Appendix C

Trigonometry

Radian and Degrees

Conversion Factor $1\pi \ radians = 180^0$

Converting degrees to radians

Example 1

Convert 120^0 to radians

$$120^0 \cdot \frac{\pi}{180} = \frac{120\pi}{180} = \frac{2\pi}{3}$$

Example 2

Convert 45^0 to radians

$$45^0 \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}$$

Example 3

Convert $\frac{3\pi}{4}$ to degrees

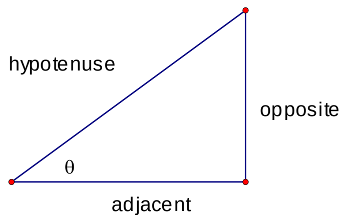
$$\frac{3\pi}{4} \cdot \frac{180}{\pi} = 3(45) = 135^0$$

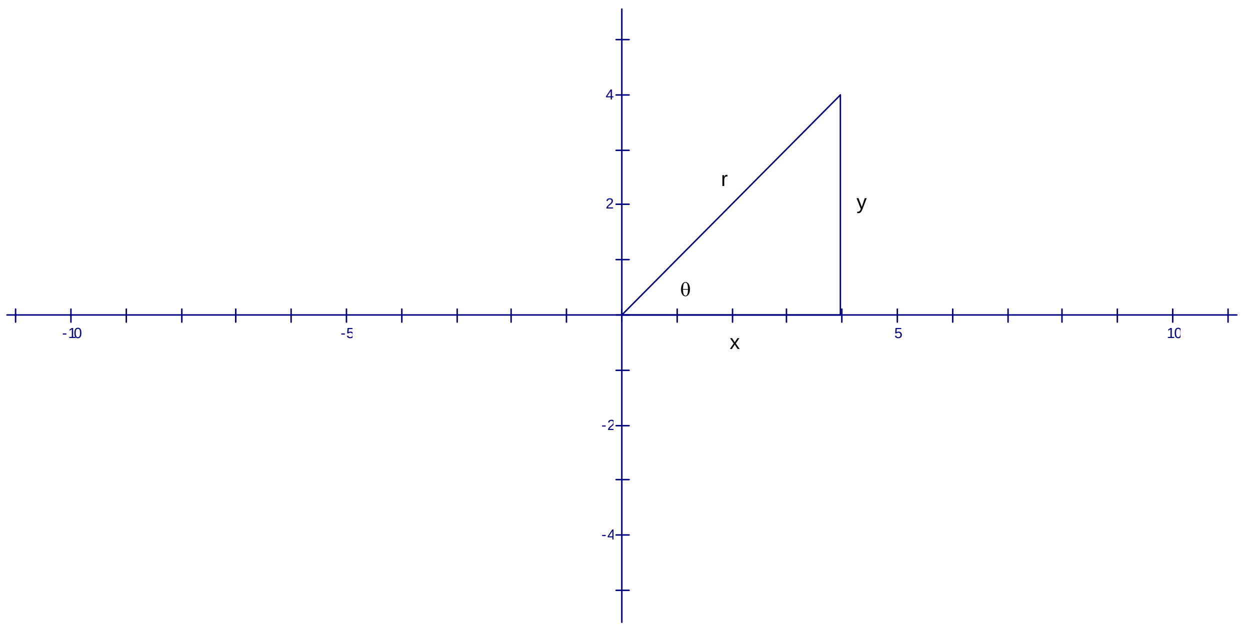
Example 4

Convert $\frac{\pi}{2}$ to degrees

$$\frac{\pi}{2} \cdot \frac{180}{\pi} = \frac{180}{2}^0 = 90^0$$

Basic Trigonometric Functions





$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{y}{r}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{x}{r}$$

$$\tan\theta = \frac{opposite}{hypotenuse} = \frac{y}{x}$$

$$\csc\theta = \frac{hypotenuse}{opposite} = \frac{r}{y}$$

$$\sec\theta = \frac{hypotenuse}{adjacent} = \frac{r}{x}$$

$$\cot\theta = \frac{adjacent}{opposite} = \frac{x}{y}$$

Note: By the Pythagorean Theorem $r^2 = x^2 + y^2$

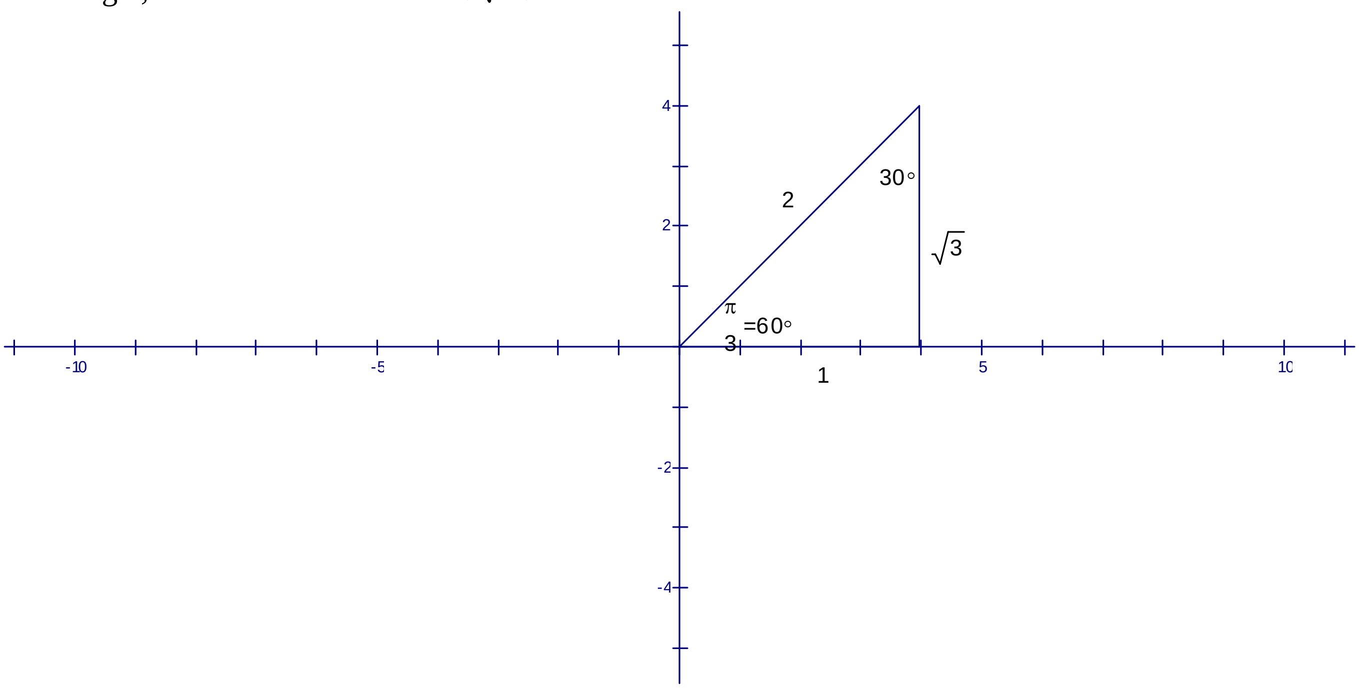
Example 5

Find exact value of the trigonometric values $\sin\theta$, $\cos\theta$, $\tan\theta$ and for the radian measure of $\frac{\pi}{3}$

First convert to degrees

$$\frac{\pi}{3}\cdot\frac{180}{\pi}=\frac{180}{3}=60^0$$

Next sketch the angle in the quadrant plane. The resulting reference triangle is a 30-60-90 triangle, so the ratio of sides is $1:\sqrt{3}:2$



The trigonometric values are:

$$\sin\theta=\frac{y}{r}=\frac{\sqrt{3}}{2}$$

$$\cos\theta=\frac{x}{r}=\frac{1}{2}$$

$$\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{1}=\sqrt{3}$$

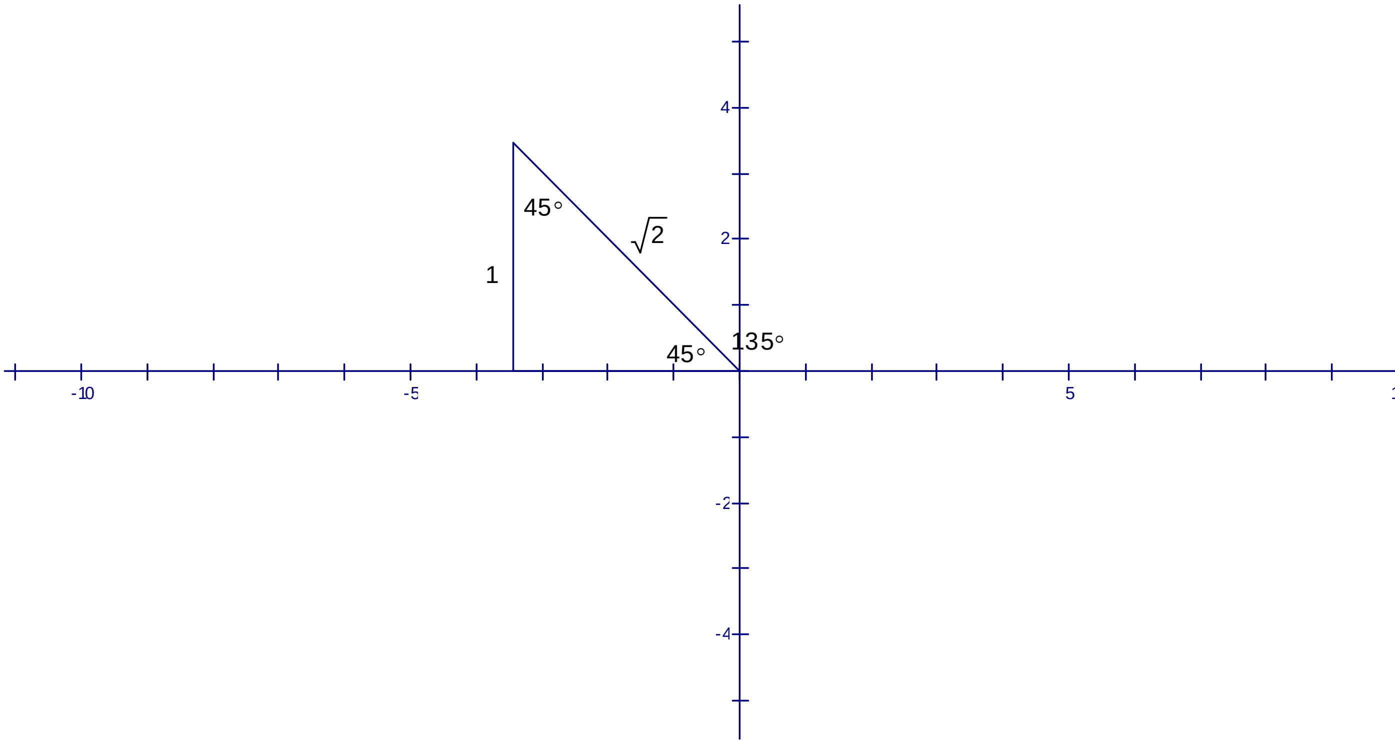
Example 6

Find exact value of the trigonometric values $\sin\theta$, $\cos\theta$, $\tan\theta$ and for the radian measure of $\frac{3\pi}{4}$

First convert to degrees

$$\frac{3\pi}{4}\cdot\frac{180}{\pi} = 3(45^0) = 135^0$$

Next sketch the angle in the quadrant plane. The resulting reference triangle is a 45-45-90 triangle, so the ratio of sides is $1:1:\sqrt{2}$



$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

Example 7

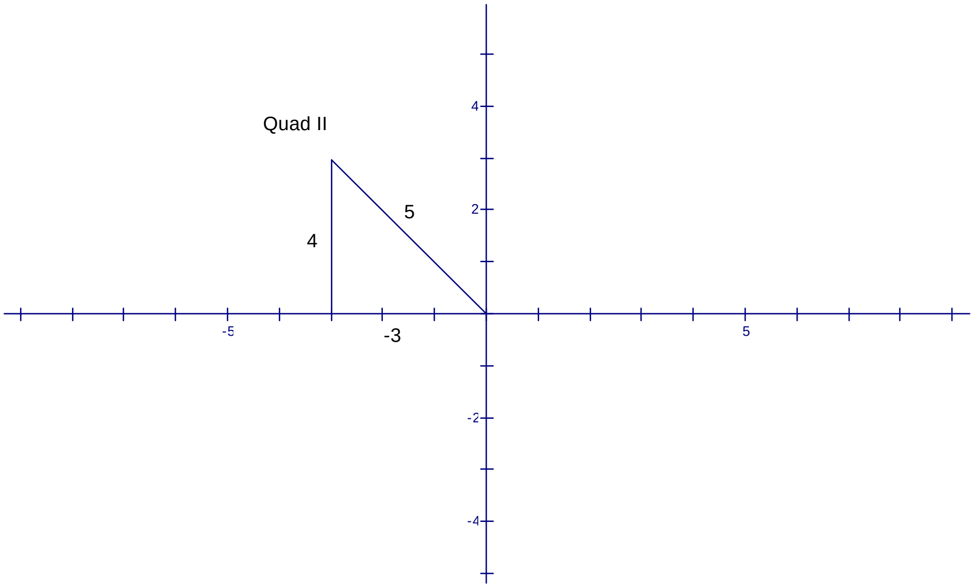
Find the remaining trigonometric functions given $\sin\theta = \frac{4}{5}, \textit{in Quadrant II}$

First find the values of x, y, and r

$$\sin\theta = \frac{4}{5} = \frac{y}{r} \Rightarrow y = 4 \text{ and } r = 5$$

Use the Pythagorean Theorem to get x

$$r^2 = x^2 + y^2$$
$$5^2 = x^2 + 4^2$$
$$25 = x^2 + 16$$
$$x^2 = 25 - 16$$
$$x^2 = 9$$
$$x = \pm 3$$



Since the triangle is in quadrant II x is negative, so x = -3

$$\cos\theta = \frac{x}{r} = -\frac{3}{5}; \tan\theta = \frac{y}{x} = -\frac{4}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{4}; \sec\theta = \frac{r}{x} = -\frac{5}{3}; \cot\theta = \frac{x}{y} = -\frac{3}{4}$$

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Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \tan^2\theta$$

Prove $\sin^2\theta + \cos^2\theta = 1$

Proof: $\sin^2\theta + \cos^2\theta = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = \frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{x^2 + y^2}{r^2} = \frac{r^2}{r^2} = 1$

Example 8

Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$

$$\sin^{-1}\left(\frac{1}{2}\right) = y \Rightarrow \sin y = \frac{1}{2}$$
$$\Rightarrow y = \frac{\pi}{3} \; and \; y = \frac{2\pi}{3}$$

Example 9

Evaluate $\arctan(-1)$

$$\arctan(-1) = \tan^{-1}(-1) = y$$
$$\tan y = -1$$

$$y = \frac{3\pi}{4} \, and \; y = \frac{7\pi}{4}$$

Trigonometric equations

Example 10

Solve $2\sin x - 1 = 0$

$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2} \Rightarrow x = 30^0$$

Example 11

Solve $2\cos^2\theta = 1$

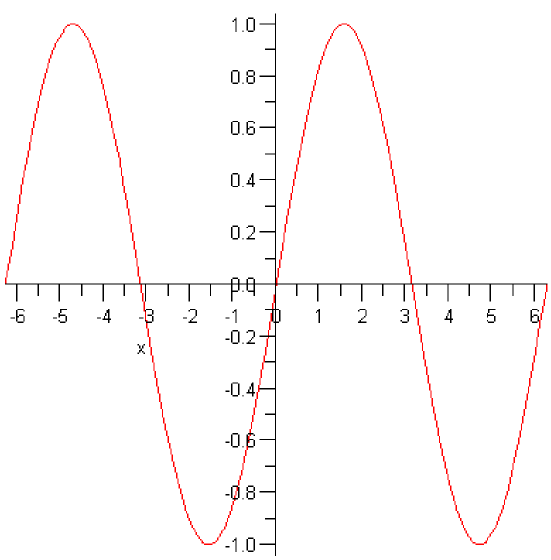
$$2\cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{2}$$
$$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{2}}$$
$$\cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = 135°, 315° \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Graphs of the Trigonometric Functions

Example 12

$y = \sin x \ on \ [-2\pi, 2\pi]$

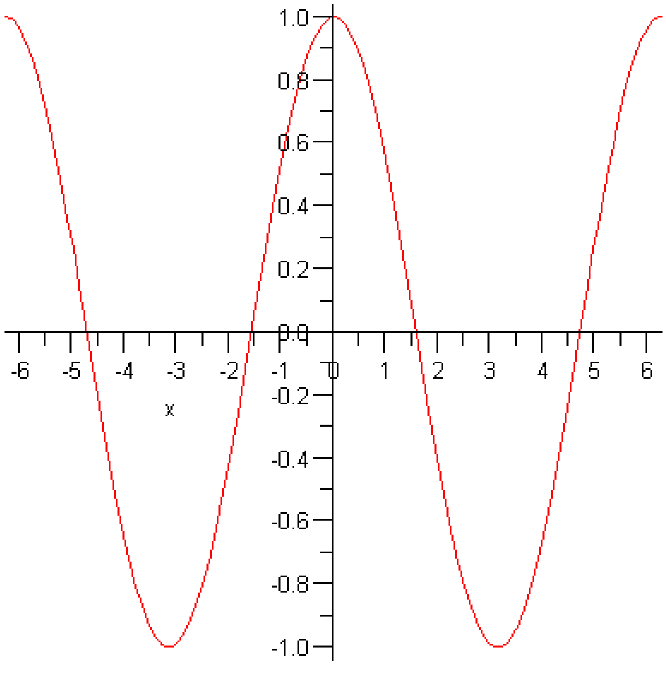
x	y
0	$y = \sin(0) = 0$
$\frac{\pi}{4}$	$y = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = .7$
$\frac{\pi}{2}$	$y = \sin\left(\frac{\pi}{2}\right) = 1$
$\frac{3\pi}{4}$	$y = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} = .7$
π	$y = \sin(\pi) = 0$
$\frac{5\pi}{4}$	$y = \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -.7$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2}\right) = -1$
$\frac{5\pi}{4}$	$y = \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -.7$
2π	$y = \sin(2\pi) = 0$



Example 13

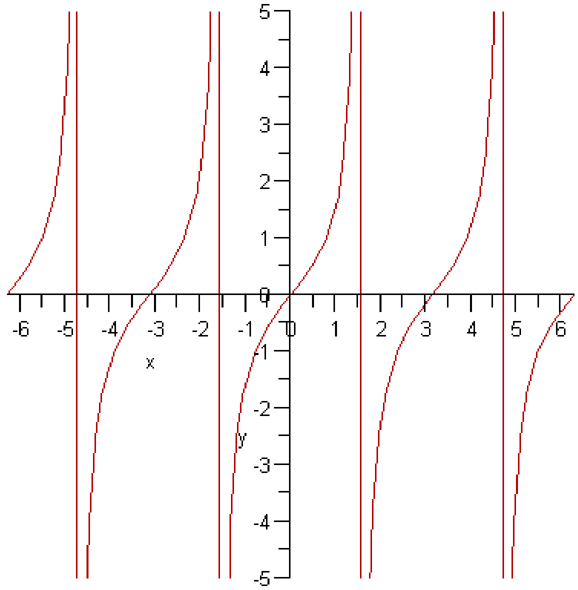
Graph $y = \cos x \; on \; [-2\pi, 2\pi]$

x	y
0	$y\cos(0) = 1$
$\frac{\pi}{4}$	$y = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = .7$
$\frac{\pi}{2}$	$y = \cos\left(\frac{\pi}{2}\right) = 0$
$\frac{3\pi}{4}$	$y = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -.7$
π	$y = \cos(\pi) = -1$
$\frac{5\pi}{4}$	$y = \cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -.7$
$\frac{3\pi}{2}$	$y = \cos\left(\frac{3\pi}{2}\right) = 0$
$\frac{5\pi}{4}$	$y = \cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2} = .7$
2π	$y = \cos(2\pi) = 1$



Example 14 Other graphs

$y = \tan x \ on \ [-2\pi, 2\pi]$



Example 15

Graph $y = \sin\left(x - \frac{\pi}{2}\right) on \ [-2\pi, 2\pi]$