00000001

Answers to Practice Test 3

Math 114 Corwin
Spring 2010

You must show your work to qualify for credit.

1. James walked behind ten different pairs of students for fifteen minutes each, recording the number of times the word "like" was used in conversation. His findings:

$$27,\ 32,\ 25,\ 27,\ 31,\ 29,\ 41,\ 39,\ 37,\ 40$$

(a) Find the mean number of times the word was used.

$\frac{27+32+\cdots+40}{10} = 32.8$

(b) Find the median number of times the word was used.

Sort: 25, 27, 27, 29, 31, 32, 37, 39, 40, 41

Ten data points $\Rightarrow$ middle position is $\frac{10+1}{2} = 5.5$, so the median is the average (mean) of the fifth and sixth data points. Thus the median is $\frac{31+32}{2} = 31.5$.

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2. Leslie's Exciting Café offers many varieties of ice cream. One evening's sales are tabulated below. (Yes, these are real flavors from around the world.)

fish	octopus	squid	ox tongue	sweet potato
eggplant	eel	crab	corn	Koshihikari
wasabi	miso	cactus	horse	goat
whale	goat	eel	octopus	corn

What is the mode of this data?

There is no mode. Octopus, eel, and corn all appear twice, and nothing appears more than twice.

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3. Jane has been obsessed with rainfall for many years, and each Halloween night she measures it. Her results, in inches:

0.1	0.0	0.2	0.15	0.13	0.7	0.52	0.33	0.1	0.45
0.3	0.09	0.25	4.2	0.3	0.41	0.17	0.52	0.82	0.0

Which measure of central tendency best represents this data?

The median. The number 4.2 is an obvious outlier.

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4. For each statistical measure listed, indicate whether it is a measure of central tendency or of spread.

   - mean — central tendency
   - range — spread
   - median — central tendency
   - mode — central tendency
   - variance — spread
   - standard deviation — spread

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5. Elle has found that the deviations from the mean for her data were $-12, -6, -2, 0, 1, 1, 3, 5$, and 7. What are the variance and standard deviation?

variance $= s^2 = \frac{\text{sum of squares of deviations}}{\#\text{data points} - 1} = \frac{(-12)^2 + (-6)^2 + \cdots + 7^2}{9-1} = \frac{269}{8} = 33.625$

standard deviation = $s = \sqrt{s^2} = \sqrt{33.625} \approx 5.8$

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6. Make a stem-and-leaf diagram for the following data. Be sure to include a key.

11	23	22	17	17	21	12	13	12
10	14	15	17	11	21	12	16	18
15	14	14	16	13	22	12	16	10

Key 1|2 = 12

Stem	Leaf
1	0 0 1 1 2 2 2 2 3 3 4 4 4 5 5 6 6 6 7 7 7 8
2	1 1 2 2 3

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7. What is the median of the data in the following frequency table?

Value	Frequency
2	2
3	2
4	0
5	2
6	3
7	1
8	3
9	1
10	4

You can recover the actual data from this table: there are two 2's, two 3's, no 4's, and so on. The median is then calculated in the usual way; it is 6.5.

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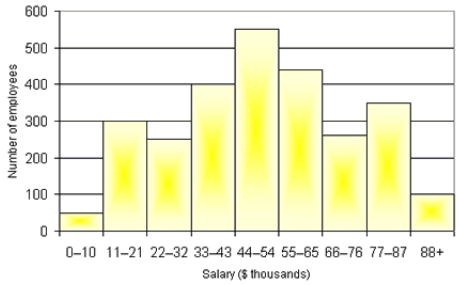
8. What is the mean of the data in the following frequency table?

Interval	Frequency
1–5	4
6–10	2
11–15	3
16–20	5
21–25	1
26–30	4

We must use midpoints here, because we cannot recover the actual data. Multiply each midpoint by the frequency of its interval, sum the results, and then divide by the number of data points: $\frac{3\cdot4+8\cdot2+\cdots28\cdot4}{19} = \frac{291}{19} \approx 15.3$

_____9. The histogram shown is



(a) bimodal

(b) skewed right

(c) skewed left

(d) uniform

(e) symmetric

(e)

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10. By answering the questions below, follow the procedure outlined in our textbook to construct a histogram of the following data.

132	132	133	133	133	133	134	134
134	135	135	135	135	135	135	136
136	136	136	137	137	137	139	140
140	140	140	141	141	141	142	142

(a) Use the rule of thumb to determine, approximately, the number and width of intervals to use.

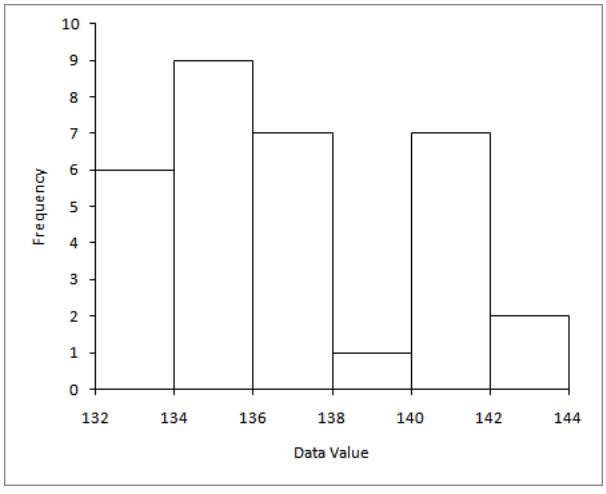
32 data points $\Rightarrow$ we should use about $\sqrt{32} \approx 6$ intervals

size of interval $\approx \frac{\text{range}}{\#\text{intervals}} = \frac{142-132}{6} \approx 2$

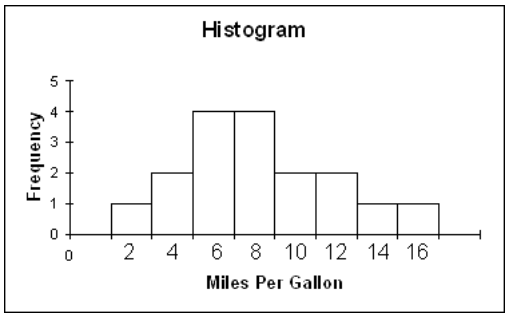
(b) Make a frequency table for the data. Begin your first interval at 132.

Interval	Frequency
132–134	6
134–136	9
136–138	7
138–140	1
140–142	7
142–144	2

(c) Sketch a histogram for the data that matches your frequency table. Be sure to label the axes appropriately.



11. What is the mean of the data represented in the histogram shown?



The histogram is marked with the midpoints of intervals: $2, 4, \ldots, 16$. Thus the mean is $\frac{2\cdot1+4\cdot2+6\cdot4+8\cdot4+10\cdot2+12\cdot2+14\cdot1+16\cdot1}{17} \approx 8.24$.

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12. Xenobia, Yancy, and Zack are running for an office. Xenobia received 312 votes and Yancy received 231 votes. How many votes does Zack need to win
(a) in a Majority Rule election?

$312 + 231 = 543$ votes have been cast so far; if Zack also got 543, he'd have 50%. Thus, he needs 544 to win.

(b) in a Plurality election?

Here he needs only one more than anyone else, so 313 will do it.

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13. A club with ten members used a Borda count to elect its president this year. Rankings were as follows.

Member	1	2	3	4	5	6	7	8	9	10
Ranking	ABC	CAB	BCA	ACB	CBA	ACB	CBA	ACB	CAB	BAC

(a) How many points did Candidate A receive?

$3+2+1+3+1+3+1+3+2+2=21$

(b) Who won?

A got 21 points, B got 17, and C got 22, so C won.

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14. In an election to be decided by True Majority Rule, the following rankings were recorded.

Ranking	ABC	ACB	BAC	BCA	CAB
# Voters	15	31	12	17	6

(a) How many voters preferred C to B?

$31+6=37$

(b) Who won?

Of the 81 voters $15+31+6=52$ preferred A to B and $15+31+12=58$ preferred A to C, so A won.

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15. Match each principle with its description.

D The Pareto Principle

B The Principle of Independence of Irrelevant Alternatives

C The Principle of Decisiveness

A The Principle of Unrestricted Domain

(a) Any set of rankings is possible.
(b) If a voter prefers A to B when C is a possible choice, then that voter prefers A to B when C is not a possible choice.
(c) A voting method must produce a winner for any set of rankings.
(d) If every voter prefers A to B, then the group prefers A to B.

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16. State the Condorcet Jury Theorem in your own words. What is its importance?

Suppose that a group of voters is using Majority Rule to decide between two alternatives, one of which is correct. Then if the probability that each individual voter is correct is greater than 50%, then the probability that the group makes the right decision increases as the number of voters increases. Also, if the probability that each individual voter is correct is less than 50%, then the probability that the group makes the right decision decreases as the number of voters increases.

The theorem is considered by many to be the theoretical basis for democracy.
