Finance Unit
Math 114
Radford University

Section 6.1

Percents

Introduction to Basic Percents

The word percent translates to mean "out of one hundred". A score of 85% on test means that you scored 85 points out of 100 possible points on the test. If you scored 44 out of 50 points on a test, then this would be a percent value of 88%. This value can be obtained by multiplying the numerator and denominator by 2 as shown in the next illustration.

$$\frac{44}{50} = \frac{2(44)}{2(50)} = \frac{88}{100} = .88 = 88\%$$

Since a percent represents the amount out of a hundred, to change a percent to a decimal, you simply drop the percent symbol and divide by 100 which can be done by moving the decimal two placing to the left as shown in the next examples.

$$45\% = \frac{45}{100} = .45$$
$$64.5\% = \frac{64.5}{100} = .645$$

Basic Percent Problems

One of the basic uses of percents is to find the percent amount of a given number. For example, how you would take 34% of 60? This would be done by changing 34% to .34, and then multiplying by .34 by 60 as shown here:

$$34\% \Rightarrow .34 \Rightarrow (.34)(60) = 20.4$$

Example 1

What is 46% of 90?

$$46\% \Rightarrow .46$$
$$(.46)(90) = 41.4$$

Mark up, mark down, and sales price

There are many common uses for percents in our society. As consumers, people use percentages to find sales prices, mark up prices, and discount. In this section, we will study how to use percents to compute discounts, mark up prices, sales prices, and sales tax. The first of these topics we will explore are discount and sales price.

Discount

Discount = (Percent Mark Down)(Retail Price)

Sale Price

Sale Price = Retail Price – Discount

Example 2

A men's sports jacket that has a retail price of $170 is discounted by 25%. What is the sale's price of the sports jacket?

$$\begin{aligned}&Mark\ Down = 25\% = .25\\&Discount = (.25)(\$170) = \$42.50\\&Sales\ price = \$170 - \$42.50 = \$127.50\end{aligned}$$

Example 3

A pair of jeans that has a retail price of $55.00 is discounted at 30%. What is the sale's price of the jeans?

$$\begin{aligned}&Mark\ Down = 30\% = .30\\&Discount = (.30)(\$55.00) = \$16.50\\&Sales\ price = \$55 - \$16.50 = \$38.50\end{aligned}$$

Example 4

The sale price of a VCR is $110.00. If the mark down is 30%, find the retail price of the VCR.

$$Let\ \ x = original\ \ price$$
$$.30x = discount$$
$$discount\ \ price = \$110.00$$
$$x - .30x = 110.00$$
$$.70x = 110.00$$
$$\frac{.70x}{.70} = \frac{110.00}{.70}$$
$$x = \$157.14$$

Mark Up Price

When stores purchase items at a whole sale price, the retail price is computed by marking up the whole sale cost using the given formulas.

Mark Up = (Percent Mark Up)(Whole Sale Price)
Retail Price = Whole Sale Price + Mark Up

Example 5

A store purchases DVD players at a whole sale price of $30 per unit which is to be marked up by 80%. What will be the retail price of the DVD player?

$$percent\ mark\ up = 80\% = .80$$
$$mark\ up = (.80)(\$30.00) = \$24.00$$
$$retail\ price = \$30.00 + \$24.00 = \$54.00$$

Example 6

The whole sale price of a pair of jeans is \$20.00. If the jeans are marked up by 65%, what is the retail price of the jeans?

$$percent\ mark\ up = 65\% = .65$$
$$mark\ up = (.65)(\$20.00) = \$13.00$$
$$retail\ price = \$20.00 + \$13.00 = \$33.00$$

Example 7

The retail price of a new television that has been marked up by 75% is \$300.00. Find the whole sale price of the television.

$$Let\ x = original\ \ price$$
$$.75x = discount$$
$$discount\ \ price = \$300.00$$
$$x + .75x = 300.00$$
$$1.75x = 300.00$$
$$\frac{1.75x}{1.75} = \frac{300.00}{1.75}$$
$$x = \$171.43$$

Sales Tax

When items are purchased at a store or place of business, a state sale's taxes is calculated and added on the price of the item. The percent rate of sale's tax in the United States is determined by each state. For example the sales tax in Virginia is 4.5%. Some states such as Delaware and Montana do not have any sale's tax. The state sale's tax is calculated by multiplying the percent rate by the purchase price. The state sale's tax is then added on the purchase price of the item.

Sales Tax Formula

Sale's Tax = (sale's tax rate)(purchase price)

Example 8

The state sale's tax rate in Virginia is 4.5%. Find the full cost to purchase a $50 pair of shoes using the Virginia tax rate of 4.5%.

$$sales\,tax = (.045)(50) = \$2.25$$
$$Cost\ including\ tax = \$50.00 + \$2.25 = \$52.25$$

Example 9

The state sale's tax rate in Ohio is 6%. Find the full cost to purchase the same pair of shoes in problem 7 using the Ohio tax rate of 6%.

$$sales\,tax = (.06)(50) = \$3.00$$
$$Cost\ including\ tax = \$50.00 + \$3.00 = \$53.00$$

Problem Set (Section 6.1)

1) Find the discount on each item if the mark down rate is 5%.
a) $90.00
b) $25.00
c) $130.00

2) Find the discount on each item if the mark down rate is 15%.
a) $100.00
b) $45.00
c) $140.00

3) Find the sale's price on each item given the mark down rate is 20%.
a) $120.00
b) $400.00
c) $215.00

4) Find the sale's price on each item given the mark down rate is 15%.
a) $60.00
b) $130.00
c) $15.00

5) A pair of jeans that has a retail price of $42.00 is discounted at 25%. What is the sale's price of the jeans?

6) A women's dress that has a retail price of $80 is discounted by 35%. What is the sale's price of the dress?

7) The sale price of a television is 200.00. If the mark down is 22%, find the retail price of the television.

8) The sale price of a laptop computer is $1100.00. If the mark down is 10%, find the retail price of the laptop computer.

9) Using a mark up rate of 30%, find the retail price given the whole sale price of each item.
a) $140.00
b) $30.00
c) $75.00

10) Using a mark up rate of 45%, find the retail price given the whole sale price of each item.
a) $200.00
b) $34.00
c) $124.00

11) The wholesale price of a pair of dress pants is $25.00. If the jeans are marked up by 60%, what is the retail price of the pants?

12) The wholesale price of a CD player is $57.00. If the CD player is marked up by 30%, what is the retail price of the CD player?

13) The retail price of a pair of dress pants is $70.00. If the jeans are marked up by 25%, what is the whole sale price of the pants?

14) The retail price of a new television that has been mark up by 55% is $420.00. Find the whole sale price of the television.

15) The sale's tax rate in North Carolina is 4.5%. Find the total cost including sale's tax for purchasing each item.
a) $150.00
b) $340.00

16) The sale's tax rate in Michigan is 6%. Find the total cost including sale's tax for purchasing each item.
a) $250.00
b) $420.00

Section 6.2

Simple Interest and Future Value

An important principle in banking and finance is the use of interest. Interest is simply when money is paid for the use of money. If you were to put money in a saving account, the bank would pay you interest in return for use of your money as investments. A special type of interest called **simple interest** is used to compute the amount of money earned or owed on the balance of money known as the **principle**. The principle is usually the amount money you have invested in the bank or the amount of money you have borrowed from the bank. Simple interest is computed form the principle, interest rate and period of time of the investment. The simple interest formula below is used for many applications of banking

Simple interest formula

$I = PRT$

$I = Interest$

$P = priniciple$

$R = Rate$

$T = Time$

In this first example, we use the simple interest formula to compute simple interest on a principle of $600.

Example 1 (Saving Account)

How much interest is earned if $600 is put in a savings account that pays 2% interest for 3 years?

Solution:

$P = \$600.00$

$I = 2\% = .02$

$T = 3 \ years$

$I = PRT$

$I = (\$600.00)(.02)(3)$

$I = (\$12.00)(3)$

$I = \$36.00$

Example 2 (Bank Loan)

You borrow $5000 from a bank that has an 8% interest rate for 4 years. How much simple interest do you owe the bank?

$$P = \$5000.00$$
$$I = 8\% = .08$$
$$T = 4 \ \ years$$

$$I = PRT$$
$$I = (\$5000.00)(.08)(4)$$
$$I = (\$400.00)(4)$$
$$I = \$1600.00$$

In the next example we will use the simple interest formula to complete the following table that consists of missing values for time, interest, and interest rate. Each of these values can be found by solving for the missing variable in the table.

Example 3 (Interest Table)

Complete the following on simple interest

Interest	Principle	Rate	Time
a) ?	$1300	2.5%	3 years
b) $50	$600	3%	?
c) $120	$1200	?	4 years

Part a) Find the interest

$$Solve \ I = PRT \ \ for \ I$$
$$Convert \ 2.5\% = .025, then \ use \ the \ \ formula$$
$$I = PRT$$
$$I = (\$1300)(.025)(3)$$
$$I = (\$32.50)(3)$$
$$I = \$97.50$$

Part b) Find the time

$$I = PRT \quad (Solve\ \ for\ T)$$
$$\$50 = (\$600)(.03)T$$
$$\$50 = \$18T$$
$$\frac{\$50}{\$18} = \frac{\$18T}{\$18}$$
$$2.7\ years = T$$

Part c)

$$I = PRT \qquad (Solve\ \ for\ \ R)$$
$$\$120 = (\$1200)R(4)$$
$$\$120 = \$4800\,R$$
$$\frac{\$120}{\$4800} = \frac{\$4800\,R}{\$4800}$$
$$R = \frac{120}{4800}$$
$$R = .025 = 2.5\%$$

Future Value

When interest is computed in a saving account, it is then added to the balance or principle. This new balance or principle is known as the **future value**.

Future Value Formula

Future Value = Principle + Interest

$$A = P + I$$
$$A = P + PRT$$
$$A = P(1 + RT)$$

Example 4   Future Value

If $15,000 is deposited in a saving account earning 2.2 % simple interest, what is the future value in 5 years?

$$P = \$15{,}000$$
$$R = 2.2\% = .022$$
$$T = 5 \ \ years$$

$$A = \$15{,}000(1 + .022 \cdot 5)$$
$$A = \$15{,}000(1 + .11)$$
$$A = \$15{,}000(1.11)$$
$$A = \$16{,}650$$

Example 5 (Finding the interest rate)

If a business borrows $18,000 and repays $26,100 in 3 years, what is the simple interest rate?

$$P = \$18{,}000$$
$$A = \$26{,}100$$
$$T = 3 \ \ years$$

$$26{,}000 = 18{,}000(1 + R(3))$$
$$26{,}100 = 18{,}000(1 + 3R)$$
$$26{,}100 = 18{,}000 + 54{,}000R$$
$$26{,}100 - 18{,}000 = 18{,}000 - 18{,}000 + 54{,}000R$$
$$8100 = 54{,}000R$$
$$\frac{8100}{54000} = \frac{54{,}000R}{54{,}000}$$
$$R = .15 \ \ or \ \ R = 15\%$$

Example 6 (Computing Time)

Suppose you wish to save $3,720. If you have $3000 and invest it at a 4% simple interest rate, how long will it take to obtain $3,720?

$P = \$3{,}000$

$A = \$3{,}720$

$R = 4\% = .04$

$$
\begin{aligned}
3{,}720 &= 3{,}000(1 + (.04)T) \\
3{,}720 &= 3{,}000(1 + .04T) \\
3{,}720 &= 3{,}000 + 120T \\
3{,}720 - 3{,}000 &= 3{,}000 - 3{,}000 + 120T \\
720 &= 120T \\
\frac{720}{120} &= \frac{120T}{120} \\
T &= 6 \ \textit{years}
\end{aligned}
$$

Problem Sets (Section 6.2)

Complete the following chart

Interest	**Principle**	**Rate**	**Time**
1) ?	$1500	3.5%	5 years
2) $50	$190	5%	?
3) $100	$1500	?	5 years
4) ?	$15,450	$2\frac{3}{4}\%$	10 years
5) $1000	$25,000	?	5years

Simple Interest

6) How much interest is earned if $4200 is put in a savings account that pays 1.5% interest for 4 years?

7) You borrow $10,000 from a bank that has a 9.4% interest rate for 5 years. How much simple interest do you owe the bank?

8) You borrow $25,000 from a bank that has a 10.5% interest rate for 5 years. How much simple interest do you owe the bank?

Future Value

9) If $23,500 is deposited in a saving account with a simple interest rate of 1.3%, what is the future value in 10 years?

10) If $1,900 is deposited in a saving account earning 2.5 % simple interest, what is the future value in 4 years?

11) If a business borrows $100,000 and repays $126,100 in 4 years, what is the simple interest rate?

12) If you borrow $10,000 from the bank and repay the bank $12,500 in 5 years, what is the simple interest rate?

13) Suppose you wish to save $5,750. If you have $5000 and invest it at a 2% simple interest rate, how long will it take to obtain $5,750?

Section 6.3

Compound Interest

Normally most banks do not use the simple interest to pay interest to their investors or costumers. Usually interest is computed and then added to the principle periodically using a schedule. This idea is referred to as **compound interest**. Interest can be compounded daily, monthly, quarterly, or semiannually. In the first example of this section, we will study how interest is compounded in saving account using a semiannual schedule.

Example 1 (Using the simple interest formula to find the compound interest)

Suppose \$4000 is placed in a saving account for 2 years that compounds interest semiannually at a rate of 2% per year. How much money would be in the saving account after 2 years?

During the time period of 2 years, interest will be computed 4 times.

Year 1 (First half of year)

$P = \$4000$

$R = .02$

$T = \frac{1}{2} = .5$

$I = PRT$

$I = (\$4000)(.02)(.5)$

$I = (\$80)(.5)$

$I = \$40$

The new principle will be $P = \$4000 + \$40 = \$4040$

Year 1 (Second half of year)

$I = PRT$

$I = (\$4040)(.02)(.5)$

$I = (\$80.80)(.5)$

$I = \$40.40$

The new principle will be $P = \$4040 + \$40.40 = \$4080.40$

Year 2 (First half of year)

$$I = PRT$$
$$I = (\$4080.40)(.02)(.5)$$
$$I = (\$81.61)(.5)$$
$$I = \$40.80$$

The new principle will be $P = \$4080.40 + \$40.80 = \$4121.20$

Year 2 (Second half of year)

$$I = PRT$$
$$I = (\$4121.20)(.02)(.5)$$
$$I = (\$82.42)(.5)$$
$$I = \$41.21$$

The new principle will be $P = \$4121.20 + \$41.21 = \$4162.41$

As shown in example 1, the simple interest formula is used to find the compound interest. Now, you will discover that there is a simpler way to compute compound interest. The compound interest formula below can actually compute the compound interest much simpler than the simple interest formula.

Compound Interest Formula

$$P = principle$$
$$r = rate$$
$$t = time$$
$$n = number\ of\ times\ \text{int}erest\ is\ computed\ in\, a\ year$$
$$A = Accumulated\ Balance$$
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example 2 (Using the compound interest formula on example 1)

Suppose \$4000 is placed in a saving account for 2 years that compound interest semiannually at a rate of 2% per year. How much money would be in the saving account after 2 years?

$$P = \$4000$$
$$T = 2$$
$$R = .02$$
$$n = 2$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$A = \$4000\left(1+\frac{.02}{2}\right)^{2\cdot 2}$$
$$A = \$4000(1+.01)^4$$
$$A = \$4000(1.01)^4$$
$$A = \$4000(1.0406)$$
$$A = \$4162.41$$

Notice that this formula gives the same result as example 1, but does require near the same number of steps.

For the rest of the section, students are encouraged to use the compound interest formula to solve most of the compound interest problems.

Example 3 (Interest Compounded Quarterly)

Mike invests \$5000 in a special saving bond that compounds interest quarterly at a rate of 5% per year. How much money would Mike have in this saving bond after 10 years?

$$P = \$5000,\ T = 10,\ R = .05,\ n = 4\ (\textit{quarterly})$$

$$A = \$5000\left(1+\frac{.05}{4}\right)^{4\cdot 10} = \$5000(1+.0125)^{40} = \$5000(1.0125)^{40} = \$5000(1.6436) = \$8218.10$$

Example 4 (Monthly Compounding)

What if Mike would invest the $5000 in a saving bond in an account that compounded monthly at the same rate of 5% per year, how much money he have after 10 years?

$P = \$5000,\ T = 10,\ R = .05,\ n = 12\ (monthly)$

$$A = \$5000\left(1 + \frac{.05}{12}\right)^{12 \cdot 10}$$
$$A = \$5000(1 + .0041667)^{120}$$
$$A = \$5000(1.0041667)^{120}$$
$$A = \$5000(1.6470)$$
$$A = \$8235.05$$

By taking the same amount of money at the same interest rate, you can actually earn a little money in interest.

Example 5 (Compounding Daily)

Now suppose Mike take his $5000 and invests it in a saving bond that compounds daily for 10 years at 5% per year. How much money would he have in the saving bond after 10 years?

$P = \$5000,\ T = 10,\ R = .05,\ n = 365\ (daily)$

$$A = \$5000\left(1 + \frac{.05}{365}\right)^{365 \cdot 10}$$
$$A = \$5000(1 + .000136986)^{3650}$$
$$A = \$5000(1.00013986)^{3650}$$
$$A = \$5000(1.648664)$$
$$A = \$8243.32$$

Example 6 (Calculating Time)

How long would it take to save $15,000, if $10,000 was placed in saving account that compounds interest quarterly at a rate of 1.0 % per year?

$P = 10{,}000$

$A = 15{,}000$

$T = ?$

$n = 4$

$R = .01$

$$A = P\left(1 + \frac{R}{n}\right)^{nt}$$

$$15{,}000 = 10{,}000\left(1 + \frac{.01}{4}\right)^{4t}$$

$$15{,}000 = 10{,}000(1 + .0025)^{4t}$$

$$15{,}000 = 10{,}000(1.0025)^{4t}$$

$$\frac{15{,}000}{10{,}000} = \frac{10{,}000(1.0025)^{4t}}{10{,}000}$$

$$1.5 = (1.0025)^{4t}$$

$$\log(1.5) = \log(1.0025)^{4t}$$

$$\log(1.5) = 4t\log(1.0025)$$

$$.17609 = 4t(.00108)$$

$$.17609 = .00432t$$

$$t = \frac{.17609}{.00432}$$

$$t \approx 40.8 \ \textit{years}$$

Example 7

How long would it take to save \$15,000, if \$10,000 was placed in saving bond that compounds interest monthly at a rate of 6.0 % per year?

$P = 10{,}000, A = 15{,}000,\ n = 12,\ R = .06,\ t = ?$

$$15{,}000 = 10{,}000\left(1 + \frac{.06}{12}\right)^{12t}$$
$$15{,}000 = 10{,}000(1 + .005)^{12t}$$
$$15{,}000 = 10{,}000(1.005)^{12t}$$
$$\frac{15{,}000}{10{,}000} = \frac{10{,}000(1.005)^{12t}}{10{,}000}$$
$$1.5 = (1.005)^{12t}$$
$$\log(1.5) = \log(1.005)^{12t}$$
$$\log(1.5) = 12t\log(1.005)$$
$$.17609 = 12t(.002166)$$
$$.17609 = .025993t$$
$$t = \frac{.17609}{.025993}$$
$$t \approx 6.8\ \ years$$

Example 8

Carmen deposits \$25,000 into a certificate of deposit that guarantees 6.6% annual interest rate, compounded quarterly. How much will be in the account at the end of five years?

$P = \$15{,}000,\ R = 6.6\% = .006, n = 4, t = 5$

$$A = \$25{,}000\left(1 + \frac{.066}{4}\right)^{4(15)}$$
$$A = \$25{,}000(1 + .00165)^{60}$$
$$A = \$25{,}000(1.00165)^{60}$$
$$A = \$25{,}000(2.6695802)$$
$$A = \$66{,}739.50$$

Problem Sets (Section 6.3)

Find the missing value

1) Given $P = \$10{,}000,\ R = .03,\ n = 4,\ T = 10,\ Find\ \ A = ?$

2) Given $P = \$17{,}000,\ R = .06,\ n = 2,\ T = 5,\ Find\ \ A = ?$

3) Given $P = \$7{,}000,\ R = .01,\ n = 12,\ T = 10,\ Find\ \ A = ?$

4) Given $A = \$10{,}000,\ R = .04,\ n = 12,\ T = 10,\ Find\ \ P = ?$

5) Given $A = \$20{,}000,\ R = .03,\ n = 12,\ T = 5,\ Find\ \ P = ?$

6) Given $P = \$30{,}000,\ R = .012,\ n = 4,\ T = 6,\ Find\ \ A = ?$

7) Given $P = \$10{,}000,\ R = .03,\ n = 4,\ A = \$14{,}000,\ T = ?$

8) Given $P = \$12{,}000,\ R = .01,\ n = 365,\ A = \$20{,}000,\ T = ?$

Saving Accounts

9) Mark puts \$30,000 in a saving account that compounds interest quarterly at 2% for 5 years. What will be the balance in Mark account after 5 years?

10) Marcia invests \$1,000 in a saving account that compounds interest monthly for 3 years at a rate of 3% per year. How much money will she have in the bank after 3 years?

11) If you leave \$16,000 in an account that earns 4% interest, compounded daily, how much money would be in the account after 5 years?

12) How money would \$18,500 earn in 10 years if the saving account compounds interest monthly at a rate of 2.3% per year?

Saving Bonds

13) If you invest \$20,000 in a saving bond that compounds interest monthly at a rate of 5% per year for 10 years, how much money will be in the saving bond after 10 years?

14) If Juan invests $30,000 in a saving bond that compounds interest monthly at a rate of 8% per year for 5 years, how much money will be in the saving bond after 5 years?

15) How much money will be in a saving bond after 20 years if you would invest $2,000 in a saving bond plan that compounds interest monthly at rate of 8.5 % per year?

16) A special saving bond earns 9% interest. How much money would be in the saving bond after 10 years if $25,000 is invested in a saving bond that compounds interest quarterly?

Computing Time

17) How much time would it take $14,000 to accumulate to $20,000 in a saving account that compounds interest quarterly at 1.5% per year?

18) How much time would it take $15,000 to grow to an accumulated balance of $25,000 in a saving bond that compounds interest quarterly at 1.5% per year?

Complete the following chart on compound interest

Principle	Interest rate	Time	Number times compounded in a year (n)	New Balance
19) $5,600	2%	5 yr	n = 12	?
20) $12,000	8.6%	?	n = 4	$15,000
21) $30,000	7.6%	10 yr	n = 365	?

Section 6.4

Loan (Financing a Home or Car)

Buying a car or house is a financial decision that most people will encounter in their life time. In this section, we will look at how banks and financial companies compute the loan payment that a costumer's will pay when they finance a home or car. We will also answer common questions about getting a loan from a bank such as how much money will you pay in interest when you finance a home with a loan, or how will interest rate and term of loan can effect your loan payment.

The loan formula

$$P = principle$$
$$r = Rate$$
$$n = Number\ of\ Times\ Interest\ is\ Computed$$
$$T = Time$$
$$PMT = Payment$$

$$PMT = \frac{P \cdot \left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Example 1

What would your monthly house payment be on a \$140,000 house if you finance the house for 30 years at an interest rate of 6%?

$$P = \$140{,}000$$
$$R = 6\%$$
$$n = 12$$
$$T = 30$$

$$PMT = \frac{(\$140{,}000)\left(\frac{.06}{12}\right)}{1 - \left(1 + \frac{.06}{12}\right)^{-12(30)}} = \frac{(\$140{,}000)(.005)}{1 - (1 + .005)^{-360}} = \frac{\$700}{1 - (1.005)^{-360}} = \frac{\$700}{1 - .166041} = \frac{\$700}{.833958} = \$839.37$$

Example 2

What would your monthly car payment be on car that cost $16,000 if you finance the car for 5 years at an interest rate of 8%?

$P = \$16,000$
$R = 8\% = .08$
$n = 12$
$T = 5$

$$PMT = \frac{(\$16,000)\left(\frac{.08}{12}\right)}{1-\left(1+\frac{.08}{12}\right)^{-12(5)}} = \frac{(\$16,000)(.0066667)}{1-(1+.0066667)^{-60}} = \frac{\$106.67}{1-(1.0066667)^{-60}} = \frac{\$106.67}{1-.67121} = \frac{\$106.67}{.3287896} = \$324.43$$

Example 3

First, calculate the monthly house payment on an $110,000 house that is financed at a rate of 6.5 % per year for 30 years. Next, calculate the interest paid over the 30 years.

$P = \$110,000$
$R = 6.5\% = .065$
$n = 12$
$T = 30$

$$PMT = \frac{(\$110,000)\left(\frac{.065}{12}\right)}{1-\left(1+\frac{.065}{12}\right)^{-12(30)}} = \frac{(\$110,000)(.00541667)}{1-(1+.00541667)^{-360}} = \frac{\$595.83}{1-(1.00541667)^{-360}} = \frac{\$595.83}{1-.1430245} = \frac{\$595.83}{.8569754} = \$695.27$$

$Total\ \ Amount = 360(695.27) = \$250297.49$
$Interest\ \ Paid = \$250,297.49 - \$110,000 = \$140,297.49$

Example 4

The available interest rate on a $200,000 house is 6% per year. Compare the monthly payments for a 30 year loan and 15 year loan, and then determine which loan the 30 year or 15 year pays the least amount of interest.

$30 - year$

$$PMT = \frac{(\$200{,}000)\left(\frac{.06}{12}\right)}{1-\left(1+\frac{.06}{12}\right)^{-12(30)}} = \frac{(\$200{,}000)(.005)}{1-(1+.005)^{-360}} = \frac{\$1{,}000}{1-(1.005)^{-360}} = \frac{\$1{,}000}{1-.166041} = \frac{\$1000}{.833959} = \$1199.10$$

$15 - year$

$$PMT = \frac{(\$200{,}000)\left(\frac{.06}{12}\right)}{1-\left(1+\frac{.06}{12}\right)^{-12(15)}} = \frac{(\$200{,}000)(.005)}{1-(1+.005)^{-180}} = \frac{\$1{,}000}{1-(1.005)^{-180}} = \frac{\$1{,}000}{1-.40748} = \frac{\$1000}{.5925176} = \$1687.71$$

Total amount of the 30 year loan

$360(\$1199.10) = \$431{,}676$

Interest Paid

$\$431{,}676 - \$200{,}000 = \$231{,}676$

Total amount of the 15 year loan

$180(\$1687.71) = \$303{,}787.80$

Interest Paid

$\$303{,}787.80 - \$200{,}000 = \$103{,}787.80$

The 30 year loan payments are less that the 15 year loan. However, the 15 year loan pays much less interest than the 30 year loan.

Example 5

The available interest rate on a $20,000 car is 10% per year. Compare the monthly payments for a 3 year loan and 5 year loan, and then determine which loan the 3 year or 5 year pays the least amount of interest.

$3-\textit{year}$

$$PMT=\frac{(\$20{,}000)\left(\frac{.10}{12}\right)}{1-\left(1+\frac{.10}{12}\right)^{-12(3)}}=\frac{(\$20{,}000)(.00833333)}{1-(1+.00833333)^{-36}}=\frac{\$166.67}{1-(1.00833333)^{-36}}=\frac{\$166.67}{1-.7417398}=\frac{\$166.67}{.2582602}=\$645.36$$

$5-\textit{year}$

$$PMT=\frac{(\$20{,}000)\left(\frac{.10}{12}\right)}{1-\left(1+\frac{.10}{12}\right)^{-12(5)}}=\frac{(\$20{,}000)(.008333333)}{1-(1+.00833333)^{-60}}=\frac{\$166.67}{1-(1.008333333)^{-60}}=\frac{\$166.67}{1-.6077886}=\frac{\$166.67}{.3922113}=\$424.24$$

Total amount of the 30 year loan

$36(\$645.36)=\$23{,}232.96$

Interest Paid

$\$23{,}232.96-\$20{,}000=\$3{,}232.96$

Total amount of the 15 year loan

$60(\$424.24)=\$25{,}454.40$

Interest Paid

$\$25{,}454.40-\$200{,}000=\$5{,}454.40$

The 5 year loan payments are less that the 3 year loan. However, the 3 year loan pays much less interest than the 5 year loan.

Problem Set (Section 6.4)

1) Find the loan payment given $P = \$130{,}000, t = 15, r = 9\%$

2) Find the loan payment given $P = \$10{,}000, t = 5, r = 10\%$

3) Find the loan payment given $P = \$3{,}000, t = 5, r = 6\%$

4) Find the amount of the loan given $PMT = \$340, t = 5, r = 6\%$

5) Find the amount of the loan given $PMT = \$840, t = 30, r = 6\%$

6) What would your monthly house payment be on $136,000 town home if you finance the house for 30 years at an interest rate of 6%?

7) What would your monthly house payment be on $240,000 house if you finance the house for 30 years at an interest rate of 8%?

8) What would your monthly car payment be on Honda Accord that cost $23,000 if you finance the car for 5 years at an interest rate of 12%?

9) What would be the payment on a $55,000 motorboat if the motorboat can be financed for 15 years at an interest rate on 9.5% per year.

10) The available interest rate on a $17,000 Pontiac Sunbird is 9% per year. Compare the monthly payments for a 3 year loan and 5 year loan, and then determine which loan the 3 year or 5 year pays the least amount of interest.

11) The available interest rate on a $195,000 house is 6% per year. Compare the monthly payments for a 30 year loan and 15 year loan, and then determine which loan the 30 year or 15 year pays the least amount of interest.

12) The available interest rate on a $290,000 house is 8% per year. Compare the monthly payments for a 30 year loan and 15 year loan, and then determine which loan the 30 year or 15 year pays the least amount of interest.

13) What would be your monthly car payment be on car that cost 12,600 if you finance the car for 5 years at an interest rate of 6%?

14) Mike purchases a Porsche for $42,300 and finances the entire amount at a rate of 8.5% per year for 5 years. What would be Mike's car payments?

Solutions to Problem Sets

Problem (Section 6.1)

1) a) $4.50
   b) $1.25
   c) $6.50

2) a) $15.00
   b) $6.75
   c) $21.00

3) a) $24.00
   b) $320.00
   c) $172.00

4) a) $51.00
   b) $110.50
   c) $12.50

5) $31.50
6) $52.00
7) $256.41
8) $1222.22
9) a) $182.00
   b) $39.00
   c) $97.50
10) a) $290.00
    b) $49.30
    c) $179.80
11) $40.00
12) $74.10
13) $56.00
14) $270.97
15) a) $156.75
    b) $355.30
16) a) $265.00
    b) $445.20

Problem Set (Section 6.2)

1) $262.50
2) 5.3 years
3) 1.33%
4) $4248.75
5) .8%
6) $252.00
7) $4700.00
8) $13,125.00
9) $26,555.00
10) $2,090.00
11) 6.5%
12) 5%
13) 7 years

Problem Set (Section 6.3)

1) $13,483.49
2) $22,846.58
3) $7,735.87
4) $6,707.81
5) $17,217.39
6) $32,236.19
7) 11 years
8) 18 years
9) $33,146.87
10) $1094.05
11) $19,542,62
12) $23,278.98
13) $32,940.19
14) $44,695.19
15) $10,882.49
16) $61,283.93
17) 24.5 years
18) 34 years
19) $6188.44
20) 3 years
21) $64,143.21

Problem Set (Section 6.4)

1) $1318.56
2) $212.46
3) $69.60
4) $17,586.69
5) $140,104.96
6) $815.39
7) $1761.03
8) $511.62
9) $574.27
10) 3 year loan: $540.60 5 year: $352.89
11) 30-year loan: $1169.12 15-year loan: $1646.47
12) 30-year loan: $2127.91 15-year loan: $2771.38
13) $243.59
14) $867.85