Practice Guide for the CMST
Core Mathematics Test
at Radford University

By William A. Case
Instructor of Mathematics
Radford University
The CMST will cover the following topics

1) Ratios
   a) ratios
   b) proportions

2) Percents and Fractions

3) Simplifying expressions
   a) substitution
   b) exponent expressions

4) Basic computation

5) Solving equations

6) Solving Inequalities

7) Basic Geometry
   a) Right triangles
   b) Similarity
   c) Pythagorean Theorem
   d) Area and Volume

8) Probability
   a) basic probability
   a) counting principles

9) Statistics
   a) mean and median

10) Lines and Slope
    a) Slope and y-intercept
    b) Slope intercept form of line

The following pages will break down each topic on the CMST into different objectives. For each objective, there will be sample problems with the correct solutions and explanations provided.
Ratios

Objectives for Core Math Exam (Ratios)

Students taking the Core Math Exam should able to do following types of problems.
  1) Convert fractions and decimals to percents.
  2) Set up and solve a proportion to find the missing value in a ratio.
  3) Find the percent of a given value
  4) Identify equivalent ratios

Examples of ratios

4 to 5
4 : 5
4
5

Ratio Problems

1) If the ratio 3:x is the same as the ratio 9:21, what is x?

   a) 6
   b) 7
   c) 10
   d) 12

Answer: b

\[
\frac{3}{x} = \frac{9}{21} \\
9x = 3(21) \\
21x = 63 \\
21 \frac{x}{21} = \frac{63}{21} \\
x = 7 \ (c)
\]
2) 20:12 is the same ratio as:

a) 5:3  
b) 3:5  
c) 4:5  
d) 12:20

**Answer:** a) 5 : 3

**Solution:** Divide 20 and 12 by a great common factor of 4, will give the ratio 5:3

3) If the ratio 14:x is the same as 7:2, what is x?

a) 3  
b) 4  
c) 5  
d) 6

**Solution:** b

\[
\frac{14}{x} = \frac{7}{2}
\]

\[
7x = 2(14)
\]

\[
x = \frac{28}{7}
\]

\[
x = 4
\]

**Answer:** 4
Percents and Fractions

1) As a fraction, 15% =

   a) \( \frac{5}{10} \)
   b) \( \frac{3}{20} \)
   c) \( \frac{15}{1000} \)
   d) \( \frac{1}{50} \)

   Answer: b

   \[ 15\% = \frac{15}{100} = \frac{3}{20} \]

2) What is 7% of 5000?

   a) 350
   b) 35
   c) 3.5
   d) 100

   Solution: a

   *Change 7% to .07, then multiply by 5000*

   \[ (.07)(5000) = 350 \]
3) What is 26% of 120?

a) 312
b) 31.2
c) 3.12
d) .312

Solution: b

Change 26% to .26, then multiply by 120

(.26)(120) = 31.2

4) In an apartment building with 40 apartments, 8 of the apartments are unoccupied. What is the percentage of apartments that are occupied?

a) 20%
b) 70%
c) 8%
d) 80%

Solution: d

Occupied apartments = 40 – 8 = 32

Percent of occupied apartments = \( \frac{32}{40} \times 100 \) = (.8)(100) = 80%

5) What is .24 as a percent?

a) 2.4%
b) 24%
c) 240%
d) .24%

Solution b)

.24 = 24%
6) What percent is \( \frac{73}{100} \)?

a) .73%
b) 7.3%
c) 73%
d) None of the above

Solution: c

\[
\frac{73}{100} = .73 = 73\%
\]

7) How should 2.5 written as a fraction?

a) \( \frac{25}{100} \)
b) \( \frac{1}{4} \)
c) \( \frac{25}{200} \)
d) \( \frac{1}{40} \)

Solution d 2.5% = \( \frac{2.5}{100} = \frac{25}{1000} \) which reduces to \( \frac{1}{40} \)

8) Rick bought a piece of pipe 7 feet long to repair his sink. He cut off a 4 feet piece of the pipe, and then used the half of what remained. What fraction of the original piece did he use?

a) \( \frac{2}{7} \)
b) \( \frac{3}{7} \)
c) \( \frac{3}{14} \)
d) \( \frac{6}{14} \)

Solution c

He cut off 4 feet from the 7 foot section of pipe which left a 3 foot piece of pipe. Half of three is 1 \( \frac{1}{2} \) or 1.5. So, the fraction of the original piece is \( \frac{1.5}{7} = \frac{3}{14} \)
Simplifying expressions

Objectives

Students taking the Core Math Exam should be able to do the following problems.

1) Combine like terms in an algebraic expression.
2) Manipulate exponent expressions using the multiplication rule for exponents
   \[ a^n a^m = a^{n+m} \]
3) Manipulate exponent expressions using the division rule.
   \[ \frac{a^m}{a^n} = a^{m-n} \]
4) Simplify parenthesis expressions using distributive property
   \[ a(b + c) = ab + ac \]

1) \(10x - 5(3 - 2x) = ?\)
   a) \(20x + 5\)
   b) \(20x + 15\)
   c) \(20x - 15\)
   d) None of the above

Answer: c

Solution: \(3x + 12\)

\(10x - 5(3 + 2x) = 10x - 15 + 10x = 20x - 15\)

Simplifying exponent expressions

2) \(4^4 4^8 = ?\)
   a) \(4^{28}\)
   b) \(4^4\)
   c) \(4^{12}\)
   d) \(16^{12}\)

Solution: a

\(4^4 4^8 = 4^{4+8} = 4^{12}\)
3) What is the value of the expression \(4\left(2 - 3x^2\right) - 5y\) when \(x = -2\) and \(y = 4\)

a) -60  
b) -156  
c) 60  
d) 44

Solution: a

\[\begin{align*}
4\left(2 - 3(-2)^2\right) - 5(4) \\
4(2 - 3(4)) - 20 \\
4(2 - 12) - 20 \\
4(-10) - 20 \\
- 40 - 20 \\
- 60
\end{align*}\]

4) Simplify \(\sqrt{24 + 40}\)

a) 7  
b) \(\sqrt{24 + 40}\)  
c) 8  
d) \(6\sqrt{7}\)

Solution: c

\[\sqrt{24 + 40} = \sqrt{64} = 8\]

5) Simplify \(\sqrt{-9}\)

a) -3  
b) 5  
c) 3  
d) Not a real number.

Solution: d (Not a real number)
Solving inequalities and linear equations

Students taking the Core Math Exam should be able to do the following problems
1) Solve a basic equation
2) Solve a basic inequality

Properties of equality

1) Addition Property of Equality  If $a = b$, then $a + c = b + c$
2) Subtraction Property of Equality  If $a = b$, then $a - c = b - c$
3) Multiplication Property of Equality  If $a = b$, then $ca = cb$
4) Division Property of Equality  If $a = b$, then $\frac{a}{c} = \frac{b}{c}$

Solving equations

Example

1) Solve $3(3x - 1) = 4x + 7$ for $x$

   a) 3
   b) -3
   c) -2
   d) 2

Answer: d

\[
3(3x - 1) = 4x + 7 \\
9x - 3 = 4x + 7 \\
9x - 4x - 3 = 4x - 4x + 7 \\
5x - 3 = 7 \\
5x - 3 + 3 = 7 + 3 \\
5x = 10 \\
\frac{5x}{5} = \frac{10}{5} \\
x = 2
\]
2) Solve $2x - 8 = 24$ for $x$

a) 7
b) 8
c) 16
d) 12

Answer: c

\[
2x - 8 = 24 \\
2x - 8 + 8 = 24 + 8 \\
2x = 32 \\
\frac{2x}{2} = \frac{32}{2} \\
x = 16
\]

Solving inequalities

Examples

3) Solve $3(x - 2) \geq x + 6$

a) $x \leq -6$
b) $x \geq -6$
c) $x \geq 6$
d) $x \geq 5$

Solution: c

\[
3(x - 2) \geq x + 6 \\
3x - 6 \geq x + 6 \\
3x - x - 6 \geq x - x + 6 \\
2x - 6 \geq 6 \\
2x - 6 + 6 \geq 6 + 6 \\
2x \geq 12 \\
\frac{2x}{2} \geq \frac{12}{2} \\
x \geq 6
\]
4) Solve $5x + 3x - 5 < 11$

a) $x > 2$

b) $x < 2$

c) $x \geq -2$

d) None of the above

Solution: b

\[
5x + 3x - 5 < 11 \\
8x - 5 < 11 \\
8x - 5 + 5 < 11 + 5 \\
8x < 16 \\
x < \frac{16}{8} \\
x < 2
\]

5) Solve $5(x - 2) = 8x - 16$

a) 1

b) 3

c) -2

d) -3

Solution: b

Answer:

\[
5(x - 2) = 8x - 16 \\
5x - 10 = 8x - 16 \\
5x - 8x - 10 = 8x - 8x - 16 \\
-3x - 10 = -16 \\
-2x - 10 + 10 = -16 + 10 \\
-2x = -6 \\
\frac{-2x}{-2} = \frac{-6}{-2} \\
x = 3
\]
6) Solve $2\left(\frac{2x - 1}{3} - \frac{1}{2}\right) = \frac{3}{5}$

a) $-\frac{9}{40}$

b) $\frac{6}{40}$

c) $-\frac{6}{5}$

d) $\frac{6}{5}$

Solution b

\[2\left(\frac{2x - 1}{3} - \frac{1}{2}\right) = \frac{3}{5}\]

\[\frac{4x}{3} - 1 = \frac{3}{5}\]

\[15\left(\frac{4x}{3} - 1\right) = 15 \cdot \frac{3}{5}\]

\[15 \cdot \frac{4x}{3} - 15 \cdot 1 = 9\]

\[20x - 15 = 9\]

\[20x - 15 + 15 = 9 + 15\]

\[20x = 24\]

\[\frac{20x}{20} = \frac{24}{20}\]

\[x = \frac{6}{5}\]
**Geometry**

**Objectives**

Students taking the Core Math Exam should be able to do the following types of Geometry problems.

1) Find the area of an object  
2) Find the volume of an object  
3) Use similar triangles to find the missing side of a triangle  
4) Use the Pythagorean Theorem to find the hypotenuse or leg of a right triangle

**Key Formulas**

**Area**

Rectangle: $A = lw$  
Triangle: $A = \frac{1}{2}bh$  
Circle: $A = \pi r^2$

Circumference of a circle: $C = 2\pi r$  

*Diameter* = $2r$

**Pythagorean Theorem**

$c^2 = a^2 + b^2$

**Volume**

Rectangular solid: $V = lwh$  
Cylinder: $V = \pi r^2 h$
Similar triangles

1) Triangles ABC and DEF are similar triangles. If triangle ABC has sides of 4,4,6 and DEF has sides of lengths 8,8,x then x=?

a) 12  
b) 16  
c) 18 
d) 20

Solution: a
Set up as an proportion to find x

\[
\frac{4}{8} = \frac{6}{x} \\
4x = 8(6) \\
x = 12
\]

2) Find the missing side c of right triangle ABC, where the leg a=10 and the other leg b=24

a) 26  
b) 20  
c) 12 
d) 16

Solution: a
Answer: hypotenuse = 26

\[ c^2 = a^2 + b^2 \]
\[ c^2 = 10^2 + 24^2 \]
\[ c^2 = 100 + 576 \]
\[ c^2 = 676 \]
\[ c = \sqrt{676} \]
\[ c = 26 \]

3) A right triangle has sides of lengths 2 and 3, and a third side that is longer than either of these. What is the length of the longest side?

a) 5
b) \( \sqrt{5} \)
c) \( \sqrt{13} \)
d) None of the above

Answer: c

\[ c^2 = a^2 + b^2 \]
\[ c^2 = 2^2 + 3^2 \]
\[ c^2 = 4 + 9 \]
\[ c^2 = 13 \]
\[ c = \sqrt{13} \]
Area

4)

Find the area of the given triangle.

Use the grid to count the number of units for the base and height

base = 9
height = 4

Area = \( \frac{1}{2} bh = \frac{1}{2} (9)(4) = \frac{1}{2} (36) = 18 \)

Solution: c
5) Use the grid to find the diameter of the circle.

![Circle on a grid]

a) 4  
b) 6  
c) 8  
d) 10

Solution: c  
\[\text{diameter} = 8 \text{ units}\]

6) The area of a square is 25 square units. What is the perimeter?

a) 15  
b) 20  
c) 25  
d) 30

Solution: b  
\[\text{Area} = s^2\]  
\[25 = s^2\]  
\[\sqrt{25} = \sqrt{s^2}\]  
\[5 = s\]

\[\text{Perimeter} = 4s = 4(5) = 20\]
7) The area of a circle is $4\pi$. What is its circumference?

a) $2\pi$
b) $4\pi$
c) $6\pi$
d) $12\pi$

Solution; b

\[ Area = \pi r^2 \]
\[ Circumference = 2\pi r \]

\[
\frac{4\pi}{\pi} = \frac{\pi r^2}{\pi} \\
4 = r^2 \\
\sqrt{4} = \sqrt{r^2} \\
r = 2 \\
\]

\[ C = 2\pi r = 2\pi(2) = 4\pi \]}
8) A square has a perimeter of 24 units. What is the area of the square?

   a) 9
   b) 16
   c) 25
   d) 36

Solution: d

Perimeter = 4s

\[
\begin{align*}
24 &= 4s \\
\frac{24}{4} &= \frac{4s}{4} \\
6 &= s
\end{align*}
\]

\[A = s^2 = 6^2 = 36 \text{ square units}\]

9) The radius of a circle is 5 units. What is its circumference?

   a) \(6\pi\)
   b) \(8\pi\)
   c) \(10\pi\)
   d) \(12\pi\)

Solution: c

Circumference = \(2\pi r\)

\[Circumference = 2\pi(5) = 10\pi\]
10) A rectangular fish tank is 20 inches by 15 inches by 16 inches. What is the volume of the fish tank?

\[ V = lwh = (20\text{ in})(15\text{ in})(16\text{ in}) = 4800\text{ in}^3 \]

11) A cylinder shaped hot water tank has a radius of 15 inches and a height of 46 inches. How much water can the hot water tank hold?

\[ V = \pi r^2 h = \pi(15\text{ in})^2(46\text{ in}) = \pi(225\text{ in}^2)(46\text{ in}) = 10350\pi\text{ in}^3 \]
12) The following circle is inscribed inside a square. Find the area of the circle, given the square has an area of 64 square inches.

a) \(64\pi\)
b) \(32\pi\)
c) \(16\pi\)
d) \(8\pi\)

Solution: c

\[A = s^2\]
\[64 = s^2\]
\[\sqrt{64} = \sqrt{s^2}\]
\[8 = s\]

diameter = 8

radius = 4

\[A = \pi r^2 = \pi (4)^2 = 16\pi\]
13) A rectangle with a perimeter of 50 cm has a length of 12 cm. Find the area of the rectangle.

a) 100 cm²
b) 140 cm²
c) 156 cm²
d) 200 cm²

Solution c

\[ A = lw \]

First find the width of the rectangle

\[ P = 2l + 2w \]
\[ 50 = 2(12) + 2w \]
\[ 50 = 24 + 2w \]
\[ 50 - 24 = 24 - 24 + 2w \]
\[ 26 = 2w \]
\[ \frac{2w}{2} = \frac{26}{2} \]
\[ w = 13 \text{ cm} \]

Using the length and width, find the area of the rectangle

\[ A = lw \]
\[ A = (12 \text{ cm})(13 \text{ cm}) = 156 \text{ cm}^2 \]
14) A rectangular box with a square base is 6 feet height and 4 feet wide. What is the volume of the box?

a) 96 ft$^3$
b) 200 ft$^3$
c) 48 ft$^3$
d) 32 ft$^3$

Solution a

\[ V = (6 \text{ ft})(4 \text{ ft})(4 \text{ ft}) = (24 \text{ ft}^2)(4 \text{ ft}) = 96 \text{ ft}^3 \]

15) Find the measure of angle $\theta$

a) 20°
b) 60°
c) 36°
d) 72°

Solution c: Since there is 10 equal sections in the circle and there is 360 degrees in a circle, the solution is

\[ \frac{360}{10} = 36° \]
**Probability and Statistics**

Students taking the Core Math Exam should be able to do the following types of probability and statistics problems

1) Basic problems using the counting principles  
2) Basic probability problems  
3) Compute the mean and median given a set of numbers.

**Probability and Counting Principles**

**Examples**

1) A piggy bank contains 4 nickels, 3 quarters, 10 dimes, 3 one dollar bills, and 5 five dollar bills. What is the probability of drawing a coin?
   a) 4 out of 25  
   b) 13 out of 25  
   c) 17 out of 25  
   d) 22 out of 25

   \[
   \text{Number of coins} = 4 + 3 + 10 = 17 \text{ coins} \\
   \text{Number of dollar bills} = 3 + 5 = 8 \text{ bills} \\
   \]

   \[
   \text{Total objects} = 17 + 8 = 25 \\
   \text{Answer: c (17 out of 25)}
   \]

   \[
   \text{Probability of drawing a coin} = \frac{17}{25}
   \]

2) A particular model of car can be purchased with one of 5 different colors, with one of 6 different transmissions, and with one of 4 different options packages. How many different configurations of this car can be purchased?
   a) 15  
   b) 24  
   c) 20  
   d) 120

   \[
   5 \cdot 6 \cdot 4 = 120 \text{ possible combinations}
   \]

Solution: d
3) The local paint store offers wallpaper in eight colors, each of which comes in 5 different patterns. How many different styles of wallpaper are available?
   a) 13
   b) 40
   c) 20
   d) 30

   Answer: b

   \[8 \times 5 = 40\] combinations

4) Assume that your car has six preset buttons each of which can be tuned to an AM or FM station. How many different stations can you preset?

   a) 8
   b) 10
   c) 12
   d) 14

   Solution: c

   \[6 \times 2 = 12\] combinations
Statistics

Mean and Median

1) Find the mean of the following numbers.

7 3 6 3 8 8 7

a) 4
b) 5
c) 6
d) 7

Answer: c

Sum of the scores = 7 + 3 + 6 + 3 + 8 + 8 + 7 = 42

\[
Mean = \frac{\text{Sum of scores}}{\text{number of scores}} = \frac{42}{7} = 6
\]

\[mean = 6\]

2) Find the mean of the following numbers

12 10 8 5 3 4

a) 6
b) 7
c) 8
d) 9

Answer: b

Sum of the scores = 12 + 10 + 8 + 5 + 3 + 4 = 42

\[
Mean = \frac{\text{Sum of scores}}{\text{number of scores}} = \frac{42}{6} = 7
\]

\[mean = 7\]
3) Find the median of the following numbers

4  3  6  5  8  9  4

a) 4
b) 5
c) 6
d) 7

Answer: b

First step is to arrange the numbers from lowest to highest

3  4  4  5  6  8  9

Second step is to find the median which is the middle value. In this problem the middle value or median is 5

4) Find the median of the following numbers

5  3  9  6  4  3  9  7

a) 5
b) 5 1/2
c) 6
d) 6 1/2

Answer: b

Arrange number from lowest to highest.

3  3  4  5  6  7  9  9

Since the data set has an even number of scores the median is found by averaging the two middle values which are 5 and 6.

\[
median = \frac{5 + 6}{2} = \frac{11}{2} = 5 \frac{1}{2}
\]
Slope and y-intercept

Students taking the Core Math Exam should be able to do the following problems.

1) Find the slope of a line given the equation of a line.
2) Find the slope of a line given two points of a line.
3) Find the equation of a line given the slope and y-intercept.

Key Formulas

Slope-intercept form of an equation: \( y = mx + b \)

Slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

Examples

1) Find the slope of a line given the equation \( 2x + 3y = 6 \)

   a) 2
   b) \( \frac{2}{3} \)
   c) \( -\frac{2}{3} \)
   d) \( -\frac{3}{2} \)

   Solution: c

Write the equation in slope-intercept form \( y = mx + b \)

\[
\begin{align*}
2x + 3y &= 6 \\
2x - 2x + 3y &= 6 - 2x \\
3y &= -2x + 6 \\
\frac{3y}{3} &= -\frac{2x}{3} + \frac{6}{3} \\
y &= -\frac{2}{3}x + 2
\end{align*}
\]

The slope is: \( m = -\frac{2}{3} \)
2) Find the slope of the line given the equation $2x + y = 5$

a) 2  
b) -2  
c) 1  
d) -1

Solution: b

Write the equation in slope-intercept form $y = mx + b$

$$2x + y = 5$$
$$2x - 2x + y = -2x + 5$$
$$y = -2x + 5$$

$m = -2$

3) A line has a slope of $-\frac{1}{2}$ and a y-intercept of (0,3). Find the equation of the line?

a) $3y = -\frac{1}{2}x$  
b) $y = -2x + 3$  
c) $y = -\frac{1}{2}x + 3$  
d) $\frac{1}{2}y = -3x$

Solution: c

$m = -\frac{1}{2}$

$b = 3$

$y = mx + b$

$\Rightarrow y = -\frac{1}{2}x + 3$
4) Find the slope of a line through the points (2,3) and (-2,5)

a) -1 
b) 1 
c) 2 
d) $-\frac{1}{2}$

Solution: d

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-2 - 2} = -\frac{2}{-4} = \frac{1}{2}$$

5) Find the equation of a line with a slope of 2 and a y-intercept of (0,-2)

a) $y = -2x + 2$ 
b) $y = 2x + 2$ 
c) $y = 2x - 2$ 
d) $y = 2x - 2$

Solution: d

$m = 2$

$b = -2$

$y = mx + b$ 

$\Rightarrow y = 2x - 2$
6) Find the equation of a line of passing through the points (1,2) and (3,5).

a) \( y = 2x + 1 \)

b) \( 2y = 3x + 1 \)

c) \( y = \frac{3}{2}x + \frac{1}{2} \)

d) \( y = \frac{3}{2}x - \frac{1}{2} \)

Solution c

Find the slope first:

\[
m = \frac{5 - 2}{3 - 1} = \frac{3}{2}
\]

Using the point (1,2) and a slope of \( \frac{3}{2} \), the point-slope formula will give you the final answer.

\[
y - y_1 = m(x - x_1)
\]

\[
y - 2 = \frac{3}{2}(x - 1)
\]

\[
y - 2 = \frac{3}{2}x - \frac{3}{2}
\]

\[
y = \frac{3}{2}x + \frac{1}{2}
\]