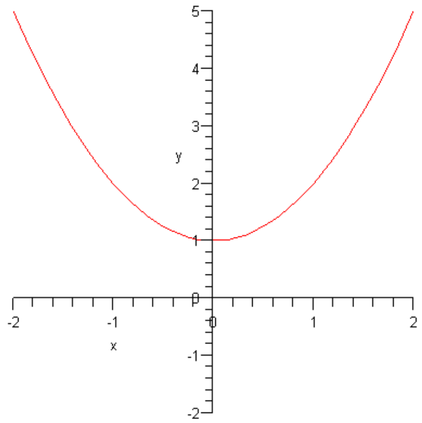
Final Exam Review

Chapter 1

1) Find the domain and range of the function, and then sketch a graph of the function.

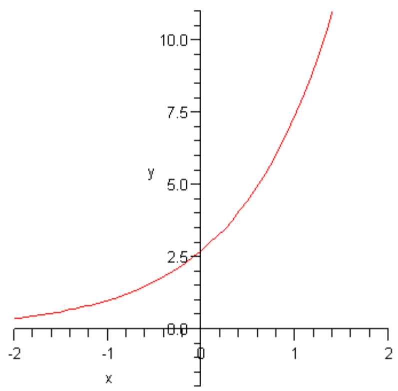
a) $f(x) = x^2 + 1$



$Domain : (-\infty, \infty)$

$Range : [1, \infty)$

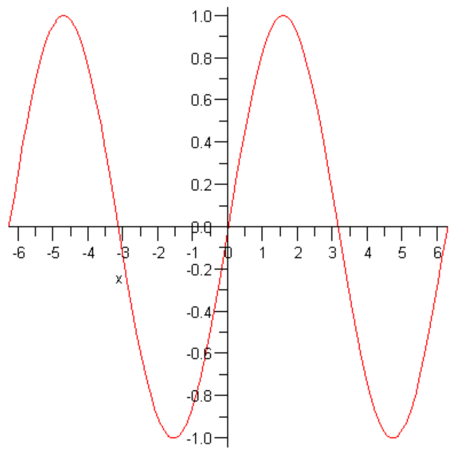
b) $f(x) = e^{x+1}$



$Domain : (-\infty, \infty)$

$Range : (0, \infty)$

c) $f(x) = \sin(x)$



$Domain: (-\infty, \infty)$
$Range: (-1,1)$

2) Given $f(x) = x^2 + 3x + 2$ and $g(x) = 2x - 3$, find the following functions.

a) $f(-2)$

$f(-2) = (-2)^2 + 3(-2) + 2 = 4 - 6 + 2 = 0$

b) $f(g(x))$

$f(g(x)) = f(2x - 3) = (2x + 3)^2 + 3(2x + 3) + 2$

c) $g(f(3))$

$g(f(3)) = g(3^2 + 3(3) + 2) = g(9 + 9 + 2) = g(20) = 2(20) + 3 = 43$

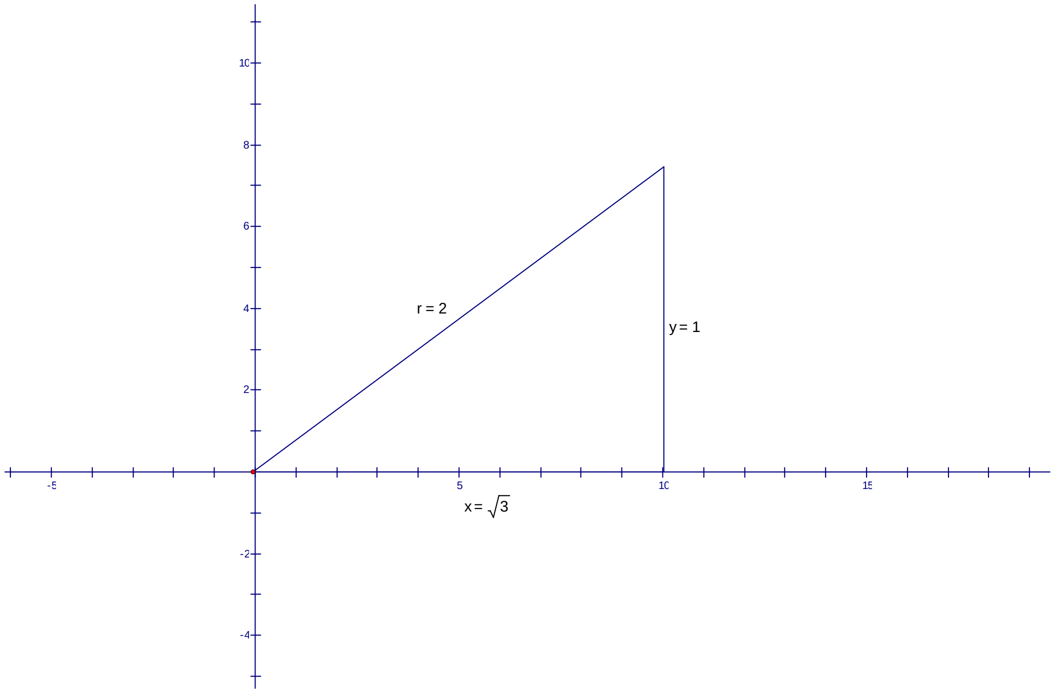
3) Convert 120^0 to radians

$$120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$$

4) Convert $\frac{5\pi}{4}$ to radians

$$\frac{5\pi}{4} \cdot \frac{180}{\pi} = 5(45^0) = 225^0$$

5) Find exact values of the trigonometric $\sin\theta$, $\cos\theta$, $\tan\theta$ and for the radian measure of $\dfrac{\pi}{6}$



$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

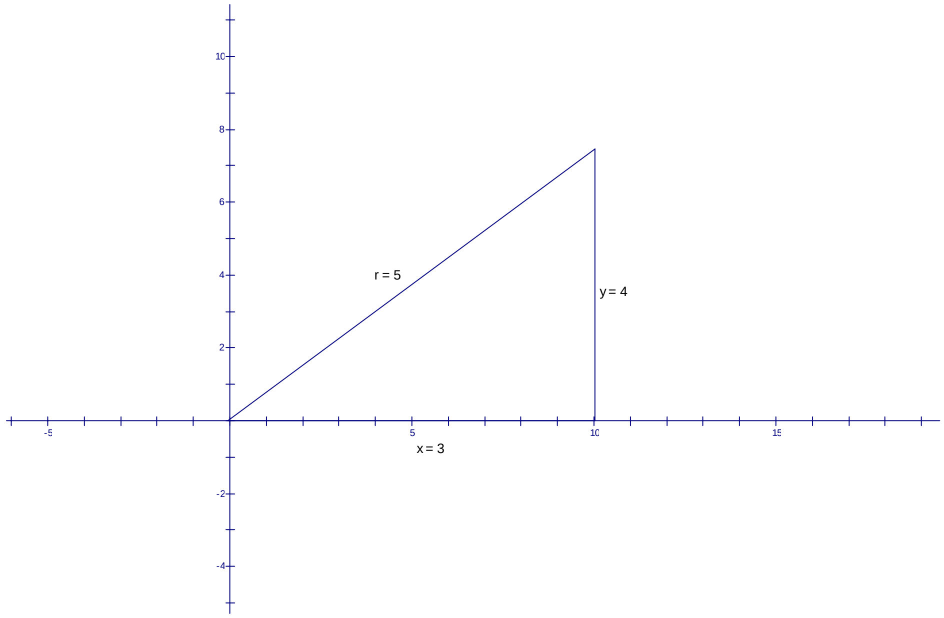
$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{1} = 2$$

$$\csc\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

6) Find the remaining trigonometric functions given $\sin\theta = \frac{4}{5}, \textit{in Quadrant I}$

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5$$



7) Write as a single logarithm. $\ln x + 3\ln y - 2\ln y$

$$\ln x + 3\ln y - 2\ln z$$
$$\ln x + \ln y^3 - \ln z^2$$
$$\ln\left(xy^3\right) - \ln(z^2)$$
$$\ln\left(\frac{xy^3}{z^2}\right)$$

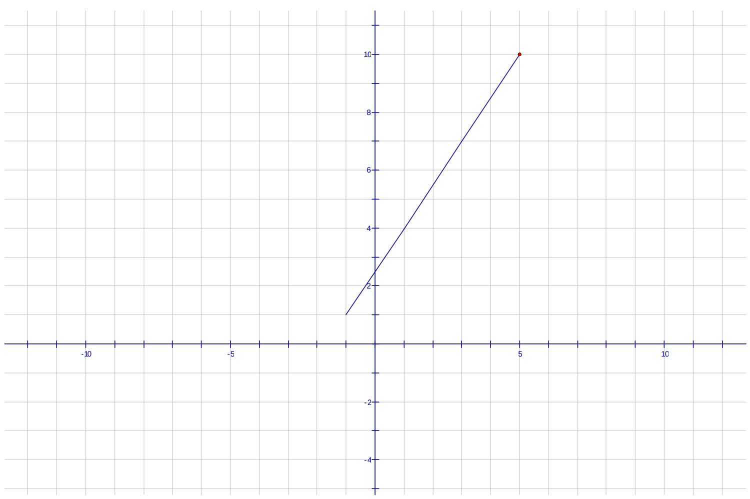
8) Find the inverse of $f(x) = x^3 - 2$, and then graph $f(x)$ and $f'(x)$.

$$f(x) = x^3 - 2$$
$$y = x^3 - 2$$
$$x = y^3 - 2$$
$$x + 2 = y^3$$
$$\sqrt[3]{x+2} = \sqrt[2]{y^3}$$
$$\sqrt[3]{x+2} = y$$

9) Sketch the following curve by using parametric equations to plot points.

$x = 2t - 3 \quad y = 3t - 2$

t	$x = 2t - 3$	$y = 3t - 2$
1	$x = 2(1) - 3 = 2 - 3 = -1$	$y = 3(1) - 2 = 3 - 2 = 1$
2	$x = 2(2) - 3 = 4 - 3 = 1$	$y = 3(2) - 2 = 6 - 2 = 4$
3	$x = 2(3) - 3 = 6 - 3 = 3$	$y = 3(3) - 2 = 9 - 2 = 7$
4	$x = 2(4) - 3 = 8 - 3 = 5$	$y = 3(4) - 2 = 12 - 2 = 10$



Chapter 2

1) Find $\lim_{x \to 2} x^2 + 2x$

$$\lim_{x \to 2} x^2 + 2x = 2^2 + 2(2) = 4 + 4 = 8$$

2) Find $\lim_{x \to 5} \frac{x-5}{x^2-25}$

$$\lim_{x \to 5} \frac{x-5}{x^2-25} = \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x \to 5} \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$$

3) Find $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

$$\lim_{th \to 0} \frac{\sqrt{t^2+9}-3}{t^2} = \lim_{h \to 0} \frac{\left(\sqrt{t^2+9}-3\right)\left(\sqrt{t^2+9}+3\right)}{t^2\left(\sqrt{t^2+9}+3\right)} = \lim_{t \to 0} \frac{\left(\sqrt{t^2+9}\right)^2 + 3\sqrt{t^2+9} - 3\sqrt{t^2+9} - 9}{t^2\left(\sqrt{t^2+9}+3\right)}$$

$$= \lim_{t \to 0} \frac{t^2+9-9}{t^2\left(\sqrt{t^2+9}+3\right)} = \lim_{t \to 0} \frac{t^2}{t^2\left(\sqrt{t^2+9}+3\right)} = \lim_{t \to 0} \frac{1}{\left(\sqrt{t^2+9}+3\right)} = \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

4) Find $\lim_{h \to 0} \frac{(h+1)^2-1}{h}$

$$\lim_{h \to 0} \frac{(h+1)^2-1}{h} = \lim_{h \to 0} \frac{(h+1)(h+1)-1}{h} = \lim_{h \to 0} \frac{h^2+h+h+1-1}{h} = \lim_{h \to 0} \frac{h^2+2h}{h} = \lim_{h \to 0} \frac{h(h+2)}{h}$$

$$= \lim_{h \to 0} h+2 = 0+2 = 2$$

5) Use the limit definition of a derivative to find the derivative of the function $f(x) = x^2 + 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2-3x)}{h} = \lim_{h \to 0} \frac{x^2+2xh+h^2+3x+3h-x^2-3x}{h}$$

$$= \lim_{h \to 0} \frac{h^2+2xh+3h}{h} = \lim_{h \to 0} \frac{h(h+2x+3)}{h} = \lim_{h \to 0} h+2x+3 = 2x+3$$

6) Evaluate $\lim_{x\to\infty}\frac{3x^2-2x-6}{2x^2-4x+2}$

$$\lim_{x\to\infty}\frac{3x^2-2x-6}{2x^2-4x+2}$$

$$\lim_{x\to\infty}\frac{\frac{3x^2-2x-6}{x^2}}{\frac{2x^2-4x+2}{x^2}}$$

$$\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{2x}{x^2}-\frac{6}{x^2}}{\frac{2x^2}{x^2}-\frac{4x}{x^2}+\frac{2}{x^2}}$$

$$\lim_{x\to\infty}\frac{3-\frac{2}{x}-\frac{6}{x^2}}{2-\frac{4}{x}+\frac{2}{x^2}}$$

$$\frac{3-\frac{2}{\infty}-\frac{6}{\infty}}{2-\frac{4}{\infty}+\frac{2}{\infty}}$$

$$\frac{3-0-0}{2-0+0}$$

$$\frac{3}{2}$$

7) Evaluate $\lim_{x\to\infty}\frac{x^2-3}{x^3-x^2+2}$

$$\lim_{x\to\infty}\frac{x^2+3x}{x^3+4x^2+x}=\lim_{x\to\infty}\frac{\frac{x^2+3x}{x^3}}{\frac{x^3+4x^2+x}{x^3}}=\lim_{x\to\infty}\frac{\frac{x^2}{x^3}+\frac{3x}{x^3}}{\frac{x^3}{x^3}+\frac{4x^2}{x^3}+\frac{x}{x^3}}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{3}{x^3}}{1+\frac{4}{x}+\frac{1}{x^2}}=\frac{\frac{1}{\infty}+\frac{3}{\infty}}{1+\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+0}{1+0+0}=0$$

8) Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}-x\right)}{\sqrt{x^2+x}+x}$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+x}\right)^2-x\sqrt{x^2+x}+x\sqrt{4x^2+x}-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}$$

$$=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{\sqrt{x^2+x}+x}{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+\frac{1}{\infty}}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{1+1}=\frac{1}{2}$$

Chapter 3

1) Find the derivative of $f(x)=x^2\sin(9x)$

$$f(x)=x^2\sin(9x)$$
$$f'(x)=\left(x^2\right)'\sin(9x)+\left(\sin(9x)\right)'\left(x^2\right)$$
$$f'(x)=2x\sin(9x)+9x^2\cos(9x)$$

2) Find the derivative of $f(x)=x^4e^{3x^2}$

$$f(x)=x^4e^{3x^2}$$
$$f'(x)=\left(x^4\right)'e^{3x^2}+\left(e^{3x^2}\right)'\left(x^4\right)$$
$$f'(x)=3x^3e^{3x^2}+\left(6xe^{3x^2}\right)\left(x^4\right)$$
$$f(x)=3x^3e^{3x^2}+6x^5e^{3x^2}$$

3) Find the derivative of $f(x)=\ln\left(4x^4+5x^2\right)$

$$f(x)=\ln\left(4x^4+5x^2\right)$$
$$f(x)=\frac{1}{4x^4+5x^2}\left(4x^4+5x^2\right)'$$
$$f(x)=\frac{16x^3+10x}{4x^4+5x^2}$$

4) Find the derivative of $y = e^{\sin x}$

$$y = e^{\sin x}$$
$$y = \cos x e^{\sin x}$$

5) Find the derivative of $y = \frac{2x^3}{4x^2 + 3x + 6}$

$$y = \frac{2x^3}{4x^2 + 3x + 6}$$

$$y' = \frac{\left(2x^3\right)\frac{d}{dx}\left(4x^2 + 3x + 6\right) + \left(4x^2 + 3x + 2\right)\frac{d}{dx}\left(2x^3\right)}{\left(4x^2 + 3x + 6\right)^2}$$
$$y' = \frac{\left(2x^3\right)\left(8x + 3\right) + \left(4x^2 + 3x + 2\right)\left(6x^2\right)}{\left(4x^2 + 3x + 2\right)^2}$$

6) Find the derivative of $y = x^4 + 6x^2 + 7x + 2$

$$y = x^4 + 6x^2 + 7x + 2$$
$$y' = 4x^3 + 12x + 7$$

7) Find the derivative using implicit differentiation.

$$x^2 + xy + 6y^2 = 5$$

$$x^2 + xy + 6y^2 = 5$$

$$\frac{d}{dx}\left(x^2 + xy + 6y^2\right) = \frac{d}{dx}(5)$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}6y^2 = 0$$

$$2x + y + xy\frac{dy}{dx} + 12y\frac{dy}{dx} = 0$$

$$xy\frac{dy}{dx} + 12y\frac{dy}{dx} = -2x - y$$

$$\left(xy + 12y\right)\frac{dy}{dx} = -\left(2x + y\right)$$

$$\frac{dy}{dx} = -\frac{2x + y}{xy + 12y}$$