Unit 3

Egyptian Geometry and Volume

Math 116

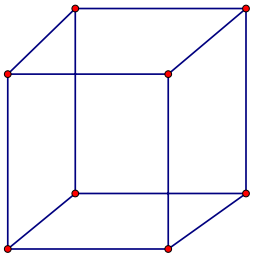
Math 116

Geometry unit

Volume

Volume and Surface Area of Different Shapes

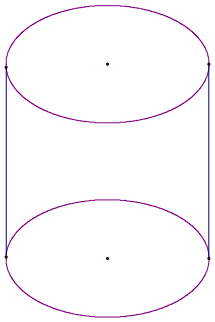
Rectangular Solid



Volume: $V = lwh$

Surface Area: $A = 2lw + 2hl + 2hw$

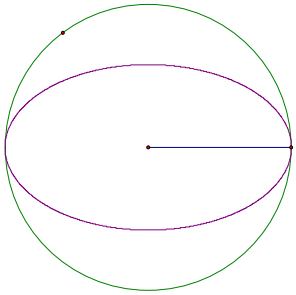
Cylinder



Volume: $V = \pi r^2 h$

Surface Area: $A = 2\pi rh + 2\pi r^2$

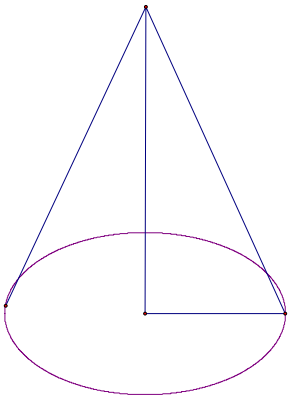
Sphere



Volume: $V = \frac{4}{3}\pi r^3$

Surface Area: $A = 4\pi r^2$

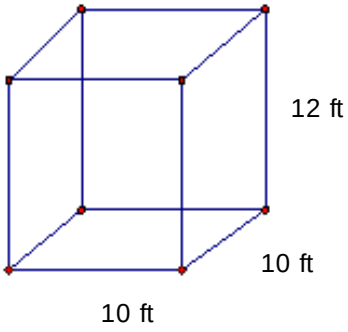
Cone



Volume: $V = \frac{1}{3}\pi r^2 h$

Examples

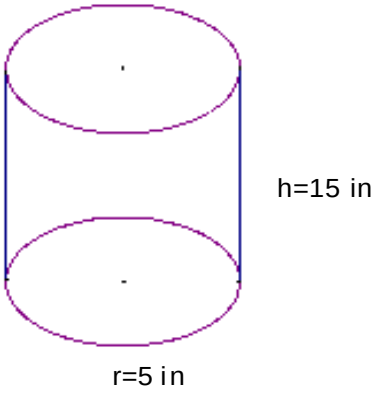
1) Find the volume and surface area



$V = lwh = (10\ ft)(10\ ft)(12\ ft) = 1200\ ft^3$

$A = 2(10)(10) + 2(10)(12) + 2(10)(12) = 200 + 240 + 240 = 680\ ft^2$

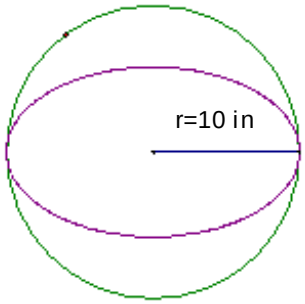
2) Find the volume and surface area



Volume: $V = \pi(5in)^2(15in) = 3.14(25in^2)(15in) = 1177.5in^3$

Surface Area: $A = 2\pi(5in)^2 + 2\pi(5in)(15in) = 157.5in^2 + 471in^2 = 628.5in^2$

3) Find the volume and surface area



$$V = \frac{4}{3}\pi(10\,in)^3 = \frac{4}{3}(3.14)(1000\,in^3) = 4186.7\,in^3$$

$$A = 4\pi(10\,in)^2 = 4(3.14)(100\,in^2) = 1256\,in^2$$

20)

Tennis Ball

$$d = 2.5\,in$$
$$r = 1.25\,in$$
$$V = \frac{4}{3}\pi(1.25)^3 = \frac{4}{3}(3.14)(1.953125\,in^3) = 8.2\ in^3$$
$$A = 4\pi(1.25\,in)^2 = 19.625\,in^2$$

Ping-pong Ball

$$d = 1.5\,in$$
$$r = .75\,in$$
$$V = \frac{4}{3}\pi(.75)^3 = \frac{4}{3}(3.14)(.421875\,in^3) = 1.8\ in^3$$

$A = 4\pi(.75in)^2 = 7.065in^2$

24)

Jupiter

$d = 88{,}640\,miles$

$r = 44{,}320\,miles$

$V = \frac{4}{3}\pi(44{,}320)^3 \approx 3.64\,x\,10^{14}\ mi^3$

Pluto

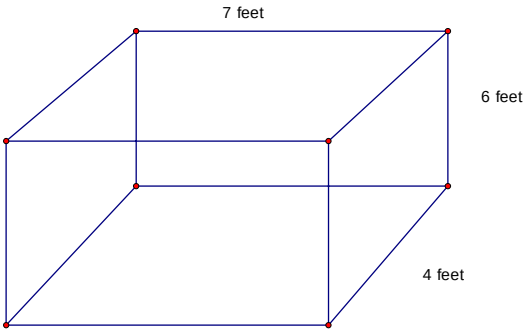
$d = 1500\,miles$

$r = 750\,miles$

$V = \frac{4}{3}\pi(750)^3 = 1767145868\,miles^3 \approx 1.77x10^9\ mi^3$

$\frac{3.64\ x\ 10^{14}\ miles}{1.77\ x\ 10^9\ miles} \approx 206355$

1) A rectangular fish tank is 7 feet by 6 feet by 4 feet. What is the surface area and volume of the fish tank



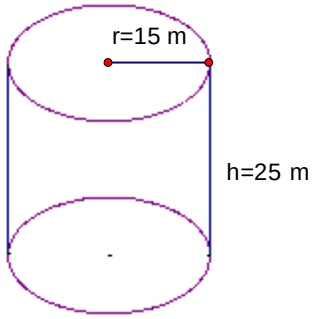
Volume

$V = lwh = (7\ ft)(4\ ft)(6\ ft) = 168\ ft^3$

Surface Area

$A = 2lw + 2hl + 2wh = 2(7\ ft)(4\ ft) + 2(6\ ft)(7\ ft) + 2(4\ ft)(6\ ft) = 56\ ft^2 + 84\ ft^2 + 48\ ft^2 = 188\ ft^2$

2) A cylinder shaped city water tower has a height of 25 meters and a radius of 15 meters. How much water can this tower hold?



$V = \pi r^2 h$

$V = (3.14)(15)^2(25)$

$V = (3.14)(225)(25)$

$V = (706.5)(25)$

$V = 17662.5\ m^3$

Egyptian Geometry

In Egyptian society, they used mathematics to survey the land. During this time period most Egyptians lived on the fertile banks on the Nile. This created problems because the river often flood it banks. As a result, the Egyptians had to know survey their property using geometric shapes. The Egyptians often used ropes to measure out 3-4-5 right triangle which could be used to measure out a perfect right angle.

Measurement

Egyptian Units of Measurement

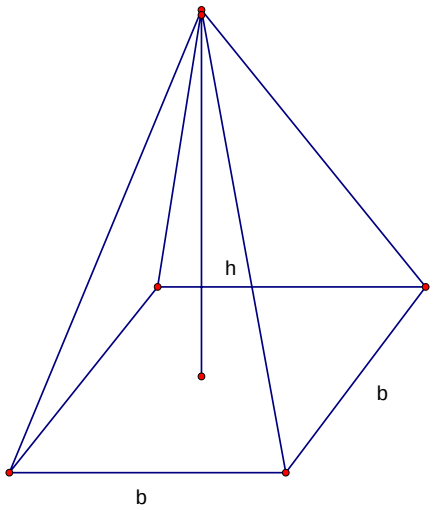
1 cubit = 7 palms
1 palms = 4 fingers
1 khet = 100 cubits "Greek Aurora"
1 setat = 1square khet = 10,000 square cubits
1 kher = $\frac{2}{3}$ cubic cubit

Cubit is the distance from the tip of your middle finger to your elbow

1 Cubit = 461 mm

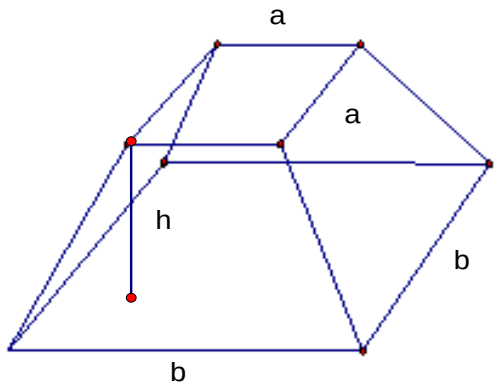
Volume

The pyramid



$$V = \frac{1}{3}b^2h$$

The Truncated Pyramid



Volume

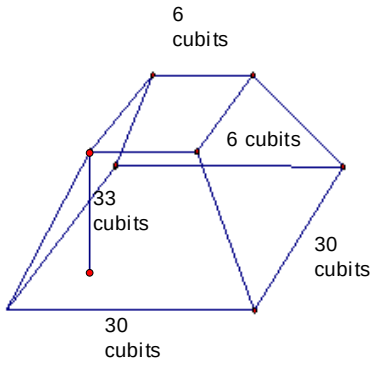
$$V = \frac{h}{3}\left(a^2 + ab + b^2\right)$$

Example from pages 181-182

6)

a) Truncated Pyramid

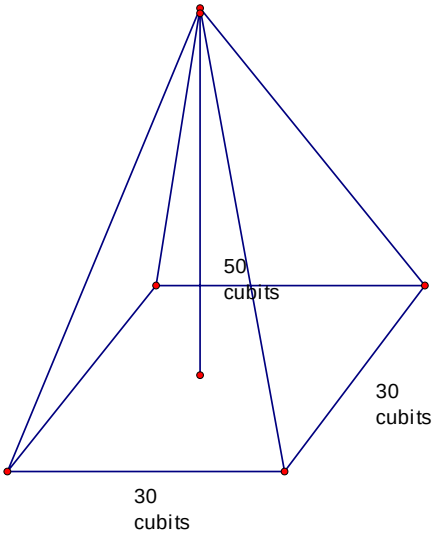
h = 33 cubits
a = 6 cubits
b = 30 cubits



$$V = \frac{h}{3}\left(b^2 + ab + a^2\right) = \frac{33}{3}\left(30^2 + 30(6) + 6^2\right) = 11(900 + 180 + 36) = 11(1116) = 12276\ \textit{cubits}$$

b)

Regular Pyramid



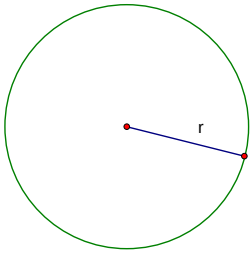
$$V = \frac{1}{3}b^2h = \frac{1}{3}(30)^2(50) = \frac{1}{3}(900)(50) = 300(50) = 15000 \ \textit{cubits}$$

The regular pyramid has a larger volume

The Egyptian Value for Pi (π)

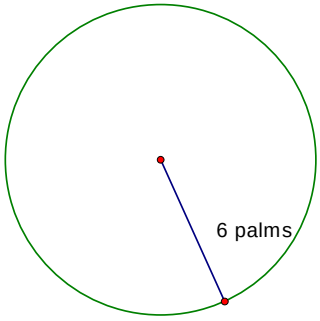
$$\pi = \frac{256}{81}$$

Area of the a circle



$$A = \pi r^2$$

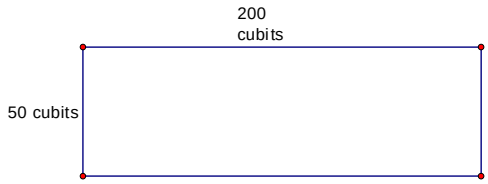
14) a) Using $\frac{256}{81}$ *as Pi*



$$A = \pi r^2 = \frac{256}{81}(6\ palms)^2 = \frac{256}{81}(36\ square\ \ palms) = \frac{1024}{9}\ palms = 113.78\ \ palms$$

b) Using 3.14 for Pi $A = \pi(6)^2 = 113.10\ \ palms$

20)

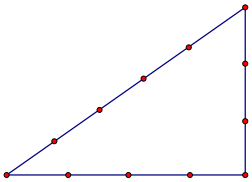


Area in cubits: $A = (200\,cubits)(50\ cubits) = 10{,}000\,cubits$

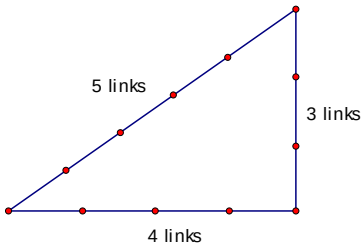
Area in setats: $10{,}000\ cubits \cdot \dfrac{1\ setat}{10000\ cubits} = 1\ setat$

Does the figure determine a right triangle?

1)



Count the links joint by the pairs of knots which gives the following measurements



Using the Pythagorean Theorem you get that the triangle is a right triangle

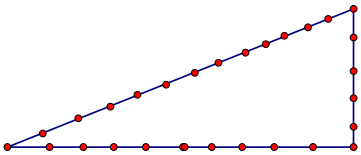
$c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

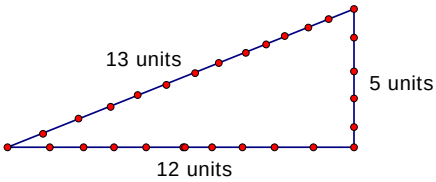
$25 = 9 + 16$

$25 = 25$

2)



Solution:



$c^2 = a^2 + b^2$

$13^2 = 5^2 + 12^2$

$169 = 25 + 144$

$169 = 169$