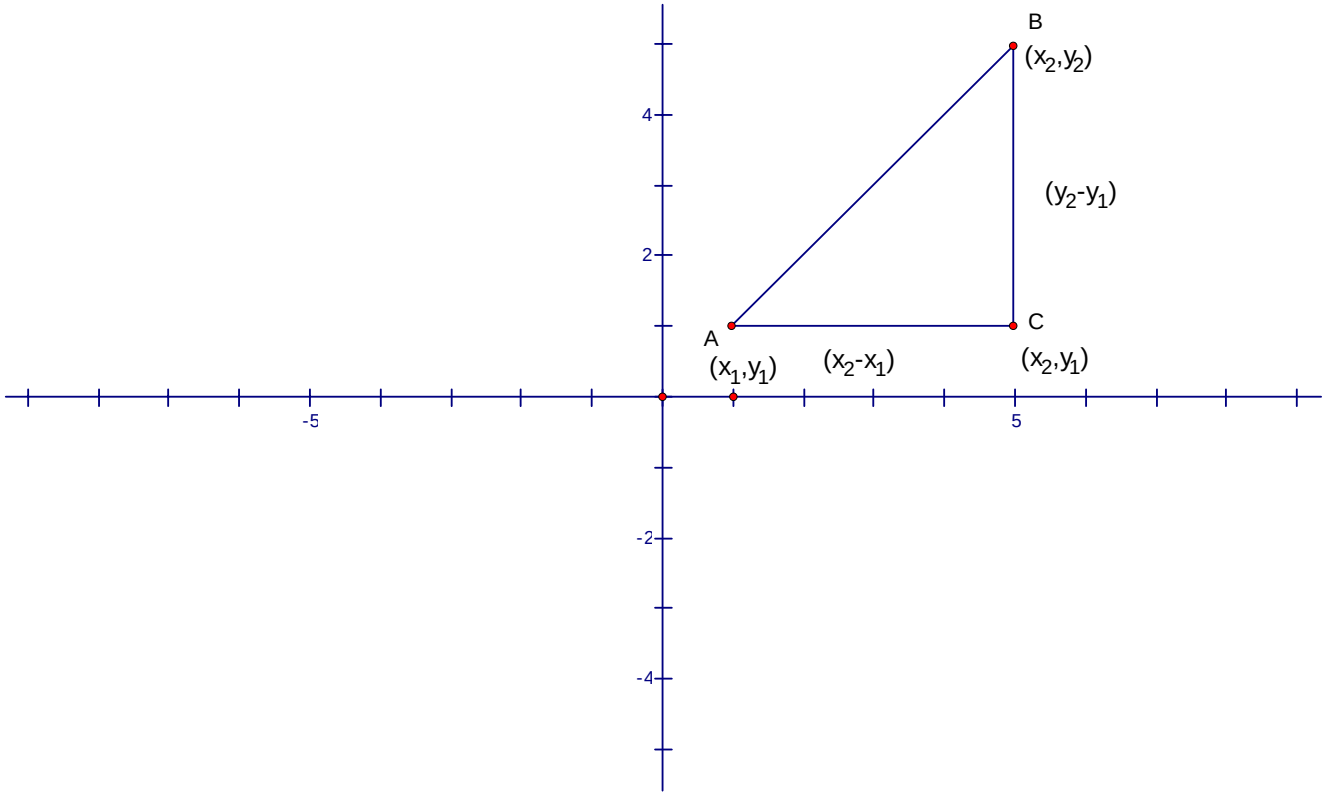
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pythagorean Theorem

If ΔABC is a right triangle, then the sum of the squares of the two legs of the right triangle is equal to the square of the hypotenuse of the right triangle. $(c^2 = a^2 + b^2)$



Proof of the distance formula



Using the Pythagorean Theorem gives the following result.

$$c^2 = a^2 + b^2$$
$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\sqrt{c^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2

Find the distance between the points (2,5) and (3,7).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(3-2)^2 + (7-5)^2}$$
$$d = \sqrt{(1)^2 + (2)^2}$$
$$d = \sqrt{1+4}$$
$$d = \sqrt{5}$$

Example 3

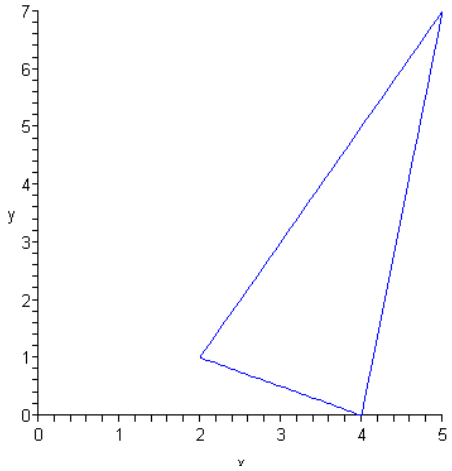
Find the distance between the points (-3,1) and (3,-2).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(3-(-3))^2 + (-2-1)^2}$$
$$d = \sqrt{(6)^2 + (-3)^2}$$
$$d = \sqrt{36+9}$$
$$d = \sqrt{45}$$
$$d = \sqrt{9 \cdot 5}$$
$$d = 3\sqrt{5}$$

Example 4 Right Triangle Problem

Show that the points A = (2,1), B = (4,0), and C = (5,7) form a right triangle, then find the area of the triangle.

i) Make a plot of the points A,B, and C



ii) **Find the distances between the points**.

$$d(A,B)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(4-2)^2+(0-1)^2}=\sqrt{2^2+(-1)^2}=\sqrt{4+1}=\sqrt{5}$$

$$d(B,C)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-4)^2+(7-0)^2}=\sqrt{1^2+7^2}=\sqrt{1+49}=\sqrt{50}$$

$$d(A,C)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-2)^2+(7-1)^2}=\sqrt{3^2+6^2}=\sqrt{9+36}=\sqrt{45}$$

iii) Use Pythagorean Theorem to check distances

$$c^2=a^2+b^2$$
$$(\sqrt{50})^2=(\sqrt{45})^2+(\sqrt{5})^2$$
$$50=45+5$$
$$50=50$$

Thus, triangle ABC is a right triangle

iv) Area of the triangle

$base = \sqrt{5}$ *and* $height = \sqrt{45}$

$$A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{5})(\sqrt{45}) = \frac{1}{2}\sqrt{225} = \frac{1}{2}(15) = 7.5$$

Example 5

Find x such that the distance between the point is 5 units, using the points (2,-1) and (x,2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$5 = \sqrt{(x-2)^2 + (2-(-1))^2}$$
$$5 = \sqrt{(x-2)^2 + 3^2}$$
$$5 = \sqrt{(x-2)^2 + 9}$$
$$5^2 = \left(\sqrt{(x-2)^2 + 9}\right)^2$$
$$25 = (x-2)^2 + 9$$
$$25 - 9 = (x-2)^2 + 9 - 9$$
$$16 = (x-2)^2$$
$$\sqrt{16} = \sqrt{(x-2)^2}$$
$$\pm 4 = x - 2$$
$$x = 2 \pm 4 \Rightarrow x = -2 \text{ or } x = 6$$