Finance Unit
Math 114
Radford University

Section 5.1

Percents

Introduction to Basic Percents

The word percent translates to mean “out of one hundred”. A score of 85% on test means that you scored 85 points out of 100 possible points on the test. If you scored 44 out of 50 points on a test, then this would be a percent value of 88%. This value can be obtained by multiplying the numerator and denominator by 2 as shown in the next illustration.

$$\frac{44}{50} = \frac{2(44)}{2(50)} = \frac{88}{100} = .88 = 88\%$$

Since a percent represents the amount out of a hundred, to change a percent to a decimal, you simply drop the percent symbol and divide by 100 which can be done by moving the decimal two placing to the left as shown in the next examples.

$$45\% = \frac{45}{100} = .45$$
$$64.5\% = \frac{64.5}{100} = .645$$

Basic Percent Problems

One of the basic uses of percents is to find the percent amount of a given number. For example, how you would take 34% of 60? This would be done by changing 34% to .34, and then multiplying by .34 by 60 as shown here:

$$34\% \Rightarrow .34 \Rightarrow (.34)(60) = 20.4$$

Example 1

What is 46% of 90?

$$46\% \Rightarrow .46$$
$$(.46)(90) = 41.4$$

Mark up, mark down, and sales price

There are many common uses for percents in our society. As consumers, people use percentages to find sales prices, mark up prices, and discount. In this section, we will study how to use percents to compute discounts, mark up prices, sales prices, and sales tax. The first of these topics we will explore are discount and sales price.

Discount

Discount = (Percent Mark Down)(Retail Price)

Sale Price

Sale Price = Retail Price – Discount

Example 2

A men’s sports jacket that has a retail price of $170 is discounted by 25%. What is the sale’s price of the sports jacket?

$$Mark\ Down = 25\% = .25$$
$$Discount = (.25)(\$170) = \$42.50$$
$$Sales\ price = \$170 - \$42.50 = \$127.50$$

Example 3

A pair of jeans that has a retail price of $55.00 is discounted at 30%. What is the sale’s price of the jeans?

$$Mark\ Down = 30\% = .30$$
$$Discount = (.30)(\$55.00) = \$16.50$$
$$Sales\ price = \$55 - \$16.50 = \$38.50$$

Example 4

The sale price of a VCR is $110.00. If the mark down is 30%, find the retail price of the VCR.

$$
\begin{aligned}
&Let \;\; x = original \;\; price \\
&.30x = discount \\
&discount \;\; price = \$110.00 \\
&x - .30x = 110.00 \\
&\quad .70x = 110.00 \\
&\quad \frac{.70x}{.70} = \frac{110.00}{.70} \\
&\qquad x = \$157.14
\end{aligned}
$$

Mark Up Price

When stores purchase items at a whole sale price, the retail price is computed by marking up the whole sale cost using the given formulas.

Mark Up = (Percent Mark Up)(Whole Sale Price)
Retail Price = Whole Sale Price + Mark Up

Example 5

A store purchases DVD players at a whole sale price of $30 per unit which is to be marked up by 80%. What will be the retail price of the DVD player?

$$
\begin{aligned}
&percent \; mark \; up = 80\% = .80 \\
&mark \; up = (.80)(\$30.00) = \$24.00 \\
&retail \; price = \$30.00 + \$24.00 = \$54.00
\end{aligned}
$$

Example 6

The whole sale price of a pair of jeans is \$20.00. If the jeans are marked up by 65%, what is the retail price of the jeans?

$$percent\ mark\ up = 65\% = .65$$
$$mark\ up = (.65)(\$20.00) = \$13.00$$
$$retail\ price = \$20.00 + \$13.00 = \$33.00$$

Example 7

The retail price of a new television that has been mark up by 75% is \$300.00. Find the whole sale price of the television.

$$Let\ x = original\ \ price$$
$$.75x = discount$$
$$discount\ \ price = \$300.00$$
$$x + .75x = 300.00$$
$$1.75x = 300.00$$
$$\frac{1.75x}{1.75} = \frac{300.00}{1.75}$$
$$x = \$171.43$$

Sales Tax

When items are purchased at a store or place of business, a state sale's taxes is calculated and added on the price of the item. The percent rate of sale's tax in the United States is determined by each state. For example the sales tax in Virginia is 4.5%. Some states such as Delaware and Montana do not have any sale's tax. The state sale's tax is calculated by multiplying the percent rate by the purchase price. The state sale's tax is then added on the purchase price of the item.

Sales Tax Formula

Sale's Tax = (sale's tax rate)(purchase price)

Example 8

The state sale's tax rate in Virginia is 4.5%. Find the full cost to purchase a $50 pair of shoes using the Virginia tax rate of 4.5%.

$sales\, tax = (.045)(50) = \$2.25$

$Cost\ including\ tax = \$50.00 + \$2.25 = \$52.25$

Example 9

The state sale's tax rate in Ohio is 6%. Find the full cost to purchase the same pair of shoes in problem 7 using the Ohio tax rate of 6%.

$sales\, tax = (.06)(50) = \$3.00$

$Cost\ including\ tax = \$50.00 + \$3.00 = \$53.00$

Section 5.2

Simple Interest and Future Value

An important principle in banking and finance is the use of interest. Interest is simply when money is paid for the use of money. If you were to put money in a saving account, the bank would pay you interest in return for use of your money as investments. A special type of interest called **simple interest** is used to compute the amount of money earned or owed on the balance of money known as the **principle**. The principle is usually the amount money you have invested in the bank or the amount of money you have borrowed from the bank. Simple interest is computed form the principle, interest rate and period of time of the investment. The simple interest formula below is used for many applications of banking

Simple interest formula

$I = PRT$

$I = Interest$

$P = priniciple$

$R = Rate$

$T = Time$

In this first example, we use the simple interest formula to compute simple interest on a principle of $600.

Example 1 (Saving Account)

How much interest is earned if $600 is put in a savings account that pays 2% interest for 3 years?

Solution:

$P = \$600.00$

$I = 1.5\% = .015$

$T = 3\ years$

$I = PRT$

$I = (\$600.00)(.015)(3)$

$I = (\$9.00)(3)$

$I = \$27.00$

Example 2 (Bank Loan)

You borrow $5000 from a bank that has an 8% interest rate for 4 years. How much simple interest do you owe the bank?

$$P = \$5000.00$$
$$I = 6\% = .06$$
$$T = 5 \ years$$

$$I = PRT$$
$$I = (\$5000.00)(.06)(5)$$
$$I = (\$300.00)(5)$$
$$I = \$1500.00$$

In the next example we will use the simple interest formula to complete the following table that consists of missing values for time, interest, and interest rate. Each of these values can be found by solving for the missing variable in the table.

Example 3 (Interest Table)

Complete the following on simple interest

Interest	**Principle**	**Rate**	**Time**
a) ?	$1300	2.5%	3 years
b) $50	$600	3%	?
c) $120	$1200	?	4 years

Part a) Find the interest

$$Solve \ I = PRT \ \ for \ I$$
$$Convert \ 2.5\% = .025, then \ use \ the \ formula$$
$$I = PRT$$
$$I = (\$1300)(.025)(3)$$
$$I = (\$32.50)(3)$$
$$I = \$97.50$$

Part b) Find the time

$$I = PRT \quad (Solve \;\; for \; T)$$
$$\$50 = (\$600)(.03)T$$
$$\$50 = \$18T$$
$$\frac{\$50}{\$18} = \frac{\$18T}{\$18}$$
$$2.7 \; years = T$$

Part c)

$$I = PRT \qquad (Solve \;\; for \;\; R)$$
$$\$120 = (\$1200)R(4)$$
$$\$120 = \$4800\,R$$
$$\frac{\$120}{\$4800} = \frac{\$4800\,R}{\$4800}$$
$$R = \frac{120}{4800}$$
$$R = .025 = 2.5\%$$

Future Value

When interest is computed in a saving account, it is then added to the balance or principle. This new balance or principle is known as the **future value**.

Future Value Formula

Future Value = Principle + Interest

$$A = P + I$$
$$A = P + PRT$$
$$A = P(1 + RT)$$

Example 4 Future Value

If $10,000 is deposited in a saving account earning 2.4 % simple interest, what is the future value in 5 years?

$$P = \$10{,}000$$
$$R = 2.4\% = .024$$
$$T = 5 \ \text{years}$$

$$A = \$10{,}000(1 + .024 \cdot 5)$$
$$A = \$10{,}000(1 + .12)$$
$$A = \$10{,}000(1.12)$$
$$A = \$11{,}200.00$$

Example 5 (Finding the interest rate)

If a business borrows $18,000 and repays $26,100 in 3 years, what is the simple interest rate?

$$P = \$18{,}000$$
$$A = \$26{,}100$$
$$T = 3 \ \text{years}$$

$$26{,}100 = 18{,}000(1 + R(3))$$
$$26{,}100 = 18{,}000(1 + 3R)$$
$$26{,}100 = 18{,}000 + 54{,}000R$$
$$26{,}100 - 18{,}000 = 18{,}000 - 18{,}000 + 54{,}000R$$
$$8100 = 54{,}000R$$
$$\frac{8100}{54000} = \frac{54{,}000R}{54{,}000}$$
$$R = .15 \ \text{or} \ \ R = 15\%$$

Example 6 (Computing Time)

Suppose you wish to save $3,720. If you have $3000 and invest it at a 4% simple interest rate, how long will it take to obtain $3,720?

$$P = \$3{,}000$$
$$A = \$3{,}720$$
$$R = 4\% = .04$$

$$\begin{aligned}
3{,}720 &= 3{,}000(1 + (.04)T) \\
3{,}720 &= 3{,}000(1 + .04T) \\
3{,}720 &= 3{,}000 + 120T \\
3{,}720 - 3{,}000 &= 3{,}000 - 3{,}000 + 120T \\
720 &= 120T \\
\frac{720}{120} &= \frac{120T}{120} \\
T &= 6 \ \textit{years}
\end{aligned}$$

Section 5.3

Compound Interest

Normally most banks do not use the simple interest to pay interest to their investors or costumers. Usually interest is computed and then added to the principle periodically using a schedule. This idea is referred to as **compound interest**. Interest can be compounded daily, monthly, quarterly, or semiannually. In the first example of this section, we will study how interest is compounded in saving account using a semiannual schedule.

Example 1 (Using the simple interest formula to find the compound interest)

Suppose \$4000 is placed in a saving account for 2 years that compounds interest semiannually at a rate of 2% per year. How much money would be in the saving account after 2 years?

During the time period of 2 years, interest will be computed 4 times.

Year 1 (First half of year)

$P = \$4000$

$R = .02$

$T = \frac{1}{2} = .5$

$I = PRT$

$I = (\$4000)(.02)(.5)$

$I = (\$80)(.5)$

$I = \$40$

The new principle will be $P = \$4000 + \$40 = \$4040$

Year 1 (Second half of year)

$I = PRT$

$I = (\$4040)(.02)(.5)$

$I = (\$80.80)(.5)$

$I = \$40.40$

The new principle will be $P = \$4040 + \$40.40 = \$4080.40$

Year 2 (First half of year)

$$I = PRT$$
$$I = (\$4080.40)(.02)(.5)$$
$$I = (\$81.61)(.5)$$
$$I = \$40.80$$

The new principle will be $P = \$4080.40 + \$40.80 = \$4121.20$

Year 2 (Second half of year)

$$I = PRT$$
$$I = (\$4121.20)(.02)(.5)$$
$$I = (\$82.42)(.5)$$
$$I = \$41.21$$

The new principle will be $P = \$4121.20 + \$41.21 = \$4162.41$

As shown in example 1, the simple interest formula is used to find the compound interest. Now, you will discover that there is a simpler way to compute compound interest. The compound interest formula below can actually compute the compound interest much simpler than the simple interest formula.

Compound Interest Formula

$$P = principle$$
$$r = rate$$
$$t = time$$
$$n = number\ of\ times\ interest\ is\ computed\ in\ a\ year$$
$$A = Accumulated\ Balance$$
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example 2 (Using the compound interest formula on example 1)

Suppose $4000 is placed in a saving account for 2 years that compound interest semiannually at a rate of 2% per year. How much money would be in the saving account after 2 years?

$P = \$4000$

$T = 2$

$R = .02$

$n = 2$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$4000\left(1 + \frac{.02}{2}\right)^{2 \cdot 2}$$

$$A = \$4000(1 + .01)^{4}$$

$$A = \$4000(1.01)^{4}$$

$$A = \$4000(1.0406)$$

$$A = \$4162.41$$

Notice that this formula gives the same result as example 1, but does require near the same number of steps.

For the rest of the section, students are encouraged to use the compound interest formula to solve most of the compound interest problems.

Example 3 (Interest Compounded Quarterly)

Mike invests $6500 in a special saving bond that compounds interest quarterly at a rate of 3.5% per year. How much money would Mike have in this saving bond after 10 years?

$P = \$6500,\ T = 10,\ R = .035,\ n = 4\ (quarterly)$

$$A = \$6500\left(1 + \frac{.035}{4}\right)^{4 \cdot 10} = \$6500(1 + .00875)^{40} = \$5000(1.00875)^{40} = \$5000(1.4169088) = \$9209.91$$

Example 4 (Monthly Compounding)

What if Mike would invest the $10000 in a saving bond in an account that compounded monthly at the same rate of 3% per year, how much money he have after 10 years?

$P = \$10000,\ T = 10,\ R = .03,\ n = 12\ (monthly)$

$$A = \$10000\left(1 + \frac{.03}{12}\right)^{12 \cdot 10}$$
$$A = \$10000(1 + .0025)^{120}$$
$$A = \$10000(1.0025)^{120}$$
$$A = \$10000(1.34935)$$
$$A = \$13493.54$$

By taking the same amount of money at the same interest rate, you can actually earn a little money in interest.

Example 5 (Compounding Daily)

Now suppose Mike take his $5000 and inverts it in a saving bond that compounds daily for 10 years at 5% per year. How much money would he have in the saving bond after 10 years?

$P = \$5000,\ T = 10,\ R = .05,\ n = 365\ (daily)$

$$A = \$5000\left(1 + \frac{.05}{365}\right)^{365 \cdot 10}$$
$$A = \$5000(1 + .000136986)^{3650}$$
$$A = \$5000(1.00013986)^{3650}$$
$$A = \$5000(1.648664)$$
$$A = \$8243.32$$

Example 6 (Calculating Time)

How long would it take to save $15,000, if $10,000 was placed in saving account that compounds interest quarterly at a rate of 1.0 % per year?

$P = 10{,}000$

$A = 15{,}000$

$T = ?$

$n = 4$

$R = .01$

$$A = P\left(1 + \frac{R}{n}\right)^{nt}$$

$$15{,}000 = 10{,}000\left(1 + \frac{.01}{4}\right)^{4t}$$

$$15{,}000 = 10{,}000(1 + .0025)^{4t}$$

$$15{,}000 = 10{,}000(1.0025)^{4t}$$

$$\frac{15{,}000}{10{,}000} = \frac{10{,}000(1.0025)^{4t}}{10{,}000}$$

$$1.5 = (1.0025)^{4t}$$

$$\log(1.5) = \log(1.0025)^{4t}$$

$$\log(1.5) = 4t\log(1.0025)$$

$$.17609 = 4t(.00108)$$

$$.17609 = .00432t$$

$$t = \frac{.17609}{.00432}$$

$$t \approx 40.8 \ years$$

Example 7

How long would it take to save $15,000, if $10,000 was placed in saving bond that compounds interest monthly at a rate of 6.0 % per year?

$P = 10{,}000, A = 15{,}000,\ n = 12,\ R = .06,\ t = ?$

$$15{,}000 = 10{,}000\left(1+\frac{.06}{12}\right)^{12t}$$
$$15{,}000 = 10{,}000(1+.005)^{12t}$$
$$15{,}000 = 10{,}000(1.005)^{12t}$$
$$\frac{15{,}000}{10{,}000} = \frac{10{,}000(1.005)^{12t}}{10{,}000}$$
$$1.5 = (1.005)^{12t}$$
$$\log(1.5) = \log(1.005)^{12t}$$
$$\log(1.5) = 12t\log(1.005)$$
$$.17609 = 12t(.002166)$$
$$.17609 = .025993t$$
$$t = \frac{.17609}{.025993}$$
$$t \approx 6.8\ \textit{years}$$

Example 8

Carmen deposits $25,000 into a certificate of deposit that guarantees 6.6% annual interest rate, compounded quarterly. How much will be in the account at the end of five years?

$P = \$15{,}000,\ R = 6.6\% = .006, n = 4, t = 5$

$$A = \$25{,}000\left(1+\frac{.066}{4}\right)^{4(15)}$$
$$A = \$25{,}000(1+.00165)^{60}$$
$$A = \$25{,}000(1.00165)^{60}$$
$$A = \$25{,}000(2.6695802)$$
$$A = \$66{,}739.50$$

Section 5.4

Loan (Financing a Home or Car)

Buying a car or house is a financial decision that most people will encounter in their life time. In this section, we will look at how banks and financial companies compute the loan payment that a costumer's will pay when they finance a home or car. We will also answer common questions about getting a loan from a bank such as how much money will you pay in interest when you finance a home with a loan, or how will interest rate and term of loan can effect your loan payment.

The loan formula

$P = principle$

$r = Rate$

$n = Number\ of\ Times\ Interest\ is\ Computed$

$T = Time$

$PMT = Payment$

$$PMT = \frac{P \cdot \left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Example 1

What would your monthly house payment be on a $160,000 house if you finance the house for 30 years at an interest rate of 6%?

$P = \$160{,}000$

$R = 6\%$

$n = 12$

$T = 30$

$$PMT = \frac{(\$160{,}000)\left(\frac{.06}{12}\right)}{1 - \left(1 + \frac{.06}{12}\right)^{-12(30)}} = \frac{(\$160{,}000)(.005)}{1 - (1 + .005)^{-360}} = \frac{\$800}{1 - (1.005)^{-360}} = \frac{\$800}{1 - .166041} = \frac{\$800}{.833958} = \$959.28$$

Example 2

What would your monthly car payment be on car that cost $16,000 if you finance the car for 5 years at an interest rate of 8%?

$P = \$16{,}000$
$R = 8\% = .08$
$n = 12$
$T = 5$

$$PMT = \frac{(\$16{,}000)\left(\frac{.08}{12}\right)}{1-\left(1+\frac{.08}{12}\right)^{-12(5)}} = \frac{(\$16{,}000)(.0066667)}{1-(1+.0066667)^{-60}} = \frac{\$106.67}{1-(1.0066667)^{-60}} = \frac{\$106.67}{1-.67121} = \frac{\$106.67}{.3287896} = \$324.43$$

Example 3

First, calculate the monthly house payment on an $110,000 house that is financed at a rate of 6.5 % per year for 30 years. Next, calculate the interest paid over the 30 years.

$P = \$110{,}000$
$R = 6.5\% = .065$
$n = 12$
$T = 30$

$$PMT = \frac{(\$110{,}000)\left(\frac{.065}{12}\right)}{1-\left(1+\frac{.065}{12}\right)^{-12(30)}} = \frac{(\$110{,}000)(.00541667)}{1-(1+.00541667)^{-360}} = \frac{\$595.83}{1-(1.00541667)^{-360}} = \frac{\$595.83}{1-.1430245} = \frac{\$595.83}{.8569754} = \$695.27$$

$Total\ \ Amount = 360(695.27) = \$250297.49$
$Interest\ \ Paid = \$250{,}297.49 - \$110{,}000 = \$140{,}297.49$

Example 4

The available interest rate on a $250,000 house is 6% per year. Compare the monthly payments for a 30 year loan and 15 year loan, and then determine which loan the 30 year or 15 year pays the least amount of interest.

$30 - year$

$$PMT = \frac{(\$250{,}000)\left(\frac{.06}{12}\right)}{1-\left(1+\frac{.06}{12}\right)^{-12(30)}} = \frac{(\$250{,}000)(.005)}{1-(1+.005)^{-360}} = \frac{\$1{,}250}{1-(1.005)^{-360}} = \frac{\$1{,}250}{1-.166041} = \frac{\$1{,}250}{.833959} = \$1498.87$$

$15 - year$

$$PMT = \frac{(\$250{,}000)\left(\frac{.06}{12}\right)}{1-\left(1+\frac{.06}{12}\right)^{-12(15)}} = \frac{(\$250{,}000)(.005)}{1-(1+.005)^{-180}} = \frac{\$1{,}250}{1-(1.005)^{-180}} = \frac{\$1{,}250}{1-.40748} = \frac{\$1{,}250}{.5925176} = \$2109.64$$

Total amount of the 30 year loan

$360(\$1199.10) = \$431{,}676$

Interest Paid

$\$431{,}676 - \$200{,}000 = \$231{,}676$

Total amount of the 15 year loan

$180(\$1687.71) = \$303{,}787.80$

Interest Paid

$\$303{,}787.80 - \$200{,}000 = \$103{,}787.80$

The 30 year loan payments are less that the 15 year loan. However, the 15 year loan pays much less interest than the 30 year loan.

Example 5

The available interest rate on a $20,000 car is 10% per year. Compare the monthly payments for a 3 year loan and 5 year loan, and then determine which loan the 3 year or 5 year pays the least amount of interest.

$$3-\textit{year}$$

$$PMT=\frac{(\$20{,}000)\left(\frac{.10}{12}\right)}{1-\left(1+\frac{.10}{12}\right)^{-12(3)}}=\frac{(\$20{,}000)(.00833333)}{1-(1+.00833333)^{-36}}=\frac{\$166.67}{1-(1.00833333)^{-36}}=\frac{\$166.67}{1-.7417398}=\frac{\$166.67}{.2582602}=\$645.36$$

$$5-\textit{year}$$

$$PMT=\frac{(\$20{,}000)\left(\frac{.10}{12}\right)}{1-\left(1+\frac{.10}{12}\right)^{-12(5)}}=\frac{(\$20{,}000)(.008333333)}{1-(1+.00833333)^{-60}}=\frac{\$166.67}{1-(1.008333333)^{-60}}=\frac{\$166.67}{1-.6077886}=\frac{\$166.67}{.3922113}=\$424.24$$

Total amount of the 30 year loan

$$36(\$645.36)=\$23{,}232.96$$

Interest Paid

$$\$23{,}232.96-\$20{,}000=\$3{,}232.96$$

Total amount of the 15 year loan

$$60(\$424.24)=\$25{,}454.40$$

Interest Paid

$$\$25{,}454.40-\$200{,}000=\$5{,}454.40$$

The 5 year loan payments are less that the 3 year loan. However, the 3 year loan pays much less interest than the 5 year loan.

Excursion

Credit Cards

People use credit cards to pay bills, buy groceries, purchase clothes, pay restaurant bills, and to pay for any other expenses they might have. Credit cards have become convenient and widely used in our society today. Unfortunately, some consumers use their credit cards too frequently and end up having what is called **credit card debt**. As a result, many consumers end up having serious financial problems due to credit card debt. In this excursion, we will look at credit card debt and some other aspects of using credit cards.

The first thing we will look at is what is known as a finance charge. Most credit card companies issue monthly billing statements. On the monthly billing statement, there is list of all transactions, the billing date, and the due date. The due date on the bill is usually one month after the billing date. If the credit card bill is paid in full by the end of the month, then the credit card company does not enforce a finance charge. If the credit card bill is not paid in full, then the credit card company will add a **finance charge** to the next bill. A common method of determining the finance charge is to compute the interest on the average daily balance. The **average daily balance** can be found by dividing the sum of the total amounts owed each day by the number of days in the billing period.

$$Average\ daily\ balance = \frac{Sum\ of\ the\ total\ amounts\ owed\ each\ day}{Number\ of\ days\ in\ the\ billing\ period}$$

Example 1 Average Daily Balance and Finance Charges

Suppose an unpaid bill for $300 had a due date of October 10. A purchase of $40 was made on October 12, and a purchase of $150 was made on October 20. A payment of $50 was made on October 17. The next billing date is November 10. The interest rate on the average daily balance is 3.5% per month. Find the finance charge on the November 10 bill.

Date	**Payment/ Purchases**	**Balance each day**	**Number of Days until the balance changes**	**Unpaid balance times the number of days**
October 10-11		$300	2	2($300) = $600
October 12-17	$40	$340	6	6($340) = $2040
October 18-20	−$50	$290	3	3($290) = $870
October 21-November 9	$150	$440	18	18($440) = $7920
Total				$11430

Solution:

$$Average\ Daily\ Balance = \frac{\$11430}{31} = \$368.71$$

Finance Charge:

$$I = PRT$$
$$I = (\$368.71)(.035)(1)$$
$$I \approx \$12.90$$

Example 2

Suppose an unpaid bill for $1530 had a due date of April 10. A purchase of $654 was made on March 14, and a purchase of $356 was made on March 22. A payment of $150 was made on March 21, and a payment of $50 was made on March 27. The next billing date is April 10. The interest rate on the average daily balance is 2.5% per month. Find the finance charge on the April 10 bill.

Date	Payment/ Purchases	Balance each day	Number of Days until the balance changes	Unpaid balance times the number of days
March 10-14		\$1530	5	5(\$1530) = \$7650
March 15-20	\$654	\$2184	6	6(\$2184) = \$13104
March 21-22	− \$150	\$2034	2	2(\$2034) = \$4068
March 23- March 27	\$356	\$2390	5	5(\$2390) = \$11950
March 28-	− \$50	\$2340	13	13(\$2340) = \$30420

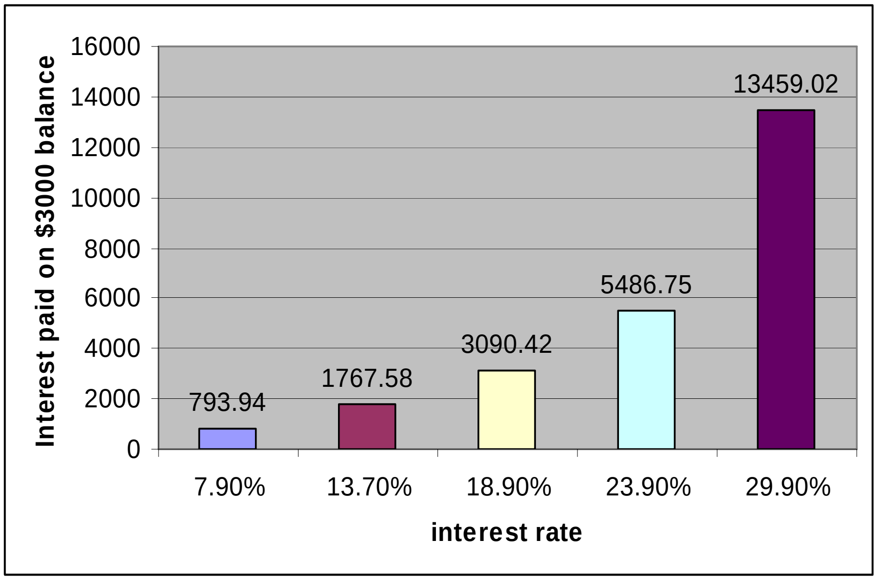
April 9				
Total				$67192

$$Average\ Daily\ Balance = \frac{\$67192}{31} = \$2167.48$$

Finance Charge:

$$I = PRT$$
$$I = (\$2167.48)(.025)(1)$$
$$I \approx \$54.19$$

The following graph shows how much total interest you would pay on a $3000 card debt, if you would make the minimum credit card payment of 3% of the credit card balance each month. (Source www.CardWeb.com, *Mathematical Excursions*, 2nd Edition, Aufmann, Lockwood, Nation, Clegg)



This is the amount of time that it would take to pay of the $3000 credit card bill given the interest rate of the credit card

Interest Rate	Time to pay off credit card debt.
7.9%	10 years 11 months
13.9%	13 years 5 months
18.9%	16 years 9 months
23.9%	22 years 9 months
29.9%	42 years

Approximate Annual Percentage Rate (APR) Formula for a Simple Interest Rate Loan.

$$APR = \frac{2Nr}{N+1}$$

$N = Number\ of\ Payments$

$r = Simple\ Interest\ Rate$

Example 3

Rick purchases a refrigerator for $700. He pays 20% down and agrees to repay the balance in 12 equal payments. The finance charge on the balance is 9% simple interest.

a) Find the finance charge

b) Estimate the annual percentage rate.

a) Down payment

$$Down\ \ Payment = (.20)(\$700) = \$140$$

Amount financed by Rick

$$Amount\ \ Financed = \$700 - \$140 = \$560$$

Interest owed

$$I = (\$560)(.09) = \$50.40$$

b) Use the APR formula to estimate the annual percentage rate.

$$
\begin{aligned}
APR &= \frac{2Nr}{N+1} \\
APR &= \frac{2(12)(.09)}{12+1} \\
APR &= \frac{(24)(.09)}{13} \\
APR &= \frac{2.16}{13} \\
APR &= .166 \approx 16.6\%
\end{aligned}
$$

Example 4 John borrows \$4500 from a bank that has a 9.5% simple interest rate and repays the loan in four equal monthly payments. Estimate the APR.

$$APR = \frac{2Nr}{N+1} = \frac{2(4)(.095)}{4+1} = \frac{(8)(.095)}{5} = \frac{.76}{5} = .152 \approx 15.2\%$$

Excursion Exercises

1) The following chart contains all transactions on a credit card for a 30 day period which includes the activity date, company, and the amount of a credit card bill. The due date is June 10. On May 10, there was an unpaid balance of $851.00. Find the finance charge if the interest rate is 2.0 % per month.

Activity Date	Company	Amount
May 10	Unpaid Balance	$851.00
May 13	Animal Clinic	$113.20
May 15	Sal's Pizzeria	$31.21
May 21	Quickie Mart	$34.12
May 23	Airplane Tickets	$296.00
May 26	Credit Card Payment	$500.00
June 5	Motown Record Store	$26.99

2) The following chart contains all transactions on a credit card for a 30 day period which includes the activity date, company, and the amount of a credit card bill. The due date is August 10. On September 10, there was an unpaid balance of $787.90. Find the finance charge if the interest rate is 1.6 % per month.

Activity Date	Company	Amount
August 10	Unpaid Balance	$787.90
August 18	Bookstore	$145.66
August 24	Quickie Mart	$23.93
August 29	Football Tickets	$91.23
August 31	Credit Card Payment	$400.00
September 2	Nick's Pub	$24.55
June 5	Carl's Record Store	$17.99

3) Mark borrows $2000 from a bank that has a 12% simple interest rate and repays the loan in six equal monthly payments. Estimate the APR.

4) Julie borrows $3000 from a bank that has a 7.5% simple interest rate and repays the loan in three equal monthly payments. Estimate the APR.