

Math 126

Functions Unit

Key Terms

Relation: A set of ordered pairs

Domain: The set of all x-values or first coordinates in a relation.

Range: The set of all y-values or second coordinates in a relation.

Example 1

Find the domain and range of the relation $\{(1,2),(2,4),(3,6),(6,7),(8,9)\}$

Domain: $\{1,2,3,6,8\}$

Range: $\{2,4,6,7,9\}$

Function: A relation where every element in the domain is paired with exactly one element in the range.

Example 2

Give the domain and range of each relation and determine if the relation is a function.

$\{(1,2),(3,4),(5,9),(6,5),(9,9)\}$

Answer: Domain: $\{1,3,5,6,9\}$

Range: $\{2,4,5,9\}$

This is a function

Example 3

Give the domain and range of each relation and determine if the relation is a function.

$\{(1,1),(2,3),(2,5),(4,6),(6,8)\}$

Answer: Domain: $\{1,2,4,6\}$

Range: $\{1,3,5,6,8\}$

This is not a function because 2 is paired with two values 3 and 5

Example 4 Find the domain and relation of the given relation, and determine if the relation is a function

Domain \Rightarrow Range

Greg \Rightarrow Jane

John \Rightarrow Jill

Bob \Rightarrow Beth

Mike \Rightarrow Carol

Mike \Rightarrow Molly

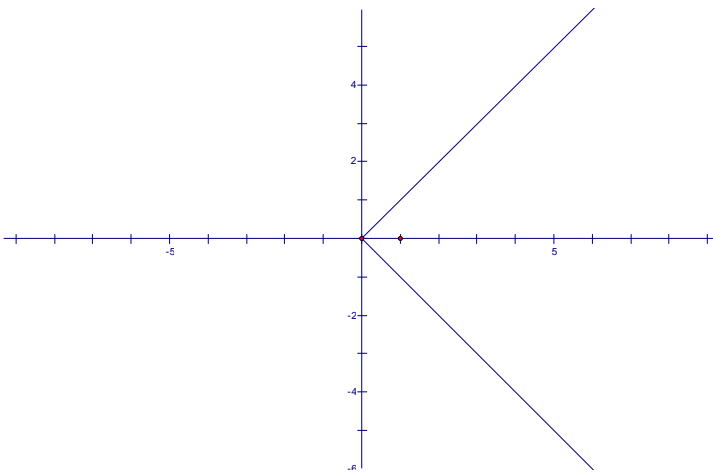
Domain: {Greg, John, Bob, Mike}, Range = {Jane, Jill, Beth, Carol, Molly}
This is not a function. (Mike is paired with Carol and Molly)

Vertical Line Test

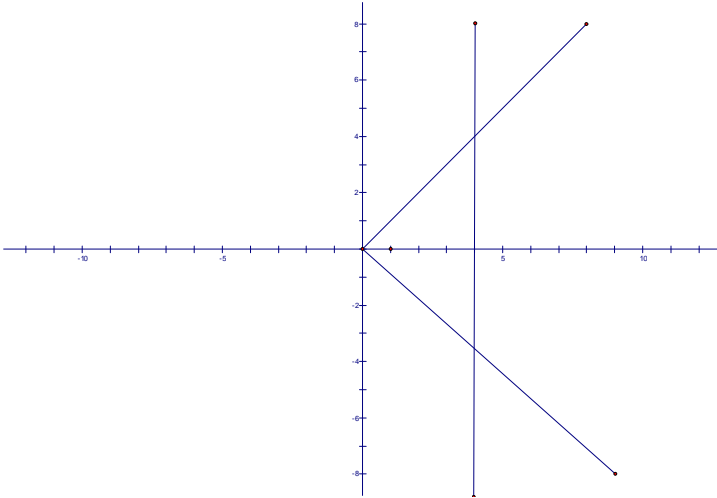
If a vertical line can be drawn so that it intersects the graph of a relation 2 or more times, then the relation is not a function.

Examples 5

Use the graph to determine if the relation is a function, and give the domain and range.



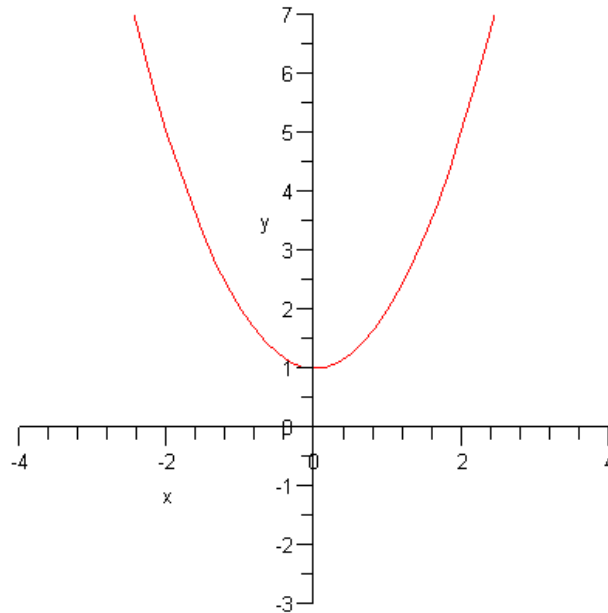
Solution:



This is not a function, since the relation fails the vertical line test.
Domain: $(0, \infty)$, **Range:** $(-\infty, \infty)$

Example 6

Use the graph to determine if the relation is a function, and give the domain and range.



$$f(x) = x^2 + 1$$

This is a function since every vertical line on the graph would intersect the relation only once. Domain: $(-\infty, \infty)$: Range: $[1, \infty)$

Function Notation

Example 7

$$f(x) = x^2 + 1$$

$$g(x) = 5x + 4$$

$$h(x) = 4x^3$$

Given the three functions above, find the following values.

1) Find $f(3)$

$$f(3) = 3^2 + 1 = 9 + 1$$

2) Find $g(-4)$

$$g(-4) = 5(-4) + 4 = -20 + 4 = -16$$

3) Find $h(-4)$

$$h(-4) = (-4)^3 = -64$$

4) Find $g(x-2)$

$$g(x-2) = 5(x-2) + 4 = 5x - 10 + 4 = 5x - 6$$

5) Find $f(x+1) = (x+1)^2 + 1 = (x+1)(x+1) + 1 = x^2 + 2x + 1 + 1 = x^2 + 2x + 2$

6) Find $f(x+h)$

$$f(x+h) = (x+h)^2 + 1 = (x+h)(x+h) + 1 = x^2 + 2xh + h^2 + 1$$

Composition of Two Functions

$f(g(x))$ and $g(f(x))$ or $f \circ g(x)$ and $g \circ f(x)$

Example 8

Given $f(x) = x^2 - 2$ and $g(x) = 2x + 3$, find $f(g(2))$ and $g(f(2))$

$$\text{Since } g(2) = 2(2) + 3 = 4 + 3 = 7,$$

$$f(g(2)) = f(7) = 7^2 - 2 = 49 - 2 = 47$$

$$\text{Since } f(2) = 2^2 - 2 = 4 - 2 = 2,$$

$$g(f(2)) = g(2) = 2(2) + 3 = 4 + 3 = 7$$

Example 9

Using the functions from **Example 8**, find $f(g(4))$ and $g(f(-3))$

$$\text{Since } g(-4) = 2(-4) + 3 = -8 + 3 = -5,$$

$$f(g(-4)) = f(-5) = (-5)^2 - 2 = 25 - 2 = 23$$

$$\text{Since } f(-3) = (-3)^2 - 2 = 9 - 2 = 7,$$

$$g(f(-3)) = g(7) = 2(7) + 3 = 14 + 3 = 17$$

Example 10

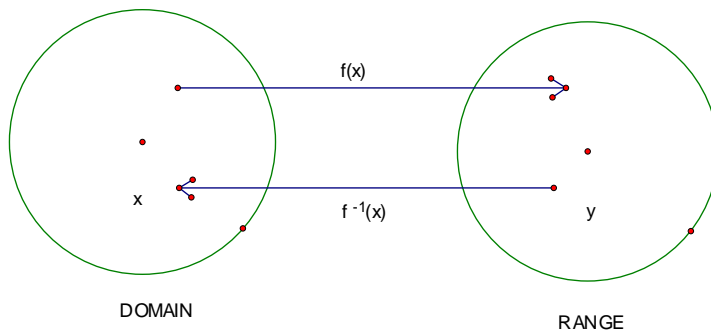
Using the functions from **Example 8**, find $f(g(x))$ and $g(f(x))$

$$f(g(x)) = f(2x + 3) = (2x + 3)^2 - 2 = (2x + 3)(2x + 3) - 2 = 4x^2 + 12x + 9 - 2 = 4x^2 + 12x + 7$$

$$g(f(x)) = g(x^2 - 2) = 3(x^2 - 2) + 3 = 3x^2 - 6 + 3 = 3x^2 - 3$$

Inverse Functions

An **inverse function** $f^{-1}(x)$ of a function $f(x)$ is function that maps each point range back to the domain where $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$



Find the inverse function of the function $f = \{(1,3), (2,5), (3,2), (4,4)\}$

Solution: Just switch the x and y components.

Thus, the inverse function would be $f^{-1} = \{(3,1), (5,2), (2,3), (4,4)\}$

Example 11 Find the inverse function of $f(x) = 3x - 4$

Solution: Replace $f(x)$ with y , switch x and y , and then solve for y :

$$f(x) = 3x - 4 \Rightarrow y = 3x - 4$$

$$\text{Switch } x \text{ and } y \Rightarrow x = 3y - 4$$

Solve for y

$$x = 3y - 4$$

$$x + 4 = 3y - 4 + 4$$

$$x + 4 = 3y$$

$$\frac{x + 4}{3} = \frac{3y}{3}$$

$$\frac{x + 4}{3} = y \Rightarrow f^{-1}(x) = \frac{x + 4}{3}$$

Example 11

Find the inverse function of $f(x) = \frac{x}{2} + 5$

Solution: Replace $f(x)$ with y , switch x and y , and then solve for y :

$$f(x) = \frac{x}{2} + 5 \Rightarrow y = \frac{x}{2} + 5$$

$$\text{Switch } x \text{ and } y \Rightarrow x = \frac{y}{2} + 5$$

Solve for y

$$x = \frac{y}{2} + 5$$

$$x - 5 = \frac{y}{2} + 5 - 5$$

$$x - 5 = \frac{y}{2}$$

$$2(x - 5) = 2\left(\frac{y}{2}\right)$$

$$2x - 10 = y \Rightarrow f^{-1}(x) = 2x - 10$$

Example 12

Find the inverse function of $f(x) = x^3 - 1$ and then sketch a graph of $f(x)$ and $f^{-1}(x)$

$$f(x) = x^3 - 1 \Rightarrow y = x^3 - 1$$

$$\text{Switch } x \text{ and } y \Rightarrow x = y^3 - 1$$

Solve for y

$$x = y^3 - 1$$

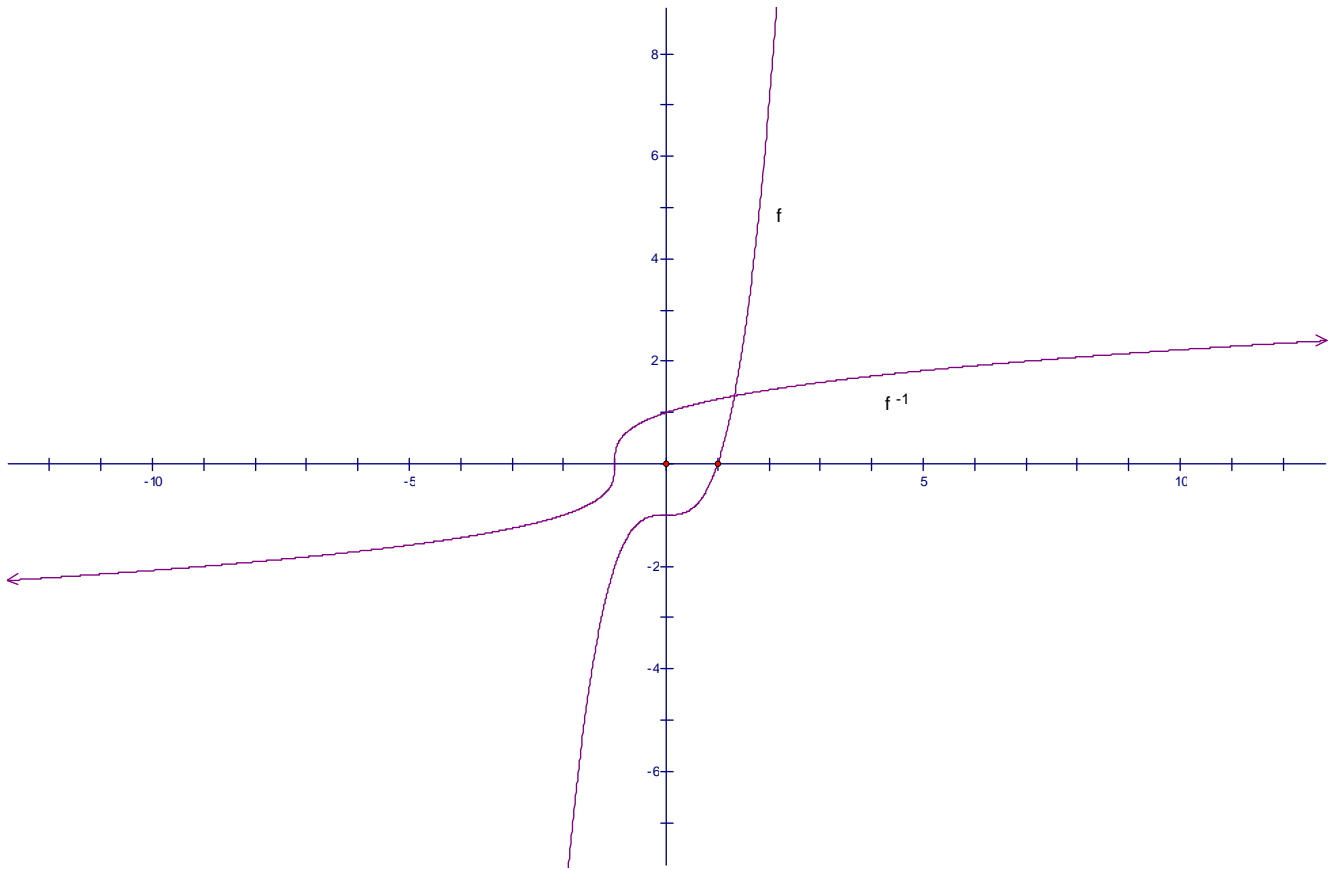
$$x + 1 = y^3 - 1 + 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x + 1} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x + 1} = y \Rightarrow f^{-1}(x) = \sqrt[3]{x + 1}$$

Graph of $f(x)$ and $f^{-1}(x)$



Example 13

Find the inverse function of $f(x) = x^2 + 3$

$$f(x) = x^2 + 3 \Rightarrow y = x^2 + 3$$

$$\text{Switch } x \text{ and } y \Rightarrow x = y^2 + 3$$

Solve for y

$$x = y^2 + 3$$

$$x - 3 = y^2 + 3 - 3$$

$$x - 3 = y^2$$

$$\sqrt{x-3} = \sqrt{y^2}$$

$$\Rightarrow \pm\sqrt{x-3} = y \text{ However, } \pm\sqrt{x-3} \text{ is not a function}$$

Therefore, f does not have an inverse function

Applications of Business Functions

Business Functions

The **cost function** is a function that projects the cost for producing x items

Symbol: $C(x)$

The **revenue function** is a function that projects the revenue that is produced for manufacturing x items.

Symbol: $R(x)$

The **profit function** is the function that project the profit earned for producing x items.

Symbol: $P(x)$

The relationship between cost, revenue, and profit

$$P(x) = R(x) - C(x)$$

Example 14

The cost for producing Nickel-metal batteries for hybrid cars is given by the cost function: $C(x) = 600x + 120,000$. Use the cost functions to answer these questions.

- 1) Find the cost to produce 100 Nickel-metal batteries.

$$C(100) = 600(100) + 120,000 = 60,000 + 120,000 = \$180,000$$

- 2) How many batteries can be produced with \$350,000

$$C(x) = 600x + 120,000$$

$$350,000 = 600x + 120,000$$

$$350,000 - 120,000 = 600x + 120,000 - 120,000$$

$$230,000 = 600x$$

$$x = \frac{230,000}{600}$$

$$x = 383 \text{ units}$$

Example 15

Use the following profit function to calculate the profit for producing 50 units.

$$P(x) = -.1x^2 + 60x$$

$$P(50) = -.1(50)^2 + 60(50)$$

$$P(50) = -.1(2500) + 3000$$

$$P(50) = -250 + 3000 = \$ 2750$$

Example 16

Suppose a computer manufacturer has a total cost function of $C(x) = 85x + 3300$ and a total revenue function of $R(x) = 385x$.

- What would be the profit function
- What is the profit on 351 items?
- How many items must be sold to avoid losing money?

Solution:

a) $P(x) = R(x) - C(x) = 385x - (85x + 3300) = 385x - 85x - 3300 = 300x - 3300$

b) $P(351) = 300(351) - 3300 = \$105,300 - \$3,300 = \$102,000$

- c) Find the breakeven point

$$R(x) = C(x)$$

$$385x = 85x + 3300$$

$$385x - 85x = 85x - 85x + 3300$$

$$300x = 3300$$

$$\frac{300x}{300} = \frac{3300}{300}$$

$$x = 100$$

Example 17

Find the breakeven point given the cost function and revenue function.

$$C(x) = 89x$$

$$R(x) = 1400 + 75x$$

$$R(x) = C(x)$$

$$1400 + 75x = 89x$$

$$1400 + 75x - 75x = 89x - 75x$$

$$1400 = 14x$$

$$\frac{14x}{14} = \frac{1400}{14}$$

$$x = 100$$

The breakeven point is 100 units

Functions Exercises

Given $f(x) = 2x^2 + 4$, $g(x) = 5x - 6$, and $h(x) = x^3 - x$, answer questions 1-10

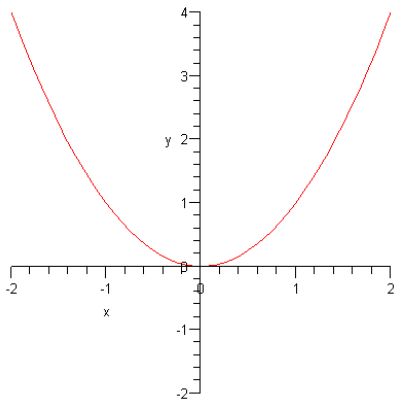
- 1) Find $f(2)$
- 2) Find $f(-3)$
- 3) Find $f(x+2)$
- 4) Find $h(2)$
- 5) Find $g(-1)$
- 6) Find $g \circ f(3)$
- 7) Find $f(g(4))$
- 8) Find $f(g(x))$
- 9) Find $f \circ g(2)$

Find the inverse function.

- 1) $f(x) = 3x - 4$
- 2) $f(x) = x^3 + 2$
- 3) $f(x) = \frac{x+2}{5}$

Find the domain and range of each relation, and then determine if the relation is a function.

- 1) $\{(1,3), (2,4), (4,2), (4,8)\}$
- 2)



3)

