Math 114

Sets Unit

Section 1.1 Introduction to sets

Definition of a set

A **set** is a collection of objects, things or numbers.

The **universal set** is the set of all possible elements of set used in the problem. Denoted by U

Examples

$\{2,4,6,8,10\}$
$\{Mike, John, Mary, Cathy\}$
$\{Virginia, West\ Virginia, Maryland\}$

Elements are the members of a given set.

$\in$ *represents is an element of*
$\notin$ *represents is not an element of*

$3 \in \{1,2,3,4,5\}$
$a \in \{a, b, c, d, e\}$

The element 3 is an element of the set $\{1,2,3,4,5\}$. This can be written as $3 \in \{1,2,3,4,5\}$
In the case where an element such as 6 that is not in the set $\{1,2,3,4,5\}$, we would write as follows: $6 \notin \{1,2,3,4,5\}$

Roster Notation

$\{a, e, i, o, u\}$
$\{Huron, Ontario, Michigan, Erie, Superior\}$
$\{2,4,6,8,.......\}$

Builder Set Notation

$\{x \mid x\ is\ a\ vowel\}$
$\{x \mid x\ is\ a\ great\ lake\}$
$\{x \mid x\ is\ an\ even\ natural\ number\}$

A set is **well defined** is the elements of the sets are clearly defined.

If a set is well defined, then there should not be any confusion of what the elements are in the set

Examples of well defined sets

$\{1,3,5,7,9,11,13\}$
$\{a,b,c,d\}$
$\{x \mid x \text{ is a even number}\}$

Examples of set that are not well defined

$\{x \mid x \text{ is something that tastes good}\}$
$\{x \mid x \text{ is a good football team}\}$

The Empty Set

The **empty set** is a set that contains no elements. The empty set is also referred to as the **null set**.

Symbol representation ϕ or { }

Special Sets

The Natural Numbers "Counting" $N = \{1,2,3,4,5,.......\}$

The Whole Numbers $\{0,1,2,3,4,5,.......\}$

The Integers $Z = \{.....-5,-4,-3,-2,-1,01,2,3,4,5,.......\}$

The Irrational Numbers $Q = \left\{\frac{a}{b}, a,b \in Z \text{ and } b \neq 0\right\}$

Section 1.2

Subsets

A set B is a subset of set C, if every element in B is an element of C. $B \subset C$

Proper Subsets

A set B is a proper subset of C, if every element of B is an element of C and there is at least one element of B that is not in C. $B \subset C$

Example

$A = \{1,2,3,4,5\}$
$C = \{1,2,3,4,5,6,7\}$

$Is\ A \subset C?$
Since every element in the set A is an element of C, A is a subset of C.

Example 1

$A = \{1,2,3,4,5\}$
$C = \{1,2,3,4,5,6,7\}$

$Is\ A \subset C?$

Solution: Since every element in the set A is an element of C, A is a subset of C.

Example 2

$Is\ \{4,5,6\}\ a\ subset\ of\ \{0,1,2,3,4,5\}$?

Solution: no, since the element 6 in not in the set $\{0,1,2,3,4,5\}$

Example 3

List all possible subsets of $\{a,c\}$
Solution: $\phi, \{a\}, \{c\}, \{a,c\}$

Example 4

List all subsets of the set {a,b,c}

Possible subsets

Solution: $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\{a,b,c\}$

Example 5

List all subsets of the set {4}

Possible sets: $\phi, \{4\}$

The pattern for subsets

Number of elements	Number of subsets
1	2
2	4
3	8
4	16

Formula to find the number of subsets s of a given set A with n elements

$s = 2^n$

Example 6

How many subsets does a set A with 13 elements have?

$s = 2^n$

$s = 2^3$

$s = 8192$

Union and Intersection

Union of Two Sets

The union of two sets is denoted by $A \cup B$ is $A \cup B = \{x \mid x \in A \ or \ x \in B\}$

Intersection of Two Sets

The intersect of two sets is denoted by $A \cap B$ is $A \cap B = \{x \mid x \in A \ and \ x \in B\}$

Example 1

Let A={1,2,3,4,5,6,7},B={1,3,5,7},C={1,2},D={3,4,5,6,7,8}, and E=ϕ

1) Is $C \subset A$?
Answer: Yes, every element in C is contained in A

2) Is $B \subset A$?
Answer: Yes, every element in C is contained in A

3) Is $D \subset A$?
Answer: No, 8 is an element of D and not an element of A?

4) Is $\phi \subset A$?
Yes, the empty set is a subset of any nonempty every set.

5) Find $A \cap B$
Answer: $A \cap B = \{1,3,5,7\}$

6) Find $A \cup B$
Answer: $A \cup B = \{1,2,3,4,5,6,7\}$

7) Find $A \cap C$
Answer: $A \cap C = \{1,2\}$

Fundamental Counting Principle

The fundamental counting principle says that if there are m ways to do task A and n ways to do task B, then there are mn ways to do task A followed by task B.

Example 1

Principles of counting

A particular model of car can be purchased with one of 3 different colors, with one of 2 different transmissions, and with one of 6 different options packages. How many different configurations of this car can be purchased?

In this problem, there essentially are 3 options followed by 2 options followed by 6 options. Thus, the answer can be found by multiplying the options.

$3 \cdot 2 \cdot 4 = 6 \cdot 4 = 24$ *possible combinations*

Example 2

The local paint store offers wallpaper in six colors, each of which comes in 8 different patterns. How many different styles of wallpaper are available?

Solution: $6 \cdot 8 = 48$ *possible combinations*

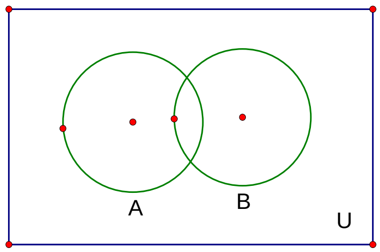
Example 3

The super sub at the local sub shop comes with a choice of one of three meats, one of four cheeses, and a choice from 5 different breads. How different super subs can be made?

$3 \cdot 4 \cdot 6 = 12 \cdot 6 = 72$ *possible combinations*

Venn Diagrams

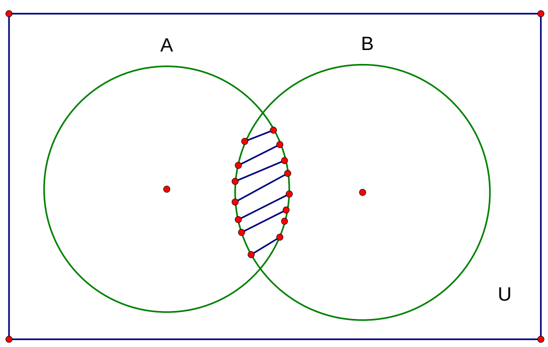
General Venn diagram for sets A and B



$\mathcal{U}$ = the universal set

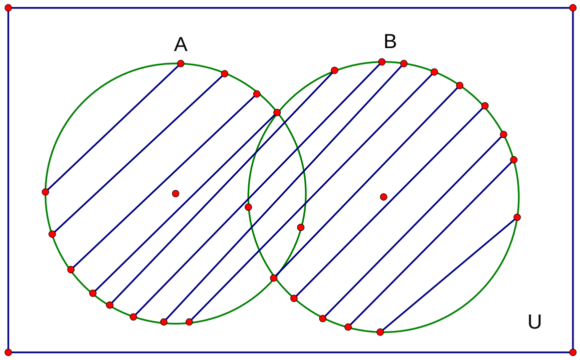
The Venn diagram for $A \cap B$

$A \cap B$



The Venn diagram for $A \cup B$

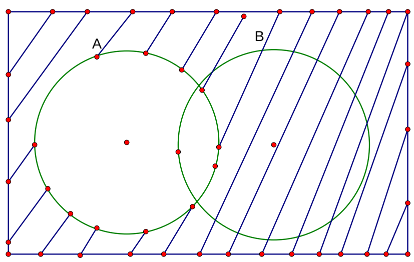
$A \cup B$



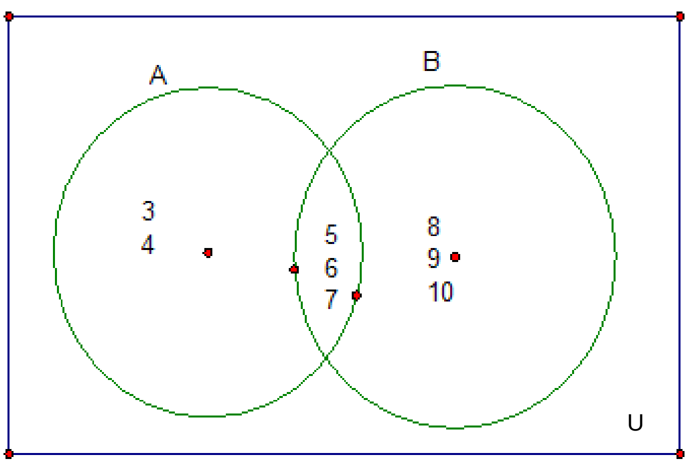
The complement of a set A

The complement of a set A is the set of all elements in the universal that are not elements of the set A.

$A' = \{x \mid x \notin A \text{ and } x \in U\}$



Example 2



1) Find $A \cap B$
   $A \cap B = \{5,6,7\}$

2) Find $A \cup B$
   $A \cup B = \{3,4,5,6,7,8,9,10\}$

3) Find A'
   $A' = \{8,9,10\}$

Example 3

Given

$A = \{1,2,3,4,5,6\}, B = \{4,5,6,7,8\}, U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

Find

1) $A \cup B$

$A \cup B = \{1,2,3,4,5,6,7,8\}$

2) $A \cap B$

$A \cap B = \{4,5,6\}$

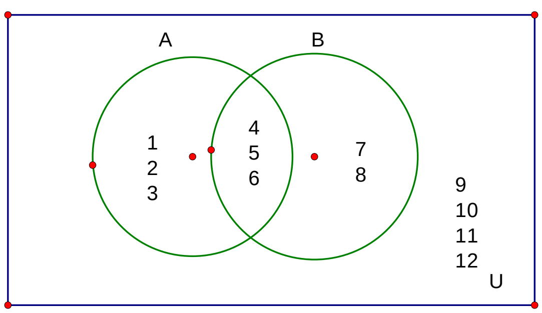
3) A'

$A' = \{7,8,9,10,11,12\}$

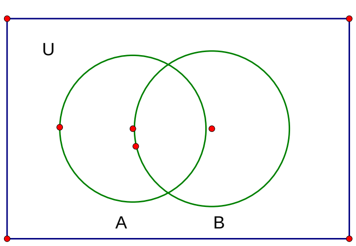
4) B'

$B' = \{1,2,3,9,10,11,10\}$

5) Make a Venn diagram of A,B, and U

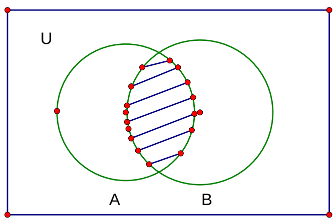


Venn diagrams

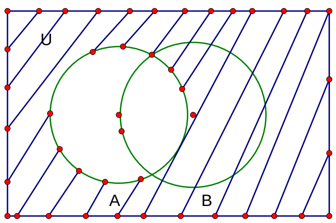


Shade the region corresponding to the indicated set.

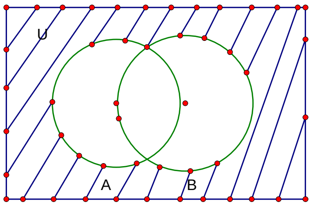
1) $A \cap B$



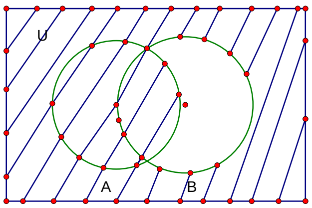
2) A'



3) $A' \cap B'$



4) $A \cup B'$

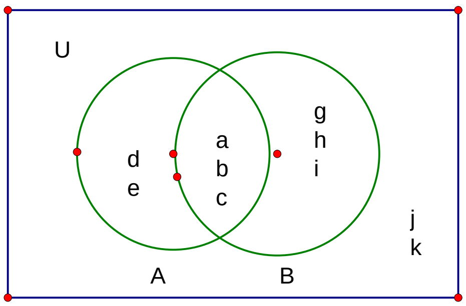


Cardinality

Definition: Cardinality is the number of elements in a given set

The number of elements in a set A is denoted by $n(A)$

$A = \{a, b, c, d, e\}, B = \{a, b, c, g, h, i\}, U = \{a, b, c, d, e, f, g, h,, i, j, k\}$



1) Find $n(A)$

$n(A) = 5$

2) Find $n(B)$

$n(B) = 6$

One-to-one correspondence

Definition: Two sets are in one-to-one correspondence if each element in the first is paired with exactly one element in the second set, and each element of the second set is paired with exactly one element form the first set.

Example 4

The sets $\{1,2,3,4\}$ *and* $\{a, b, c, d\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{cccc} \{1, & 2, & 3, & 4\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{a, & b, & c, & d\} \end{array}$$

Example 5

The sets $\{Viriginia,\ Maryland, North\ Carolina\}$ and $\{Richmond, Annapolis, Raleigh\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{ccc} \{Viriginia, & Maryland, & North\ Carolina\} \\ \updownarrow & \updownarrow & \updownarrow \\ \{Richmond, & Annapolis, & Raleigh\} \end{array}$$

Two sets are equivalent if they have the same cardinality or they contain the same number of elements.

Therefore, the sets from example 4 and example 5 are equivalent.

$\{1,2,3,4\}\ is\ an\ equivalent\ set\ to \{a,b,c,d\}$

$\{Viriginia,\ Maryland, North\ Carolina\}\ is\ an\ equivalent\ set\ to\ \{Richmond,\ Annapolis,\ Raleigh\}$

Example 6

Are these sets equivalent?

$\{3,4,5,6\}\ and\ \{a,b,c,d\}$

Solution: Yes

Problem Set 1

I) Which of the following sets are well defined?

1) $\{1,2,3,4,5,6,7\}$
2) $\{x \mid x \text{ is a U S state}\}$
3) $\{x \mid x \text{ is a fun game}\}$

II) Write each set in roster form

1) $\{x \mid x \text{ is a state that begins with the letter } V\}$
2) $\{x \mid x \text{ is a vowel}\}$

III) Subsets

1) List all subsets of $\{a, i\}$
2) List all subsets of $\{v, p, i\}$
3) List all subsets of $\{4,5,6,7\}$
4) A set of 15 elements would have how many possible subsets

IV) Union and Intersection

$Let\ A = \{a,b,c,d\},\ B = \{b,d,f,h\},\ C = \{h,i,j,k\},\ D = \{e,f,g,h,i\}$

5) Find $A \cup B$
6) Find $A \cap B$
7) Find $B \cup C$
8) Find $A \cap D$
9) Is $C \subset A$?
10) Is $B \subset A$?
11) Make a Venn diagram for sets $A, B,\ and\ U$

V) Union and Intersection

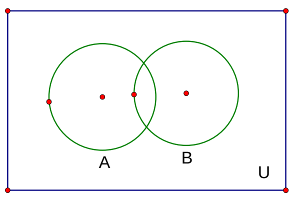
$Let\ A = \{2,4,6,8,10\},\ B = \{3,5,7,9\},\ C = \{2,3,4,5\},\ D = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

$U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

12) Find $A \cup B$
13) Find $A \cap B$
14) Find A'
15) Find $(A \cap B)'$
16) Make a Venn diagram for sets $A, B,\ and\ U$

VI) Venn Diagrams

Use the general Venn diagram to answer the following questions



17) Shade the Venn diagram of $A \cup B$
18) Shade the Venn diagram of $A \cap B$
19) Shade the Venn diagram of A'
20) Shade the Venn diagram of $A \cup B'$