

Math 114

Logic Unit

Test Review

1) Fill in the blank

| Connector | Symbol |
|-----------|---------------|
| Or | \vee |
| And | \wedge |
| If-then | \rightarrow |
| Negation | \sim |

Negation each statement

2) All cats are friendly

Answer: **Negation:** Some cats are not friendly

3) John likes ice cream.

Answer: **Negation:** John does not like ice cream

4) Dave likes Algebra and Geometry

$$A \wedge G$$

$$\sim (A \wedge G)$$

$$\sim A \vee \sim G$$

Negation: Dave does not like Algebra or Geometry

Using the symbolic representations

p: The cat is friendly

q: The cat is playful

5) Express the following compound statements in symbolic form.

a) The cat is friendly and the cat is playful

$$p \wedge q$$

b) Either the cat is playful or the cat isn't friendly

$$q \vee \sim p$$

c) If the cat is playful, then the cat is friendly.

$$q \rightarrow p$$

Complete the each true table.

6) $p \vee q$

| p | q | $p \vee q$ |
|---|---|------------|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

7) $p \wedge q$

| P | q | $p \wedge q$ |
|---|---|--------------|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

8) $p \rightarrow q$

| p | q | $p \rightarrow q$ |
|---|---|-------------------|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Complete each truth table

9) $(p \vee q) \rightarrow \sim p$

| p | q | $p \vee q$ | $\sim p$ | $(p \vee q) \rightarrow \sim p$ |
|---|---|------------|----------|---------------------------------|
| T | T | T | F | F |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | T |

$$10) (p \wedge q) \rightarrow q$$

| p | q | $p \wedge q$ | $(p \wedge q) \rightarrow q$ |
|---|---|--------------|------------------------------|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

Validity

- 1) Write the argument in symbolic form, then determine if the argument is valid
 p: you exercise regularly
 q: you are healthy

If you are healthy, you exercise regularly.

You don't exercise regularly

Therefore, you are not healthy.

If you are healthy, you exercise regularly. $q \rightarrow p$

You don't exercise regularly ($\sim p$)

Therefore, you are not healthy. ($\sim q$)

$(q \rightarrow p) \wedge (\sim p)$: hypothesis

$(\sim q)$: conclusion

$$(q \rightarrow p) \wedge (\sim p) \rightarrow (\sim q)$$

| p | q | $q \rightarrow p$ | $(\sim p)$ | $(q \rightarrow p) \wedge (\sim p)$ | $(\sim q)$ | $(q \rightarrow p) \wedge (\sim p) \rightarrow (\sim q)$ |
|---|---|-------------------|------------|-------------------------------------|------------|--|
| T | T | T | F | F | F | T |
| T | F | T | F | F | T | T |
| F | T | F | T | F | F | T |
| F | F | T | T | T | T | T |

The argument is not valid

Determine if the argument uses inductive reasoning or deductive reasoning

- 1) Math 114 is a math course

Math courses are fun

Math 114 is fun

Deductive reasoning

- 2) John had a chemistry quiz on Monday.
John had a chemistry quiz on Wednesday
 Therefore, John will have a quiz on Friday

Inductive reasoning

Equivalent Statements

- 1) Show that the following statements are equivalent.

$$q \rightarrow p \text{ and } \sim p \rightarrow \sim q$$

$$q \rightarrow p$$

| p | q | $q \rightarrow p$ |
|---|---|-------------------|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | F |

$$\sim p \rightarrow \sim q$$

| p | q | $\sim p$ | $\sim q$ | $\sim p \rightarrow \sim q$ |
|---|---|----------|----------|-----------------------------|
| T | T | F | F | T |
| T | F | F | T | F |
| F | T | T | F | T |
| F | F | T | T | T |

The truth tables have identical results in the last column. Thus, the statements are equivalent.

Solutions to the test 1 Review

Test 1 Review (Math 114) Radford University

Solutions

I) Reasoning and statements

1) Do the following arguments use deductive reasoning or inductive reasoning

- a) Stan noticed that a plane flew over his house at 5:00 PM Monday
Stan noticed that a plane flew over his house at 5:00 PM Tuesday
Stan noticed that a plane flew over his house at 5:00 PM Wednesday
Therefore, Stan concludes that a plane will fly over his house everyday at 5:00 PM

(Inductive reasoning)

- b) All mountain ranges are scenic.
The Blue Ridge Mountains are a mountain range
Therefore, the Blue Ridge Mountains are scenic.

(Deductive reasoning)

2) Negate the following statements.

- a) All trees have green leaves

Negation: Some trees do not have green leaves

- b) Some food at Greasy Nicks taste good.

Negation: All food at Greasy Nicks does not taste good.

II) Complete a truth table for each statement

- 3) $p \vee \sim q$

Solution:

| p | q | $\sim q$ | $p \vee \sim q$ |
|---|---|----------|-----------------|
| T | T | F | T |
| T | F | T | T |
| F | T | F | F |
| F | F | T | T |

$$4) (p \rightarrow q) \wedge \sim p$$

Solution:

| p | q | $\sim p$ | $p \rightarrow q$ | $(p \rightarrow q) \wedge \sim p$ |
|---|---|----------|-------------------|-----------------------------------|
| T | T | F | T | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

$$5) (\sim q \vee p) \wedge q$$

Solution:

| p | q | $\sim q$ | $(\sim q \vee p)$ | $(\sim q \vee p) \wedge q$ |
|---|---|----------|-------------------|----------------------------|
| T | T | F | T | T |
| T | F | T | T | F |
| F | T | F | F | F |
| F | F | T | T | F |

III) Use De Morgan's Law to negate each statement

$$6) p \wedge q$$

Negation: $\sim (p \wedge q) = \sim p \vee \sim q$

$$7) \sim p \vee \sim q$$

Negation: $\sim (\sim p \vee \sim q) = \sim (\sim p) \wedge \sim (\sim q) = p \wedge q$

IV) Determine if the argument is valid or invalid.

- 8) If John studies for his math test, he will pass the math test. $p \rightarrow q$
John did not pass his math test $\sim q$
 Therefore, John did not study for his math test $\sim p$

Argument: $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

| p | q | $\sim p$ | $\sim q$ | $(p \rightarrow q)$ | $(p \rightarrow q) \wedge \sim q$ | $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$ |
|---|---|----------|----------|---------------------|-----------------------------------|--|
| T | T | F | F | T | F | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | F | T |
| F | F | T | T | T | T | T |

Valid argument

- 9) If you work hard, then you will go far in life $p \rightarrow q$
You work hard p
 Therefore, you will go far in life. q

Argument: $((p \rightarrow q) \wedge p) \rightarrow q$

| p | q | $p \rightarrow q$ | $((p \rightarrow q) \wedge p)$ | $((p \rightarrow q) \wedge p) \rightarrow q$ |
|---|---|-------------------|--------------------------------|--|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

This argument is a tautology, so the argument is valid