

## Math 116

## Practice Final Exam Fall 2007

1. Given  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 0\}$

$$A = \{1, 3, 6\}$$

$$B = \{1, 4, 7, 8\}$$

Find  $(A \cap B)'$

a.  $\{1, 3, 4, 6, 7, 8\}$

b.  $\emptyset$

c.  $\{2, 3, 4, 5, 6, 7, 8, 0\}$

d.  $\{1\}$

**Solution: D**

$$A \cap B = \{1\} \Rightarrow (A \cap B)' = \{2, 3, 4, 5, 6, 7, 8, 0\}$$

2. Given:  $U = \{i, l, u, v, m, a, t, h\}$

$$A = \{l, u, v\}$$

$$B = \{v, m, a, h\}$$

Find  $(A \cup B)'$

a.  $\{l, u\}$

b.  $\{i, v, m, a, t, h\}$

c.  $\{l, u, v, i, t\}$

d.  $\{m, a, t, h\}$

**Solution: C**

$$B' = \{i, l, u, t\}$$

$$(A \cup B') = \{l, u, v, i, t\}$$

3. How many subsets are there for set  $\{1, 3, 6\}$ ?

a. 2

b. 3

c. 7

d. 8

$$2^3 = 8$$

**Solution: D**

4. List all of the subsets of the set  $\{1, 3, 6\}$
- $\emptyset, \{1\}, \{3\}, \{6\}, \{1,3\}, \{1,6\}, \{3,6\}, \{1,3,6\}$
  - $\{1\}, \{3\}, \{6\}, \{1,3\}, \{1,6\}, \{3,6\}, \{1,3,6\}$
  - $\emptyset, \{1,3,6\}$
  - $\{1\}, \{3\}, \{6\}$

$\emptyset, \{1\}, \{3\}, \{6\}, \{1,3\}, \{3,6\}, \{1,6\}, \{1,3,6\}$

**Solution: A**

5. Which of the following sentences is not a statement?
- Virginia is in the United States.
  - The United States is part of Central America.
  - Radford is located in Virginia.
  - Don't go to Central America.

**Solution: D (It's a Command)**

6. Which of the following is the negation of 'Some dogs wag their tails'?
- Some dogs do not wag their tails.
  - All dogs wag their tails.
  - Not all dogs do not wag their tails.
  - No dogs wag their tails.

**Solution: C**

Negation: All dogs do not wag their tails or Not all dogs wag their tails

7. Let  $P$  = 'you are smart' and  $Q$  = 'you are rich', and consider the following argument:

If you are smart, then you are rich.

You are not rich.

Therefore, you are not smart.

Which of the following best symbolizes this argument

- |  |  |
|--|--|
| <p>a. <math>P \rightarrow \sim Q</math><br/> <math>Q</math><br/>           -----<br/> <math>\therefore \sim P</math></p> | <p>c. <math>P \rightarrow Q</math><br/> <math>\sim Q</math><br/>           -----<br/> <math>\therefore \sim P</math></p> |
| <p>b. <math>\sim Q \rightarrow P</math><br/> <math>Q</math><br/>           -----<br/> <math>\therefore \sim P</math></p> | <p>d. <math>P \wedge Q</math><br/> <math>\sim Q</math><br/>           -----<br/> <math>\therefore P</math></p>           |

**Solution: C**

If you are smart, then you are rich.  $P \rightarrow Q$

You are not rich.  $\sim Q$

Therefore, you are not smart.  $\sim P$

$$P \rightarrow Q$$

$$\frac{\sim Q}{\text{-----}}$$

$$\therefore \sim P$$

8. Which of the statements below completes a correct argument for the following?

If you are a mechanic, then you are an engineer.

Sarah is a mechanic.

- Therefore, if Sarah is an engineer, then she is a mechanic.
- Therefore, Sarah is an engineer.
- Therefore, some engineers are mechanics.
- Therefore, some mechanics are engineers.

**Solution: B**

9. Let  $P$  = 'The ball is red,'  $Q$  = 'The grass is green,' and  $R$  = 'The opera is wonderful.'  
Which of the following is symbolized by  $(\sim R \wedge P) \wedge Q$ ?
- a. The opera is wonderful or the ball is not red, or the grass is green.
  - b. The opera is wonderful and the ball is not red, and the grass is green.
  - c. The opera is not wonderful and the ball is red, and the grass is green.
  - d. The opera is not wonderful or the ball is red, or the grass is green.

**Solution: D**

10. Which of the following would be the correct truth table for  $\sim(\sim P \vee Q)$ ?

a.

| $P$ | $Q$ | $\sim(\sim P \vee Q)$ |
|-----|-----|-----------------------|
| T   | T   | T                     |
| T   | F   | T                     |
| F   | T   | F                     |
| F   | F   | T                     |

b.

| $P$ | $Q$ | $\sim(\sim P \vee Q)$ |
|-----|-----|-----------------------|
| T   | T   | F                     |
| T   | F   | T                     |
| F   | T   | F                     |
| F   | F   | F                     |

c.

| $P$ | $Q$ | $\sim(\sim P \vee Q)$ |
|-----|-----|-----------------------|
| T   | T   | F                     |
| T   | F   | T                     |
| F   | T   | T                     |
| F   | F   | T                     |

d.

| $P$ | $Q$ | $\sim(\sim P \vee Q)$ |
|-----|-----|-----------------------|
| T   | T   | T                     |
| T   | F   | T                     |
| F   | T   | T                     |
| F   | F   | F                     |

**Solution: B**

| $P$ | $Q$ | $\sim P$ | $(\sim P \vee Q)$ | $\sim(\sim P \vee Q)$ |
|-----|-----|----------|-------------------|-----------------------|
| T   | T   | F        | T                 | <b>F</b>              |
| T   | F   | F        | F                 | <b>T</b>              |
| F   | T   | T        | T                 | <b>F</b>              |
| F   | F   | T        | T                 | <b>F</b>              |

11. Let  $P$  = 'you are smart' and  $Q$  = 'you are rich', and consider the following argument:

If you are smart, then you are rich.

You are not rich.

Therefore, you are not smart.

Is the argument valid or invalid, or neither?

- a. Valid
- b. Invalid
- c. Neither

$$((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$$

| $P$ | $Q$ | $\sim P$ | $\sim Q$ | $P \rightarrow Q$ | $((P \rightarrow Q) \wedge \sim Q)$ | $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$ |
|-----|-----|----------|----------|-------------------|-------------------------------------|--|
| T   | T   | F        | F        | T                 | F                                   | <b>T</b>   |
| T   | F   | F        | T        | F                 | F                                   | <b>T</b>   |
| F   | T   | T        | F        | T                 | F                                   | <b>T</b>   |
| F   | F   | T        | T        | T                 | T                                   | <b>T</b>   |

The argument is valid because it is a tautology

**Solution: A**

12. With the help of the table

| Hindu-Arabic Numeral | Roman Numeral |
|----------------------|---------------|
| <b>1</b>             | <b>I</b>      |
| <b>5</b>             | <b>V</b>      |
| <b>10</b>            | <b>X</b>      |
| <b>50</b>            | <b>L</b>      |
| <b>100</b>           | <b>C</b>      |
| <b>500</b>           | <b>D</b>      |
| <b>1000</b>          | <b>M</b>      |

convert the Roman numeral **MCMLXXIV** to its equivalent Hindu-Arabic (base 10) form

- a. 1964
- b. 1974
- c. 2076
- d. 2176

1974

**Solution: B**

13. What decimal (base 10) number is represented by the binary number 101111?

- a. 33
- b. 10
- c. 47
- d. 94

101111

$$1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 32 + 0 + 8 + 4 + 2 + 1 = 47$$

**Solution: C**

14. The base-10 number 207 is represented by what binary number?
- 11110011
  - 11001111
  - 11111111
  - 1110001

$$207 = 128 + 79$$

$$207 = 1 \cdot 2^7 + 64 + 15$$

$$207 = 1 \cdot 2^7 + 1 \cdot 2^6 + 8 + 7$$

$$207 = 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^3 + 4 + 2 + 1$$

$$207 = 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

11001111

**Solution: B**

15. Which of the following number is prime?
- 105
  - 139
  - 125
  - 119

**Solution: B**

16. In canonical form, find the prime factorization of the number 5929?
- $7^2 11^2$
  - $7^3 11^2$
  - $2 \cdot 7^2 \cdot 11$
  - $2^3 \cdot 7^2 \cdot 11$

$$5929$$

$$7 \cdot 847$$

$$7 \cdot 7 \cdot 121$$

$$7 \cdot 7 \cdot 11 \cdot 11$$

$$7^2 11^2$$

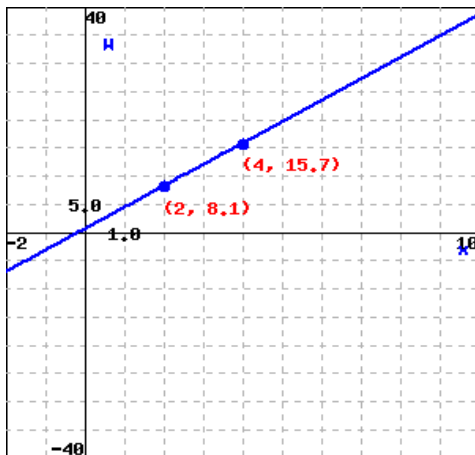
**Solution: A**

17. Suppose that the weight  $W$  in pounds of a  $\frac{3}{4}$  in. galvanized pipe with protective thread caps is given by the equation

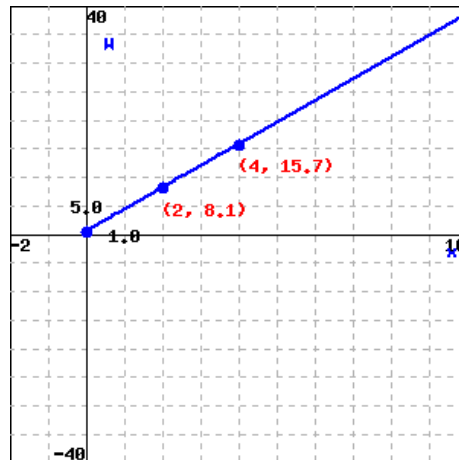
$$W = 3.8x + 0.5$$

where  $x$  represents the length of the pipe in feet. Enter the letter of graph below that best matches this equation.

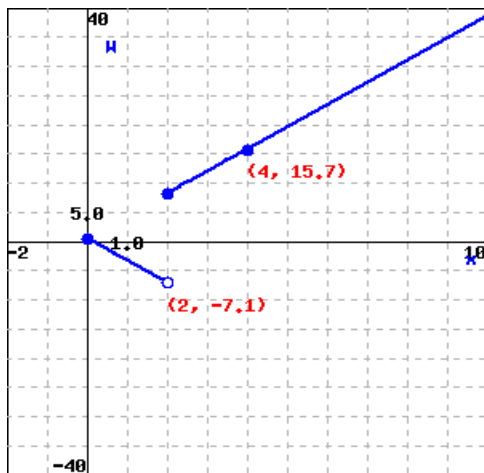
a.



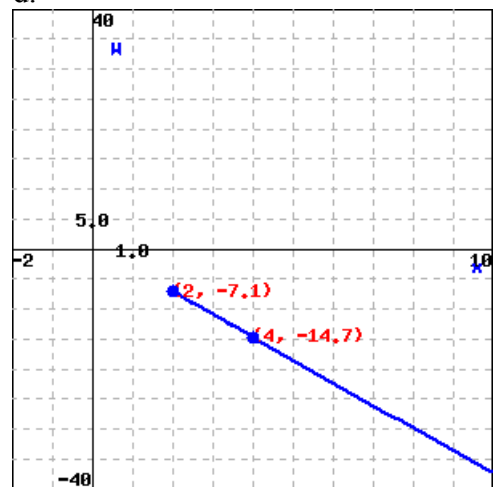
c.



c.



d.



**Solution: B**

18. Find the slope of the line that passes through the points (2,1) and (4,9).

- a. 4
- b.  $-\frac{1}{4}$
- c. -4
- d.  $\frac{1}{4}$

$$m = \frac{9-1}{4-2} = \frac{8}{2} = 4$$

**Solution: A**

19. Suppose that the weight  $W$  in pounds of a  $\frac{3}{4}$  in. galvanized pipe with protective thread caps is given by the equation

$$W = 3.8x + 0.5$$

where  $x$  represents the length of the pipe in feet. Find the weight of a 34 ft long piece of pipe.

- a. 3.8 lbs
- b. 129.7 lbs
- c. 8.8 lbs
- d. 25.5 lbs

$$W = 3.8(34) + 0.5 = 129.2 + 0.5 = 129.7 \text{ lbs}$$

**Solution: B**

20. Suppose that the weight  $W$  in pounds of a  $\frac{3}{4}$  in. galvanized pipe with protective thread caps is given by the equation

$$W = 3.8x + 0.5$$

where  $x$  represents the length of the pipe in feet. If the pipe weighs 125.9 lbs., how long is the pipe?

- a. 33 ft
- b. 94.4 ft
- c. 167.9 ft
- d. 478.9 ft

$$W = 3.8x + 0.5$$

$$125.9 = 3.8x + .5$$

$$125.9 - .5 = 3.8x$$

$$3.8x = 125.4$$

$$x = 33 \text{ ft}$$

**Solution: A**

21. Find the y-intercept of the equation  $4x + 5y = 20$ .

- a. (5, 0)
- b. (0, 5)
- c. (4, 0)
- d. (0, 4)

$$4x + 5y = 20$$

$$4x - 4x + 5y = -4x + 20$$

$$5y = -4x + 20$$

$$\frac{5y}{5} = \frac{-4x}{5} + \frac{20}{5}$$

$$y = -\frac{4}{5}x + 4 \Rightarrow b = 4$$

**Solution: C**

22. Suppose that the path traveled by a golf ball can be modeled with the quadratic equation  $y = -0.02x^2 + 0.58x$ , where  $x$  is the distance in yards from the point it was hit and  $y$  is the height of the golf ball in yards. Assuming that the ground is level and rounding your final answer to 1 digit to the right of the decimal, determine the maximum height of the golf ball.
- a. 0.0 yards
  - b. 4.2 yards
  - c. 12.6 yards
  - d. 14.5 yards

$$y = -0.02x^2 + .58x$$

$$x = -\frac{b}{2a} = -\frac{.58}{2(-.02)} = \frac{.58}{.04} = 14.5$$

$$y = -.02(14.5)^2 + .58(14.5) = -.02(210.25) + 8.41 = -4.205 + 8.41 = 4.2$$

**Solution: B**

23. Suppose that the path traveled by a golf ball can be modeled with the quadratic equation  $y = -0.02x^2 + 0.58x$ , where  $x$  is the distance in yards from the point it was hit and  $y$  is the height of the golf ball in yards. Assuming that the ground is level and rounding your final answer to 1 digit to the right of the decimal, determine how far from where it was hit does the ball reach the ground?

- a. 0.0 yards
- b. 9.6 yards
- c. 62.0 yards
- d. 29.0 yards

$$y = -0.02x^2 + .58x$$

$$x = -\frac{b}{2a} = -\frac{.58}{2(-.02)} = \frac{.58}{.04} = 14.5$$

$$2(14.5) = 29 \text{ ft}$$

**Solution: D**

24. Determine the vertex,  $x$ -intercepts, and sketch the graph of the equation  $y = -x^2 - 4x$ ?

$$y = -x^2 - 4x$$

$x$  - int

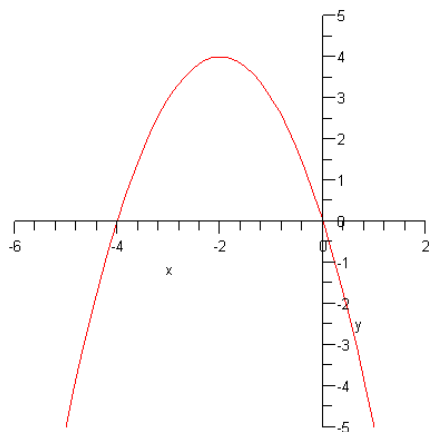
$$0 = -x^2 - 4x$$

$$0 = -x(x + 4)$$

$$-x = 0 \text{ or } x + 4 = 0$$

$$x = 0 \text{ or } x = -4$$

$$x = \frac{-(-4)}{2(-1)} = -2 : y = -(-2)^2 - 4(-2) = -4 + 8 = 4 : \text{Vertex} : (-2, 4)$$



25. Suppose a company is trying to stimulate sales of a product in an area that has 488000 viewers. The number of viewers  $V$  who are made aware of the product after  $t$  days is given by the exponential equation  $V = 488000 - 488000e^{-0.3t}$ . How many viewers have been exposed to the new ad campaign after 16 days? Round to the nearest whole number.
- 4016
  - 100000
  - 483984
  - 488000

$$V = 488000 - 488000e^{-0.3t} = 488000 - 488000e^{-0.3(16)} = 488000 - 488000e^{-4.8} = 483984$$

**Solution: C**

26. Suppose \$ 3500 was invested into an account at 5% compounded monthly, use the following function to find the amount in the account after 15 years. Round your answer to the nearest cent (2 decimal places).

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

- \$7397.96
- \$3725.27
- \$3675.00
- \$4958.33

$$A = \$3500 \left( 1 + \frac{.05}{12} \right)^{12(15)} = \$3500(1 + .00416667)^{180} = 7397.96$$

**Solution: A**

27. Solve  $\log_4 x = 3$  for  $x$ .
- 4
  - 16
  - 64
  - 256

$$\log_4 x = 3$$

$$4^3 = x$$

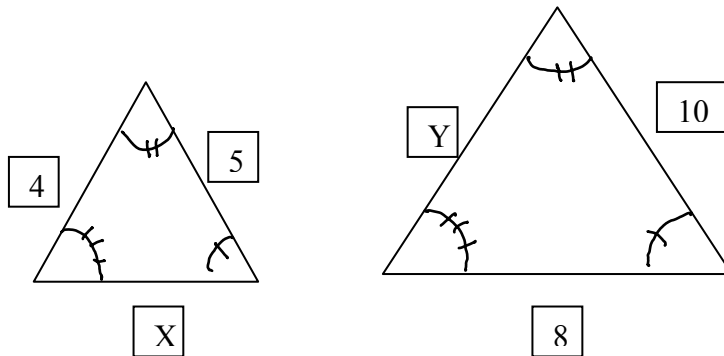
$$64 = x$$

**Solution: C**

28. Find the Richter Scale magnitude of an earthquake that hit Japan. It had an intensity of  $18,765,985 I_0$ . Use the model below and round your answer to one decimal place.

- a. 7.3
- b. 9.2
- c. 8.1
- d. 6.5

29. Given the following similar triangles.



Find the length of the missing sides  $x$  and  $y$ .

- a.  $x = 4, y = 8$
- b.  $x = 3, y = 6$
- c.  $x = 3, y = 4$
- d.  $x = 6, y = 8$

$$\frac{4}{y} = \frac{5}{10}$$

$$5y = 4(10)$$

$$5y = 40$$

$$y = 8$$

$$\frac{x}{8} = \frac{5}{10}$$

$$10x = 40$$

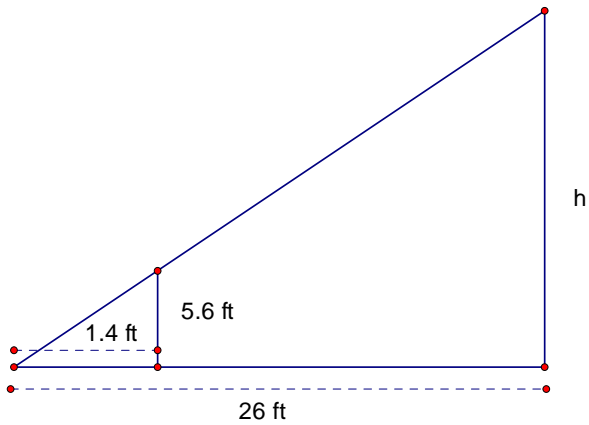
$$x = 4$$

**Solution: A**

30. Name two properties that similar triangles have?

Congruent angles and proportional sides

31. A person who is 5.6 feet tall casts a 1.4 foot shadow at the same instant that a tree casts a shadow of 26 feet. How tall is the tree?
- a. 0.3 ft
  - b. 6.5 ft
  - c. 27.4 ft
  - d. 104 ft



$$\frac{1.4}{5.6} = \frac{26}{h}$$

$$1.4h = (5.6)(26)$$

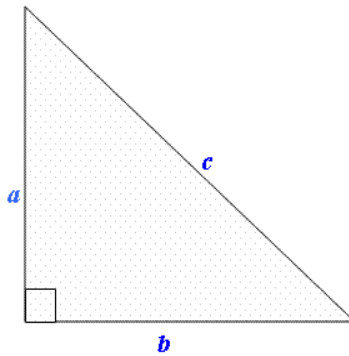
$$1.4h = 145.6$$

$$\frac{1.4h}{1.4} = \frac{145.6}{1.4}$$

$$h = 104 \text{ ft}$$

**Solution D**

32. For the right triangle given by



determine  $c$  if  $a = 13$  and  $b = 22$ .

- a. 653
- b.  $\sqrt{53}$
- c. 35
- d. 25.55

$$c^2 = a^2 + b^2$$

$$c^2 = 13^2 + 22^2$$

$$c^2 = 169 + 484$$

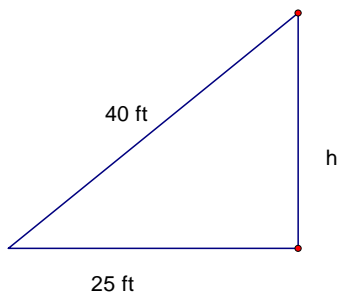
$$c^2 = 653$$

$$c = \sqrt{653} = 25.55$$

**Solution: D**

33. Suppose a carpenter has a 40 ft diagonal brace to put on a rectangular wall that is 25 feet long. How high does the wall need to be for the brace to fit?

- a. 25 ft
- b. 31.2 ft
- c. 47.2 ft
- d. 65 ft



$$c^2 = a^2 + b^2$$

$$40^2 = 25^2 + h^2$$

$$1600 = 625 + h^2$$

$$h^2 = 975$$

$$c = \sqrt{975} = 31.2 \text{ ft}$$

**Solution: B**

34. Given the geometric solid for a cube, find the volume of a cube when the side has a length of 3 inches.
- 9 cubic inches
  - 27 cubic inches
  - 18 cubic inches
  - 36 cubic inches

$$V = 3 \text{ in} \cdot 3 \text{ in} \cdot 3 \text{ in} = 27 \text{ in}^3$$

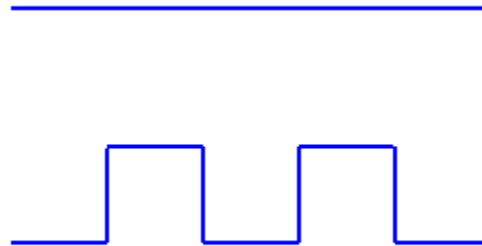
**Solution: B**

35. Suppose a spherical globe has a radius for 12 in. Find the approximate volume of the globe (recall that the volume of a globe is given by the formula  $V = \frac{4}{3}\pi r^3$ )
- $2304 \text{ in}^3$
  - $1728 \text{ in}^3$
  - $7238.2 \text{ in}^3$
  - $14476.5 \text{ in}^3$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3 = 7238.2 \text{ in}^3$$

**Solution: C**

36. In the figure below, a fractal is created by starting with a line segment, dividing it into equal parts and replacing the middle with the indicated geometric shape. Draw the next iteration of the fractal.



**Solution**

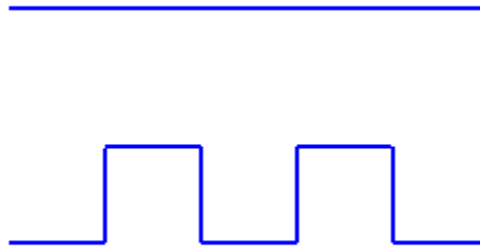


37. Rounded to two decimal places, find the dimension,  $d$ , of the fractal if the scaling ratio,  $r$ , = 5 with a replacement ratio,  $N$ , of 9.
- 1.37
  - 0.73
  - 1.26
  - 1.46

$$d = \frac{\log(9)}{\log(5)} = 1.37$$

**Solution: A**

38. In the figure below, a fractal is created by starting with a line segment, dividing it into equal parts and replacing the middle with the indicated geometric shapes. Find the dimension of this fractal.



- a. 0.73
- b. 1.22
- c. 1.29
- d. 1.37

$$N = 9$$

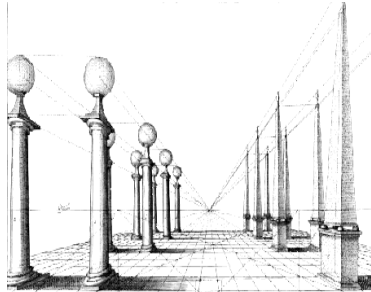
$$r = \frac{1}{5}$$

$$s = 5$$

$$d = \frac{\log(9)}{\log(5)} = 1.37$$

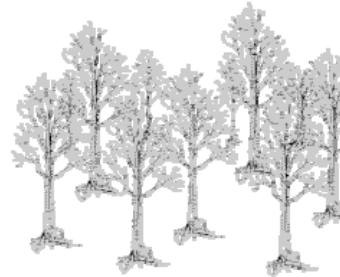
**Solution: D**

39. For each the following pictures, the dominant perspective is either overlapping shapes, atmospheric, diminishing sizes, one point perspective, or very little perspective is used. In the blank following the picture, list the best answer for the dominant perspective (there should be just one answer for each problem).



a.

Answer: one point perspective



b.

Answer: Overlapping shapes



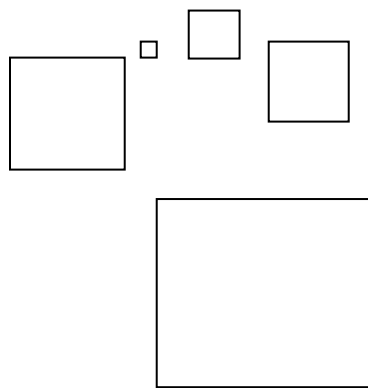
c.

Answer: none



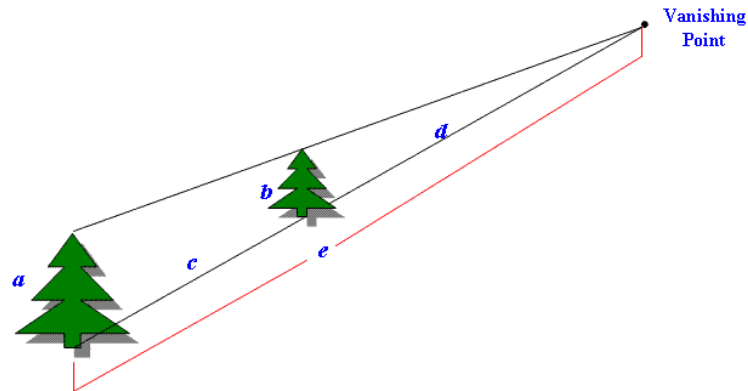
d.

Answer: atmospheric Perspective



e. Answer; diminishing sizes

40. Suppose a series of trees of the same size are a part of a one-point perspective drawing. The letters  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  represents heights or distances given by the following figure



If  $a = 215$  ft,  $b = 175$  ft, and  $e = 325$  ft, find  $d$ . Round your answer to 2 decimal places.

- 264.5 ft.
- 195.0 ft.
- 205.5 ft.
- 128.2 ft

$$\frac{215}{325} = \frac{175}{d}$$

$$215d = (325)(175)$$

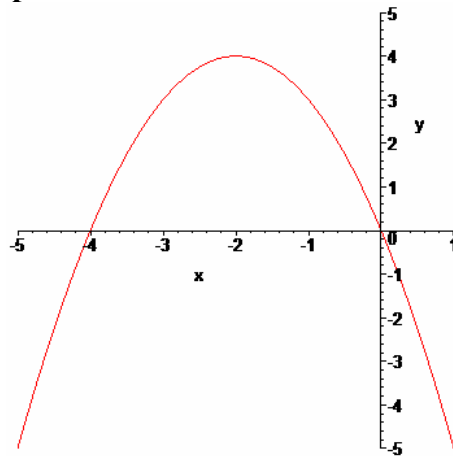
$$215d = 56875$$

$$d = 264.5$$

**Solution: A**

**Practice Final Exam Fall 2007 – Answers**

1. c
2. c
3. d
4. a
5. d
6. d
7. c
8. b
9. c
10. b
11. a
12. b
13. c
14. b
15. b
16. a
17. c
18. a
19. b
20. a
21. d
22. b
23. d
24. Vertex:  $(-2, 4)$ ,  $x$  intercepts:  $(-4, 0)$ ,  $(0, 0)$ , Graph:



25. c
26. a
27. c
28. a
29. a
30. Corresponding angles are equal and corresponding sides are proportional.
31. d

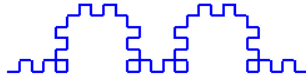
32. d

33. b

34. b

35. c

36. Next Iteration is



37. a

38. d

39. a: One Point, b: Overlapping shapes, c: Very little, d: Atmospheric,  
e: Diminishing Sizes

40. a